27. (a) $s(t) = -16t^2 - 30t + 600$
$v(t) = -32t - 30$
(b) -94 ft/sec
(c) $v'(1) = -62$ ft/sec; $v'(3) = -126$ ft/sec
(d) About 5.258 sec (e) About -198.256 ft/sec

29. $4(5x^3 - 15x^2 - 11x - 8)$ **31.** $\sqrt{x}\cos x + \sin x/(2\sqrt{x})$

33. $\dfrac{-(x^2+1)}{(x^2-1)^2}$ **35.** $\dfrac{4x^3\cos x + x^4 \sin x}{\cos^2 x}$

37. $3x^2 \sec x \tan x + 6x \sec x$ **39.** $-x \sin x$

41. $y = 4x + 10$ **43.** $y = -8x + 1$ **45.** $-48t$

47. $\frac{225}{4}\sqrt{x}$ **49.** $6 \sec^2 \theta \tan \theta$

51. $v(3) = 11$ m/sec; $a(3) = -6$ m/sec^2 **53.** $28(7x+3)^3$

55. $-\dfrac{2x}{(x^2+4)^2}$ **57.** $-45\sin(9x+1)$

59. $\frac{1}{2}(1 - \cos 2x) = \sin^2 x$ **61.** $(36x+1)(6x+1)^4$

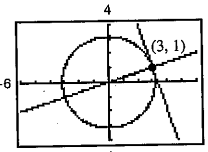
63. $\dfrac{3}{(x^2+1)^{3/2}}$ **65.** $\dfrac{-3x^2}{2\sqrt{1-x^3}}$; -2 **67.** $-\dfrac{8x}{(x^2+1)^2}$; 2

69. $-\csc 2x \cot 2x$; 0 **71.** $384(8x+5)$ **73.** $2\csc^2 x \cot x$

75. (a) $-18.667°$/h (b) $-7.284°$/h
(c) $-3.240°$/h (d) $-0.747°$/h

77. $-\dfrac{x}{y}$ **79.** $\dfrac{y(y^2 - 3x^2)}{x(x^2 - 3y^2)}$ **81.** $\dfrac{y \sin x + \sin y}{\cos x - x \cos y}$

83. Tangent line: $3x + y - 10 = 0$
Normal line: $x - 3y = 0$



85. (a) $2\sqrt{2}$ units/sec (b) 4 units/sec (c) 8 units/sec

87. 450π km/h

P.S. Problem Solving *(page 159)*

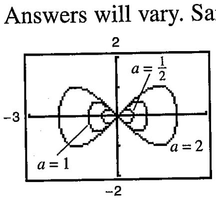
1. (a) $r = \frac{1}{2}$; $x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$
(b) Center: $(0, \frac{5}{4})$; $x^2 + (y - \frac{5}{4})^2 = 1$

3. $p(x) = 2x^3 + 4x^2 - 5$

5. (a) $y = 4x - 4$ (b) $y = -\frac{1}{4}x + \frac{9}{2}$; $\left(-\frac{9}{4}, \frac{81}{16}\right)$
(c) Tangent line: $y = 0$ (d) Proof
Normal line: $x = 0$

7. (a) Graph $\begin{cases} y_1 = \dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \\ y_2 = -\dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \end{cases}$ as separate equations.

(b) Answers will vary. Sample answer:



The intercepts will always be $(0, 0)$, $(a, 0)$, and $(-a, 0)$, and the maximum and minimum y-values appear to be $\pm\frac{1}{2}a$.

(c) $\left(\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right)$, $\left(\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$

9. (a) When the man is 90 ft from the light, the tip of his shadow is $112\frac{1}{2}$ ft from the light. The tip of the child's shadow is $111\frac{1}{9}$ ft from the light, so the man's shadow extends $1\frac{7}{18}$ ft beyond the child's shadow.
(b) When the man is 60 ft from the light, the tip of his shadow is 75 ft from the light. The tip of the child's shadow is $77\frac{7}{9}$ ft from the light, so the child's shadow extends $2\frac{7}{9}$ ft beyond the man's shadow.
(c) $d = 80$ ft
(d) Let x be the distance of the man from the light, and let s be the distance from the light to the tip of the shadow.
If $0 < x < 80$, then $ds/dt = -50/9$.
If $x > 80$, then $ds/dt = -25/4$.
There is a discontinuity at $x = 80$.

11. (a) $v(t) = -\frac{27}{5}t + 27$ ft/sec (b) 5 sec; 73.5 ft
$a(t) = -\frac{27}{5}$ ft/sec^2
(c) The acceleration due to gravity on Earth is greater in magnitude than that on the moon.

13. Proof. The graph of L is a line passing through the origin $(0, 0)$.

15. (a) j would be the rate of change of acceleration.
(b) $j = 0$. Acceleration is constant, so there is no change in acceleration.
(c) a: position function, d: velocity function,
b: acceleration function, c: jerk function

Chapter 3

Section 3.1 *(page 167)*

1. $f'(0) = 0$ **3.** $f'(2) = 0$ **5.** $f'(-2)$ is undefined.

7. 2, absolute maximum (and relative maximum)

9. 1, absolute maximum (and relative maximum);
2, absolute minimum (and relative minimum);
3, absolute maximum (and relative maximum)

11. $x = 0, x = 2$ **13.** $t = 8/3$ **15.** $x = \pi/3, \pi, 5\pi/3$

17. Minimum: $(2, 1)$
Maximum: $(-1, 4)$

19. Minimum: $(2, -8)$
Maximum: $(6, 24)$

21. Minimum: $\left(-1, -\frac{5}{2}\right)$
Maximum: $(2, 2)$

23. Minimum: $(0, 0)$
Maximum: $(-1, 5)$

25. Minimum: $(0, 0)$
Maxima: $\left(-1, \frac{1}{4}\right)$ and $\left(1, \frac{1}{4}\right)$

27. Minimum: $(1, -1)$
Maximum: $\left(0, -\frac{1}{2}\right)$

29. Minimum: $(-1, -1)$
Maximum: $(3, 3)$

31. Minimum value is -2 for $-2 \le x < -1$.
Maximum: $(2, 2)$

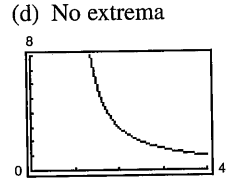
33. Minimum: $(3\pi/2, -1)$
Maximum: $(5\pi/6, 1/2)$

35. Minimum: $(\pi, -3)$
Maxima: $(0, 3)$ and $(2\pi, 3)$

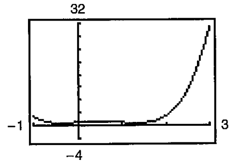
37. (a) Minimum: $(0, -3)$;
Maximum: $(2, 1)$
(b) Minimum: $(0, -3)$
(c) Maximum: $(2, 1)$
(d) No extrema

39. (a) Minimum: $(1, -1)$;
Maximum: $(-1, 3)$
(b) Maximum: $(3, 3)$
(c) Minimum: $(1, -1)$
(d) Minimum: $(1, -1)$

41.

