Key

Ch. 3 Review AP Practice Problems (p.264) – Chain Rule, Implicit Diff. & Inverse Derivatives

1. Find an equation of the tangent line to the graph of $y = 3^{\sin x} - 4$ when $x = 0$.

(A) $y = (\ln 3)x - 4$ $\boxed{\text{(B)}}$ $y = (\ln 3)x - 3$

(C) $y = x - 3$ (D) $y = 3x - 3$

$y(0) = 3^{\sin 0} - 4 = 3^0 - 4 = 1 - 4 = -3$

$y'(x) = \ln 3 \cdot 3^{\sin x} \cdot \cos x - 0$

$y'(0) = \ln 3 \cdot 3^{\sin 0} \cdot \cos 0 = \ln 3 (1)(1) =$

$y'(0) = \ln 3$

point: $(0, -3)$

slope: $m = \ln 3$

$\boxed{y + 3 = \ln 3 (x - 0)}$

2. If $f(x) = \sin u$, $u = v - \frac{1}{v^2}$, and $v = \ln x$, find $f'(e)$.

(A) 0 $\boxed{\text{(B)}}$ $\frac{3}{e}$ (C) $\frac{1}{e} + \frac{2}{e \ln 3}$ (D) $\frac{2}{e^2}$

$f(x) = \sin\left[\ln x - (\ln x)^{-2}\right] \rightarrow f'(x) = \cos\left[\ln x - (\ln x)^{-2}\right] \cdot \left[\frac{1}{x} + 2(\ln x)^{-3}\left(\frac{1}{x}\right)\right]$

$f'(e) = \cos\left[\ln e - (\ln e)^{-2}\right]\left[\frac{1}{e} + 2(\ln e)^{-3}\left(\frac{1}{e}\right)\right]$

$f'(e) = \cos(0)\left[\frac{1}{e} + \frac{2}{e}\right] = \boxed{\frac{3}{e}}$

3. If $e^{f(x)} = 2 + x^4$, then $f'(x) =$

(A) $\frac{4x^3}{e^x}$ (B) $4x^3 e^x$

$\boxed{\text{(C)}}$ $\frac{4x^3}{2 + x^4}$ (D) $\frac{e^x}{2 + x^4}$

$e^{f(x)} \cdot f'(x) = 0 + 4x^3$

$\boxed{f'(x) = \frac{4x^3}{e^{f(x)}} \rightarrow \frac{4x^3}{2 + x^4}}$

* since $e^{f(x)} = 2 + x^4$

4. Find y' if $y = \sin^{4/3}(4x - x^2)$.

(A) $y' = \frac{16 - 8x}{3} \sin^{1/3}(4x - x^2)$

$\boxed{\text{(B)}}$ $y' = \frac{16 - 8x}{3} \sin^{1/3}(4x - x^2) \cdot \cos(4x - x^2)$

(C) $y' = \frac{4}{3} \sin^{1/3}(4x - x^2) \cdot \cos(4x - x^2)$

(D) $y' = -\frac{8x}{3} \sin^{1/3}(4x - x^2) \cdot \cos(4 - 2x)$

$y = \left[\sin(4x - x^2)\right]^{4/3}$

*chain Rule

out: $[\ ]^{4/3}$

in: $\sin(\ )$

inner: $4x - x^2$

$y' = \frac{4}{3}\left[\sin(4x - x^2)\right]^{1/3} \cdot \cos(4x - x^2) \cdot (4 - 2x)$

$\boxed{y' = \frac{4(4 - 2x)}{3} \cdot \left[\sin(4x - x^2)\right]^{1/3} \cdot \cos(4x - x^2)}$

5. The slope of the normal line to the graph of $x^2 + (xy - 2)^2 = 20$ at the point $(2, -1)$ is

perpendicular to tangent line

(A) $\frac{3}{4}$ (B) -4 (C) $\frac{4}{3}$ $\boxed{\text{(D)}}$ $-\frac{4}{3}$

$2x + 2[xy - 2]\left[1y + x\left(\frac{dy}{dx}\right)\right] = 0$

*product Rule

$y + x\left(\frac{dy}{dx}\right) = \frac{-2x}{2[xy - 2]}$

$-1 + 2\left(\frac{dy}{dx}\right) = \frac{-2(2)}{2[-2 - 2]}$

$-1 + 2\left(\frac{dy}{dx}\right) = \frac{1}{2}$

$2\left(\frac{dy}{dx}\right) = \frac{3}{2}$

$\frac{dy}{dx} = \frac{3}{4}$

$m_1 = 3/4$

$\boxed{m_2 = -4/3}$

6. If $y = \sin(3x + 2y)$, find the rate of change of y with respect to x at the origin.

(A) 0 (B) 3 (C) -1 $\boxed{\text{(D)}}$ -3

$*$ find $\frac{dy}{dx}$ at $(0,0)$

$\frac{dy}{dx} = \cos(3x+2y)\cdot\left(3+2\left(\frac{dy}{dx}\right)\right)$

$\frac{dy}{dx} = \cos(0+0)\cdot\left[3+2\left(\frac{dy}{dx}\right)\right]$

$\frac{dy}{dx} = 1\left(3+2\left(\frac{dy}{dx}\right)\right)$

$1\left(\frac{dy}{dx}\right) = 3+2\left(\frac{dy}{dx}\right)$

$-3 = 1\left(\frac{dy}{dx}\right)$

$\boxed{\left.\frac{dy}{dx}\right|_{(0,0)} = -3}$

7. The points (1, 1), (2, 3), and (3, 13) are on the graph of the function $f(x) = x^3 - 2x^2 + x + 1,\ x \geq 1$. If the function g is the inverse function of f, then $g'(3) =$

$\boxed{\text{(A)}}$ $\frac{1}{5}$ (B) $\frac{1}{16}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{5}$

$f(\ ) = 3$	$g(3) =$ __
$f'(\ ) =$ __	$g'(3) =$ __

$f(2) = 3$	$g(3) = 2$
$f'(2) = 5$	$g'(3) = \boxed{1/5}$

$f'(x) = 3x^2 - 4x + 1$

$f'(2) = 3(2)^2 - 4(2) + 1$

$f'(2) = 12 - 8 + 1 = 5$

8. Find $\frac{d}{dx}\ln(\ln x)$.

(A) 1 (B) $\frac{x}{\ln x}$ $\boxed{\text{(C)}}$ $\frac{1}{x\ln x}$ (D) $\frac{1}{(\ln x)^2}$

$y' = \frac{\frac{1}{x}}{\ln x} \rightarrow y' = \frac{1}{x}\cdot\frac{1}{\ln x} = \boxed{\frac{1}{x\ln x}}$

9. The derivative of $y = \tan^{-1}(xe^x)$ equals

(A) $\frac{xe^x + e^x}{\sqrt{1 + x^2e^{2x}}}$ (B) $\frac{e^x + 1}{1 + xe^x}$

(C) $\frac{2x^2e^{2x} + 2xe^x}{1 + x^2e^{2x}}$ $\boxed{\text{(D)}}$ $\frac{xe^x + e^x}{1 + x^2e^{2x}}$

$*\ \frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$

product Rule

$y = \tan^{-1}(\overset{f}{x}\cdot\overset{g}{e^x})$

$y' = \frac{1\cdot e^x + x\cdot e^x}{1+(xe^x)^2}$

$\boxed{y' = \frac{e^x + xe^x}{1+x^2e^{2x}}}$