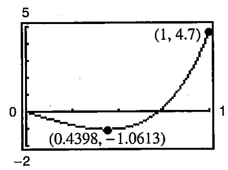


Minimum: $(4, 1)$

43. 

Minima: $\left(\frac{-\sqrt{3}+1}{2}, \frac{3}{4}\right)$ and $\left(\frac{\sqrt{3}+1}{2}, \frac{3}{4}\right)$

Maximum: $(3, 31)$

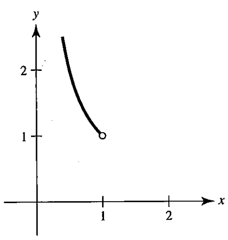
45. (a) 

(b) Minimum: $(0.4398, -1.0613)$

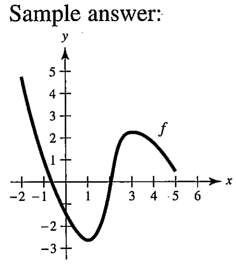
47. Maximum: $\left|f''\left(\sqrt[3]{-10+\sqrt{108}}\right)\right| = f''(\sqrt{3}-1) \approx 1.47$

49. Maximum: $|f^{(4)}(0)| = \frac{56}{81}$

51. Answers will vary. Sample answer: Let $f(x) = 1/x$. f is continuous on $(0, 1)$ but does not have a maximum or minimum.



53. Answers will vary. Sample answer:



55. (a) Yes (b) No **57.** (a) No (b) Yes

59. Maximum: $P(12) = 72$; No. P is decreasing for $I > 12$.

61. $\theta = \operatorname{arcsec} \sqrt{3} \approx 0.9553$ rad

63. True **65.** True **67.** Proof

69. Putnam Problem B3, 2004

Section 3.2 *(page 174)*

1. $f(-1) = f(1) = 1$; f is not continuous on $[-1, 1]$.

3. $f(0) = f(2) = 0$; f is not differentiable on $(0, 2)$.

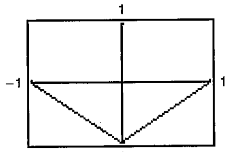
5. $(2, 0), (-1, 0)$; $f'(\frac{1}{2}) = 0$ **7.** $(0, 0), (-4, 0)$; $f'(-\frac{8}{3}) = 0$

9. $f'(\frac{3}{2}) = 0$ **11.** $f'\left(\frac{6-\sqrt{3}}{3}\right) = 0$; $f'\left(\frac{6+\sqrt{3}}{3}\right) = 0$

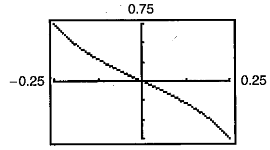
13. Not differentiable at $x = 0$ **15.** $f'(-2+\sqrt{5}) = 0$

17. $f'\left(\frac{\pi}{2}\right) = 0$; $f'\left(\frac{3\pi}{2}\right) = 0$ **19.** $f'\left(\frac{\pi}{6}\right) = 0$

21. Not continuous on $[0, \pi]$

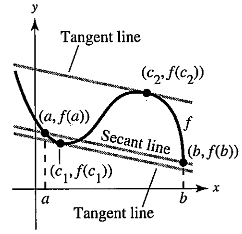
23. 

Rolle's Theorem does not apply.

25. 

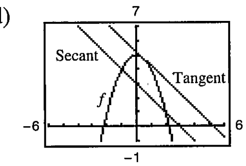
Rolle's Theorem does not apply.

27. (a) $f(1) = f(2) = 38$
(b) Velocity $= 0$ for some t in $(1, 2)$; $t = \frac{3}{2}$ sec

29. 

31. The function is not continuous on $[0, 6]$.

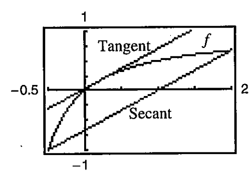
33. The function is not continuous on $[0, 6]$.

35. (a) Secant line: $x + y - 3 = 0$ (b) $c = \frac{1}{2}$
(c) Tangent line: $4x + 4y - 21 = 0$
(d) 

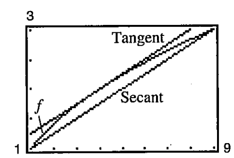
37. $f'(-1/2) = -1$ **39.** $f'(1/\sqrt{3}) = 3, f'(-1/\sqrt{3}) = 3$

41. $f'(\frac{8}{27}) = 1$ **43.** f is not differentiable at $x = -\frac{1}{2}$.

45. $f'(\pi/2) = 0$

47. (a)–(c) 

(b) $y = \frac{2}{3}(x - 1)$
(c) $y = \frac{1}{3}(2x + 5 - 2\sqrt{6})$

49. (a)–(c) 

(b) $y = \frac{1}{4}x + \frac{3}{4}$
(c) $y = \frac{1}{4}x + 1$

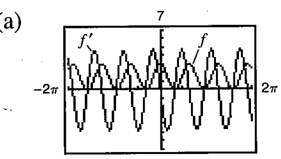
51. (a) -14.7 m/sec (b) 1.5 sec

53. No. Let $f(x) = x^2$ on $[-1, 2]$.

55. No. $f(x)$ is not continuous on $[0, 1]$. So it does not satisfy the hypothesis of Rolle's Theorem.

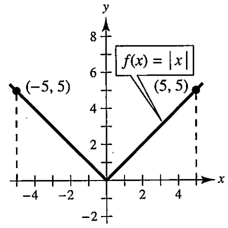
57. By the Mean Value Theorem, there is a time when the speed of the plane must equal the average speed of 454.5 miles/hour. The speed was 400 miles/hour when the plane was accelerating to 454.5 miles/hour and decelerating from 454.5 miles/hour.

59. Proof

61. (a) 

(b) Yes; yes
(c) Because $f(-1) = f(1) = 0$, Rolle's Theorem applies on $[-1, 1]$. Because $f(1) = 0$ and $f(2) = 3$, Rolle's Theorem does not apply on $[1, 2]$.
(d) $\lim_{x\to 3^-} f'(x) = 0$; $\lim_{x\to 3^+} f'(x) = 0$

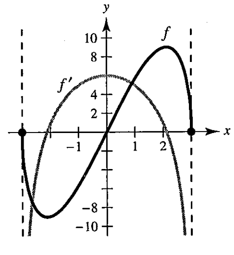
63.



65–67. Proofs **69.** $f(x) = 5$ **71.** $f(x) = x^2 - 1$
73. False. f is not continuous on $[-1, 1]$. **75.** True
77–85. Proofs

Section 3.3 *(page 183)*

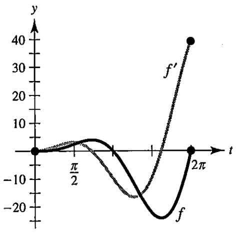
1. (a) $(0, 6)$ (b) $(6, 8)$
3. Increasing on $(3, \infty)$; Decreasing on $(-\infty, 3)$
5. Increasing on $(-\infty, -2)$ and $(2, \infty)$; Decreasing on $(-2, 2)$
7. Increasing on $(-\infty, -1)$; Decreasing on $(-1, \infty)$
9. Increasing on $(1, \infty)$; Decreasing on $(-\infty, 1)$
11. Increasing on $(-2\sqrt{2}, 2\sqrt{2})$;
Decreasing on $(-4, -2\sqrt{2})$ and $(2\sqrt{2}, 4)$
13. Increasing on $(0, \pi/2)$ and $(3\pi/2, 2\pi)$;
Decreasing on $(\pi/2, 3\pi/2)$
15. Increasing on $(0, 7\pi/6)$ and $(11\pi/6, 2\pi)$;
Decreasing on $(7\pi/6, 11\pi/6)$
17. (a) Critical number: $x = 2$
(b) Increasing on $(2, \infty)$; Decreasing on $(-\infty, 2)$
(c) Relative minimum: $(2, -4)$
19. (a) Critical number: $x = 1$
(b) Increasing on $(-\infty, 1)$; Decreasing on $(1, \infty)$
(c) Relative maximum: $(1, 5)$
21. (a) Critical numbers: $x = -2, 1$
(b) Increasing on $(-\infty, -2)$ and $(1, \infty)$;
Decreasing on $(-2, 1)$
(c) Relative maximum: $(-2, 20)$;
Relative minimum: $(1, -7)$
23. (a) Critical numbers: $x = -\frac{5}{3}, 1$
(b) Increasing on $\left(-\infty, -\frac{5}{3}\right), (1, \infty)$;
Decreasing on $\left(-\frac{5}{3}, 1\right)$
(c) Relative maximum: $\left(-\frac{5}{3}, \frac{256}{27}\right)$;
Relative minimum: $(1, 0)$
25. (a) Critical numbers: $x = \pm 1$
(b) Increasing on $(-\infty, -1)$ and $(1, \infty)$;
Decreasing on $(-1, 1)$
(c) Relative maximum: $\left(-1, \frac{4}{5}\right)$; Relative minimum: $\left(1, -\frac{4}{5}\right)$
27. (a) Critical number: $x = 0$
(b) Increasing on $(-\infty, \infty)$
(c) No relative extrema
29. (a) Critical number: $x = -2$
(b) Increasing on $(-2, \infty)$; Decreasing on $(-\infty, -2)$
(c) Relative minimum: $(-2, 0)$
31. (a) Critical number: $x = 5$
(b) Increasing on $(-\infty, 5)$; Decreasing on $(5, \infty)$
(c) Relative maximum: $(5, 5)$
33. (a) Critical numbers: $x = \pm\sqrt{2}/2$; Discontinuity: $x = 0$
(b) Increasing on $(-\infty, -\sqrt{2}/2)$ and $(\sqrt{2}/2, \infty)$;
Decreasing on $(-\sqrt{2}/2, 0)$ and $(0, \sqrt{2}/2)$
(c) Relative maximum: $(-\sqrt{2}/2, -2\sqrt{2})$;
Relative minimum: $(\sqrt{2}/2, 2\sqrt{2})$
35. (a) Critical number: $x = 0$; Discontinuities: $x = \pm 3$
(b) Increasing on $(-\infty, -3)$ and $(-3, 0)$;
Decreasing on $(0, 3)$ and $(3, \infty)$
(c) Relative maximum: $(0, 0)$
37. (a) Critical number: $x = 0$
(b) Increasing on $(-\infty, 0)$; Decreasing on $(0, \infty)$
(c) Relative maximum: $(0, 4)$
39. (a) Critical number: $x = 1$
(b) Increasing on $(-\infty, 1)$; Decreasing on $(1, \infty)$
(c) Relative maximum: $(1, 4)$
41. (a) Critical numbers: $x = \pi/6, 5\pi/6$;
Increasing on $(0, \pi/6), (5\pi/6, 2\pi)$;
Decreasing on $(\pi/6, 5\pi/6)$
(b) Relative maximum: $(\pi/6, (\pi + 6\sqrt{3})/12)$;
Relative minimum: $(5\pi/6, (5\pi - 6\sqrt{3})/12)$
43. (a) Critical numbers: $x = \pi/4, 5\pi/4$;
Increasing on $(0, \pi/4), (5\pi/4, 2\pi)$;
Decreasing on $(\pi/4, 5\pi/4)$
(b) Relative maximum: $(\pi/4, \sqrt{2})$;
Relative minimum: $(5\pi/4, -\sqrt{2})$
45. (a) Critical numbers:
$x = \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$;
Increasing on $(\pi/4, \pi/2), (3\pi/4, \pi), (5\pi/4, 3\pi/2), (7\pi/4, 2\pi)$;
Decreasing on $(0, \pi/4), (\pi/2, 3\pi/4), (\pi, 5\pi/4), (3\pi/2, 7\pi/4)$;
(b) Relative maxima: $(\pi/2, 1), (\pi, 1), (3\pi/2, 1)$;
Relative minima: $(\pi/4, 0), (3\pi/4, 0), (5\pi/4, 0), (7\pi/4, 0)$
47. (a) Critical numbers: $\pi/2, 7\pi/6, 3\pi/2, 11\pi/6$;
Increasing on $(0, \pi/2)$, $(7\pi/6, 3\pi/2)$, $(11\pi/6, 2\pi)$;
Decreasing on $(\pi/2, 7\pi/6)$, $(3\pi/2, 11\pi/6)$
(b) Relative maxima: $(\pi/2, 2)$, $(3\pi/2, 0)$;
Relative minima: $(7\pi/6, -1/4)$, $(11\pi/6, -1/4)$
49. (a) $f'(x) = 2(9 - 2x^2)/\sqrt{9 - x^2}$
(b)



(c) Critical numbers: $x = \pm 3\sqrt{2}/2$
(d) $f' > 0$ on $(-3\sqrt{2}/2, 3\sqrt{2}/2)$;
$f' < 0$ on $(-3, -3\sqrt{2}/2)$, $(3\sqrt{2}/2, 3)$
f is increasing when f' is positive and decreasing when f' is negative.

51. (a) $f'(t) = t(t\cos t + 2\sin t)$

(b)

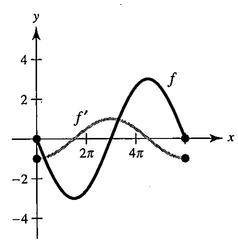


(c) Critical numbers: $t = 2.2889, 5.0870$

(d) $f' > 0$ on $(0, 2.2889), (5.0870, 2\pi)$;
$f' < 0$ on $(2.2889, 5.0870)$
f is increasing when f' is positive and decreasing when f' is negative.

53. (a) $f'(x) = -\cos(x/3)$

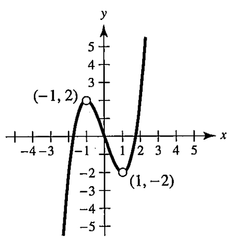
(b)



(c) Critical numbers: $x = 3\pi/2, 9\pi/2$

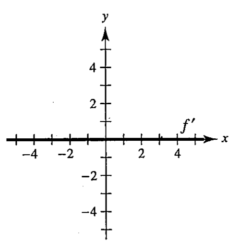
(d) $f' > 0$ on $\left(\frac{3\pi}{2}, \frac{9\pi}{2}\right)$;
$f' < 0$ on $\left(0, \frac{3\pi}{2}\right), \left(\frac{9\pi}{2}, 6\pi\right)$
f is increasing when f' is positive and decreasing when f' is negative.

55. $f(x)$ is symmetric with respect to the origin.
Zeros: $(0, 0), (\pm\sqrt{3}, 0)$

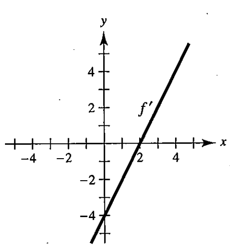


$g(x)$ is continuous on $(-\infty, \infty)$, and $f(x)$ has holes at $x = 1$ and $x = -1$.

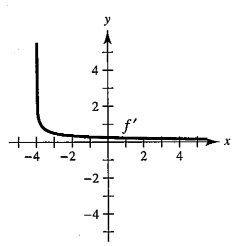
57.



59.

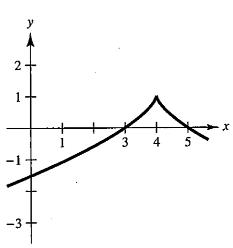


61.



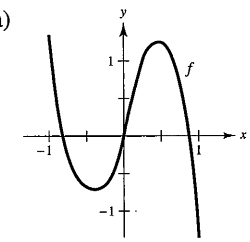
63. $g'(0) < 0$ **65.** $g'(-6) < 0$ **67.** $g'(0) > 0$

69. Answers will vary. Sample answer:



71. $(5, f(5))$ is a relative minimum.

73. (a)



(b) Critical numbers: $x \approx -0.40$ and $x \approx 0.48$
(c) Relative maximum: $(0.48, 1.25)$;
Relative minimum: $(-0.40, 0.75)$

75. (a) $s'(t) = 9.8(\sin\theta)t$; speed $= |9.8(\sin\theta)t|$
(b)

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$s'(t)$	0	$4.9\sqrt{2}t$	$4.9\sqrt{3}t$	$9.8t$	$4.9\sqrt{3}t$	$4.9\sqrt{2}t$	0

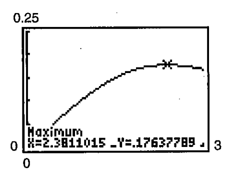
The speed is maximum at $\theta = \pi/2$.

77. (a)

t	0	0.5	1	1.5	2	2.5	3
$C(t)$	0	0.055	0.107	0.148	0.171	0.176	0.167

$t = 2.5$ h

(b)



$t \approx 2.38$ h (c) $t \approx 2.38$ h

79. $r = 2R/3$

81. (a) $v(t) = 6 - 2t$ (b) $[0, 3)$ (c) $(3, \infty)$ (d) $t = 3$

83. (a) $v(t) = 3t^2 - 10t + 4$
(b) $\left[0, (5 - \sqrt{13})/3\right)$ and $\left((5 + \sqrt{13})/3, \infty\right)$
(c) $\left(\frac{5 - \sqrt{13}}{3}, \frac{5 + \sqrt{13}}{3}\right)$ (d) $t = \frac{5 \pm \sqrt{13}}{3}$

85. Answers will vary.

87. (a) Minimum degree: 3
(b) $a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0$
$a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 2$
$3a_3(0)^2 + 2a_2(0) + a_1 = 0$
$3a_3(2)^2 + 2a_2(2) + a_1 = 0$
(c) $f(x) = -\frac{1}{2}x^3 + \frac{3}{2}x^2$

89. (a) Minimum degree: 4
(b) $a_4(0)^4 + a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0$
$a_4(2)^4 + a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 4$
$a_4(4)^4 + a_3(4)^3 + a_2(4)^2 + a_1(4) + a_0 = 0$
$4a_4(0)^3 + 3a_3(0)^2 + 2a_2(0) + a_1 = 0$
$4a_4(2)^3 + 3a_3(2)^2 + 2a_2(2) + a_1 = 0$
$4a_4(4)^3 + 3a_3(4)^2 + 2a_2(4) + a_1 = 0$
(c) $f(x) = \frac{1}{4}x^4 - 2x^3 + 4x^2$

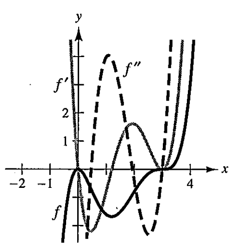
91. True **93.** False. Let $f(x) = x^3$.

95. False. Let $f(x) = x^3$. There is a critical number at $x = 0$, but not a relative extremum.

97–99. Proofs **101.** Putnam Problem A3, 2003

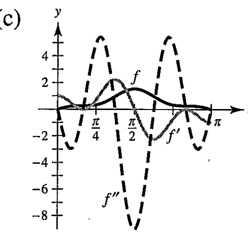
Section 3.4 *(page 192)*

1. $f' > 0, f'' < 0$ **3.** Concave upward: $(-\infty, \infty)$
5. Concave upward: $(-\infty, 2)$; Concave downward: $(2, \infty)$
7. Concave upward: $(-\infty, -2), (2, \infty)$;
Concave downward: $(-2, 2)$
9. Concave upward: $(-\infty, -1), (1, \infty)$;
Concave downward: $(-1, 1)$
11. Concave upward: $(-2, 2)$;
Concave downward: $(-\infty, -2), (2, \infty)$
13. Concave upward: $(-\pi/2, 0)$; Concave downward: $(0, \pi/2)$
15. Point of inflection: $(2, 8)$; Concave downward: $(-\infty, 2)$;
Concave upward: $(2, \infty)$
17. Points of inflection: $(-2, -8)$, $(0, 0)$;
Concave upward: $(-\infty, -2)$, $(0, \infty)$;
Concave downward: $(-2, 0)$
19. Points of inflection: $(2, -16)$, $(4, 0)$;
Concave upward: $(-\infty, 2), (4, \infty)$;
Concave downward: $(2, 4)$
21. Concave upward: $(-3, \infty)$
23. Points of inflection: $\left(-\sqrt{3}/3, 3\right), \left(\sqrt{3}/3, 3\right)$;
Concave upward: $\left(-\infty, -\sqrt{3}/3\right), \left(\sqrt{3}/3, \infty\right)$;
Concave downward: $\left(-\sqrt{3}/3, \sqrt{3}/3\right)$
25. Point of inflection: $(2\pi, 0)$;
Concave upward: $(2\pi, 4\pi)$; Concave downward: $(0, 2\pi)$
27. Concave upward: $(0, \pi)$, $(2\pi, 3\pi)$;
Concave downward: $(\pi, 2\pi)$, $(3\pi, 4\pi)$
29. Points of inflection: $(\pi, 0), (1.823, 1.452), (4.46, -1.452)$
Concave upward: $(1.823, \pi), (4.46, 2\pi)$
Concave downward: $(0, 1.823), (\pi, 4.46)$
31. Relative maximum: $(3, 9)$
33. Relative maximum: $(0, 3)$; Relative minimum: $(2, -1)$
35. Relative minimum: $(3, -25)$
37. Relative minimum: $(0, -3)$
39. Relative maximum: $(-2, -4)$; Relative minimum: $(2, 4)$
41. No relative extrema, because f is nonincreasing.
43. (a) $f'(x) = 0.2x(x-3)^2(5x-6)$;
$f''(x) = 0.4(x-3)(10x^2 - 24x + 9)$
(b) Relative maximum: $(0, 0)$;
Relative minimum: $(1.2, -1.6796)$;
Points of inflection: $(0.4652, -0.7048)$,
$(1.9348, -0.9048)$, $(3, 0)$
(c)



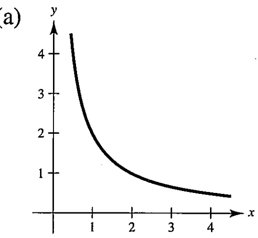
f is increasing when f' is positive and decreasing when f' is negative. f is concave upward when f'' is positive and concave downward when f'' is negative.

45. (a) $f'(x) = \cos x - \cos 3x + \cos 5x$;
$f''(x) = -\sin x + 3 \sin 3x - 5 \sin 5x$
(b) Relative maximum: $(\pi/2, 1.53333)$;
Points of inflection: $(\pi/6, 0.2667), (1.1731, 0.9637)$,
$(1.9685, 0.9637), (5\pi/6, 0.2667)$
(c)

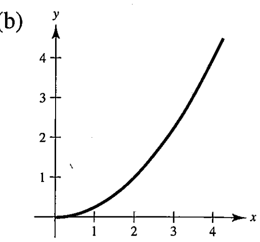


f is increasing when f' is positive and decreasing when f' is negative. f is concave upward when f'' is positive and concave downward when f'' is negative.

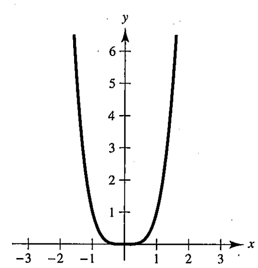
47. (a)



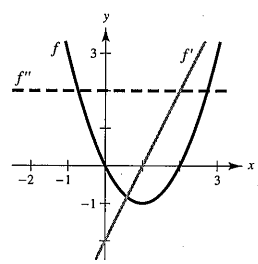
(b)



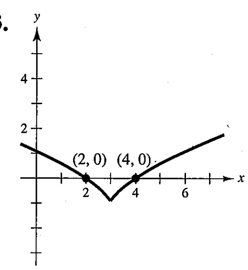
49. Answers will vary. Sample answer: $f(x) = x^4$; $f''(0) = 0$, but $(0, 0)$ is not a point of inflection.



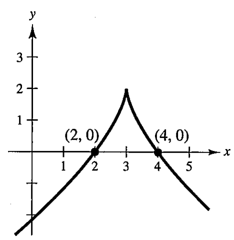
51.



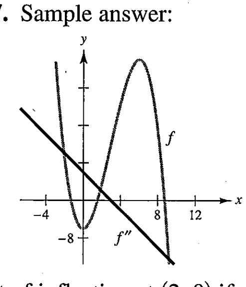
53.



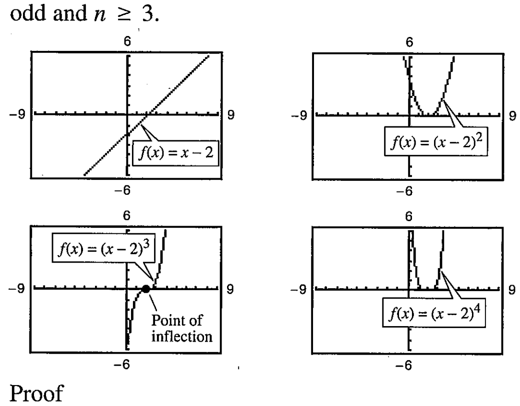
55.



57. Sample answer:



59. (a) $f(x) = (x-2)^n$ has a point of inflection at $(2, 0)$ if n is odd and $n \geq 3$.

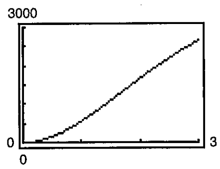


(b) Proof

61. $f(x) = \frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 24$
63. (a) $f(x) = \frac{1}{32}x^3 + \frac{3}{16}x^2$ (b) Two miles from touchdown
65. $x = 100$ units
67. (a)

t	0.5	1	1.5	2	2.5	3
S	151.5	555.6	1097.6	1666.7	2193.0	2647.1

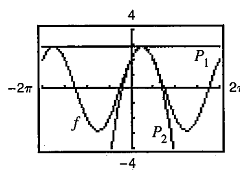
$1.5 < t < 2$

(b)


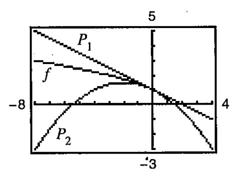
(c) About 1.633 yr

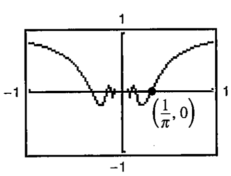
$t \approx 1.5$

69. $P_1(x) = 2\sqrt{2}$
$P_2(x) = 2\sqrt{2} - \sqrt{2}(x - \pi/4)^2$
The values of f, P_1, and P_2 and their first derivatives are equal when $x = \pi/4$. The approximations worsen as you move away from $x = \pi/4$.



71. $P_1(x) = 1 - x/2$
$P_2(x) = 1 - x/2 - x^2/8$
The values of f, P_1, and P_2 and their first derivatives are equal when $x = 0$. The approximations worsen as you move away from $x = 0$.



73.


75. True

77. False. f is concave upward at $x = c$ if $f''(c) > 0$.
79. Proof

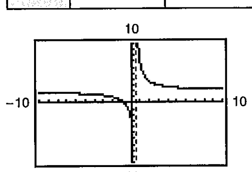
Section 3.5 *(page 202)*

1. f **2.** c **3.** d **4.** a **5.** b **6.** e

7.

x	10^0	10^1	10^2	10^3
$f(x)$	7	2.2632	2.0251	2.0025

x	10^4	10^5	10^6
$f(x)$	2.0003	2.0000	2.0000

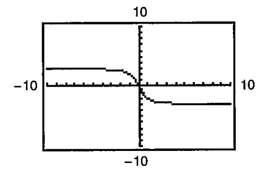


$$\lim_{x\to\infty} \frac{4x + 3}{2x - 1} = 2$$

9.

x	10^0	10^1	10^2	10^3
$f(x)$	-2	-2.9814	-2.9998	-3.0000

x	10^4	10^5	10^6
$f(x)$	-3.0000	-3.0000	-3.0000

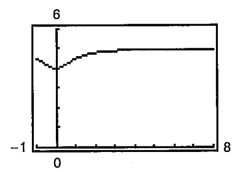


$$\lim_{x\to\infty} \frac{-6x}{\sqrt{4x^2 + 5}} = -3$$

11.

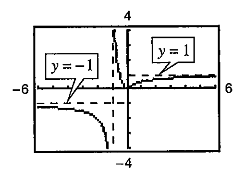
x	10^0	10^1	10^2	10^3
$f(x)$	4.5000	4.9901	4.9999	5.0000

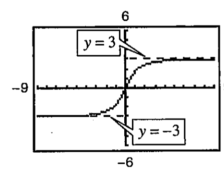
x	10^4	10^5	10^6
$f(x)$	5.0000	5.0000	5.0000



$$\lim_{x\to\infty} \left(5 - \frac{1}{x^2 + 1}\right) = 5$$

13. (a) ∞ (b) 5 (c) 0 **15.** (a) 0 (b) 1 (c) ∞
17. (a) 0 (b) $-\frac{2}{3}$ (c) $-\infty$ **19.** 4 **21.** $\frac{2}{3}$ **23.** 0
25. $-\infty$ **27.** -1 **29.** -2 **31.** $\frac{1}{2}$ **33.** ∞
35. 0 **37.** 0

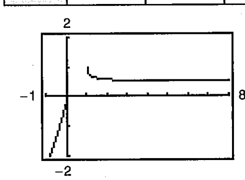
39.


41.


43. 1 **45.** 0 **47.** $\frac{1}{6}$

49.

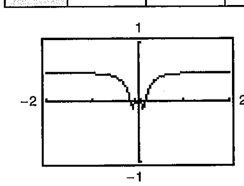
x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1.000	0.513	0.501	0.500	0.500	0.500	0.500



$$\lim_{x\to\infty} \left[x - \sqrt{x(x - 1)}\right] = \frac{1}{2}$$

51.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	0.479	0.500	0.500	0.500	0.500	0.500	0.500



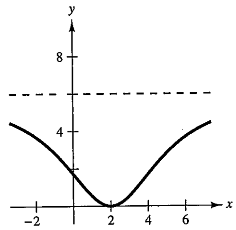
The graph has a hole at $x = 0$.

$$\lim_{x\to\infty} x \sin \frac{1}{2x} = \frac{1}{2}$$

53. As x becomes large, $f(x)$ approaches 4.

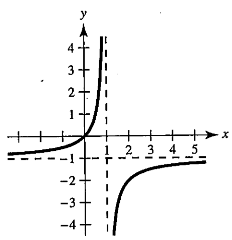
55. Answers will vary. Sample answer: Let

$$f(x) = \frac{-6}{0.1(x-2)^2+1} + 6.$$

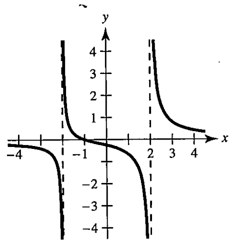


57. (a) 5 (b) -5

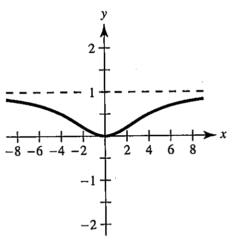
59.



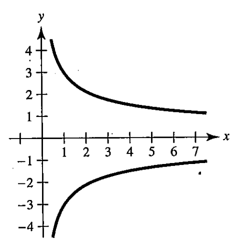
61.



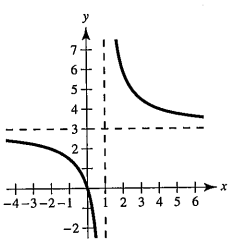
63.



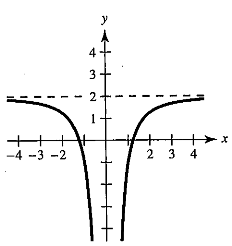
65.



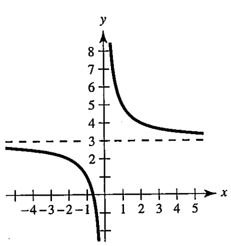
67.



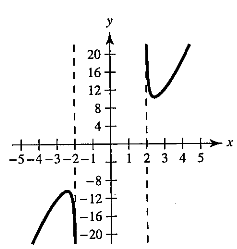
69.



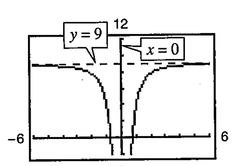
71.



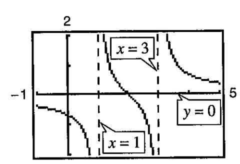
73.



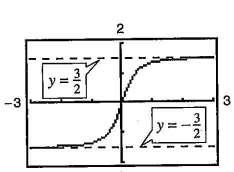
75.



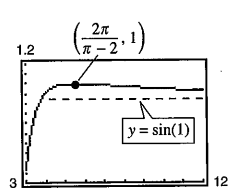
77.



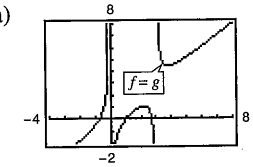
79.



81.

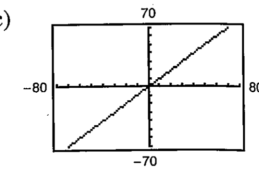


83. (a)



(b) Proof

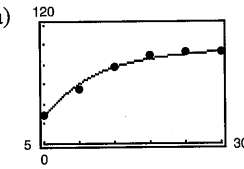
(c)



Slant asymptote: $y = x$

85. 100% **87.** $\lim_{t\to\infty} N(t) = +\infty$; $\lim_{t\to\infty} E(t) = c$

89. (a)



(b) Yes. $\lim_{t\to\infty} S = \frac{100}{1} = 100$

91. (a) $\lim_{x\to\infty} f(x) = 2$

(b) $x_1 = \sqrt{\frac{4-2\varepsilon}{\varepsilon}}$, $x_2 = -\sqrt{\frac{4-2\varepsilon}{\varepsilon}}$

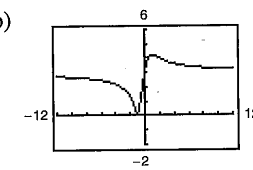
(c) $\sqrt{\frac{4-2\varepsilon}{\varepsilon}}$ (d) $-\sqrt{\frac{4-2\varepsilon}{\varepsilon}}$

93. (a) Answers will vary. $M = \frac{5\sqrt{33}}{11}$ **95–97.** Proofs

(b) Answers will vary. $M = \frac{29\sqrt{177}}{59}$

99. (a) $d(m) = \frac{|3m+3|}{\sqrt{m^2+1}}$

(b)



(c) $\lim_{m\to\infty} d(m) = 3$;
$\lim_{m\to-\infty} d(m) = 3$;
As m approaches $\pm\infty$, the distance approaches 3

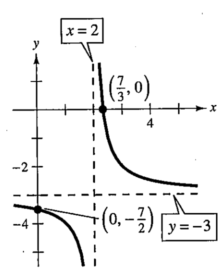
101. Proof

103. False. Let $f(x) = \frac{2x}{\sqrt{x^2+2}}$. $f'(x) > 0$ for all real numbers.

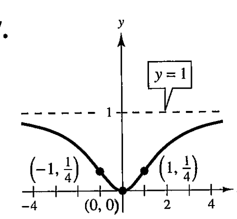
Section 3.6 *(page 212)*

1. d **2.** c **3.** a **4.** b

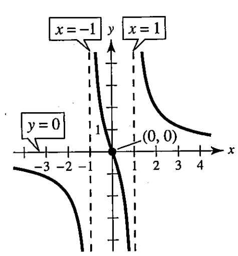
5.



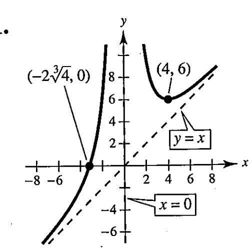
7.



9.



11.



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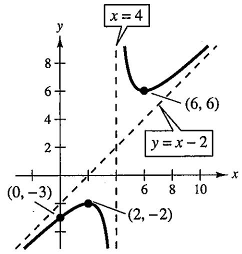

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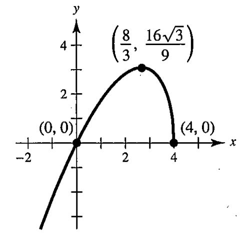

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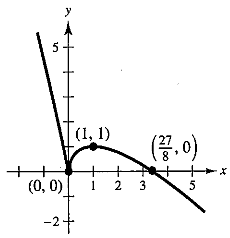

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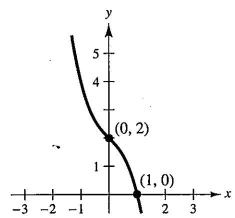

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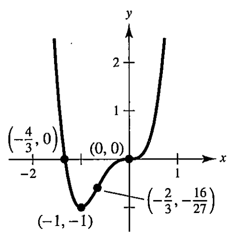
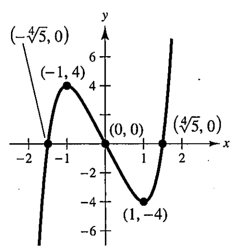
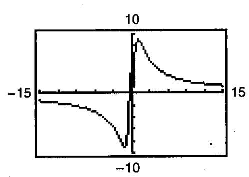


23.

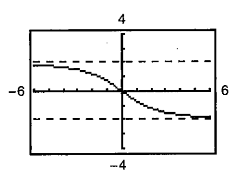


25.



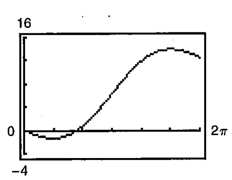
Minimum: $(-1.10, -9.05)$;
Maximum: $(1.10, 9.05)$;
Points of inflection:
$(-1.84, -7.86), (1.84, 7.86)$;
Vertical asymptote: $x = 0$;
Horizontal asymptote: $y = 0$

27.



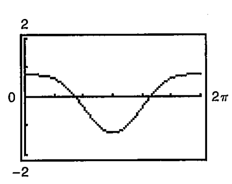
Point of inflection: $(0, 0)$;
Horizontal asymptotes: $y = \pm 2$

29.



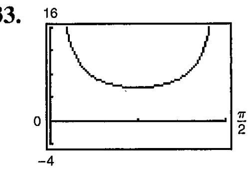
Relative minimum:
$\left(\frac{\pi}{3}, \frac{2\pi}{3} - 2\sqrt{3}\right)$;
Relative maximum:
$\left(\frac{5\pi}{3}, \frac{10\pi}{3} + 2\sqrt{3}\right)$;
Points of inflection: $(0, 0)$, $(\pi, 2\pi)$, $(2\pi, 4\pi)$

31.



Relative minimum: $\left(\pi, -\frac{5}{4}\right)$;
Points of inflection:
$\left(\frac{2\pi}{3}, -\frac{3}{8}\right), \left(\frac{4\pi}{3}, -\frac{3}{8}\right)$

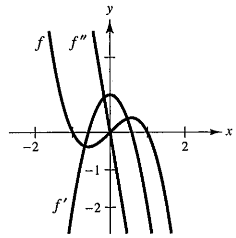
33.



Relative minimum: $\left(\frac{\pi}{4}, 4\sqrt{2}\right)$;
Vertical asymptotes: $x = 0, \frac{\pi}{2}$

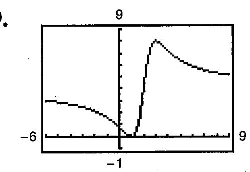
35. f is decreasing on $(2, 8)$, and therefore $f(3) > f(5)$.

37.



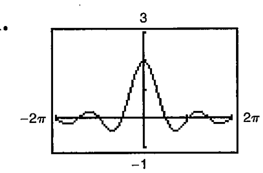
The zeros of f' correspond to the points where the graph of f has horizontal tangents. The zero of f'' corresponds to the point where the graph of f' has a horizontal tangent.

39.



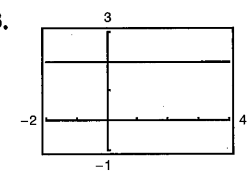
The graph crosses the horizontal asymptote $y = 4$.
The graph of a function f does not cross its vertical asymptote $x = c$ because $f(c)$ does not exist.

41.



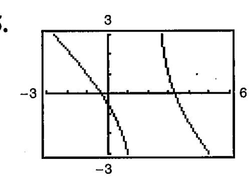
The graph has a hole at $x = 0$.
The graph crosses the horizontal asymptote $y = 0$.
The graph of a function f does not cross its vertical asymptote $x = c$ because $f(c)$ does not exist.

43.



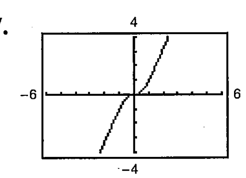
The graph has a hole at $x = 3$.
The rational function is not reduced to lowest terms.

45.



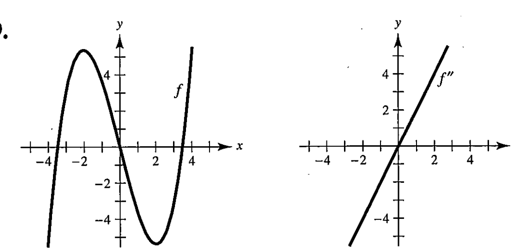
The graph appears to approach the line $y = -x + 1$, which is the slant asymptote.

47.

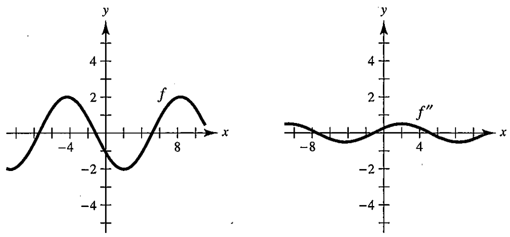


The graph appears to approach the line $y = 2x$, which is the slant asymptote.

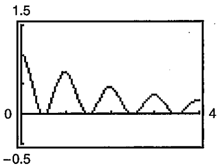
49.



51.



53. (a)



The graph has holes at $x = 0$ and at $x = 4$.
Visually approximated critical numbers: $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$

(b) $f'(x) = \dfrac{-x\cos^2(\pi x)}{(x^2+1)^{3/2}} - \dfrac{2\pi\sin(\pi x)\cos(\pi x)}{\sqrt{x^2+1}}$;
Approximate critical numbers: $\frac{1}{2}, 0.97, \frac{3}{2}, 1.98, \frac{5}{2}, 2.98, \frac{7}{2}$;
The critical numbers where maxima occur appear to be integers in part (a), but by approximating them using f', you can see that they are not integers.

55. Answers will vary. Sample answer: $y = 1/(x-3)$

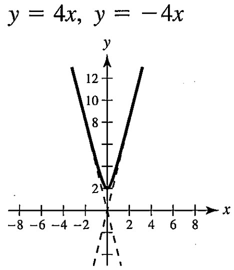
57. Answers will vary.
Sample answer: $y = (3x^2 - 7x - 5)/(x - 3)$

59. (a) x_0, x_2, x_4 (b) x_2, x_3 (c) x_1 (d) x_1 (e) x_2, x_3

61. (a)–(h) Proofs

63. Answers will vary. Sample answer: The graph has a vertical asymptote at $x = b$. If a and b are both positive or both negative, then the graph of f approaches ∞ as x approaches b, and the graph has a minimum at $x = -b$. If a and b have opposite signs, then the graph of f approaches $-\infty$ as x approaches b, and the graph has a maximum at $x = -b$.

65. $y = 4x,\ y = -4x$



67. Putnam Problem 13(i), 1939

Section 3.7 *(page 220)*

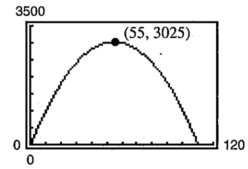
1. (a) and (b)

First Number, x	Second Number	Product, P
10	$110 - 10$	$10(110 - 10) = 1000$
20	$110 - 20$	$20(110 - 20) = 1800$
30	$110 - 30$	$30(110 - 30) = 2400$
40	$110 - 40$	$40(110 - 40) = 2800$
50	$110 - 50$	$50(110 - 50) = 3000$
60	$110 - 60$	$60(110 - 60) = 3000$
70	$110 - 70$	$70(110 - 70) = 2800$
80	$110 - 80$	$80(110 - 80) = 2400$
90	$110 - 90$	$90(110 - 90) = 1800$
100	$110 - 100$	$100(110 - 100) = 1000$

The maximum is attained near $x = 50$ and 60.

(c) $P = x(110 - x)$

(d)



(e) 55 and 55

3. $S/2$ and $S/2$ **5.** 21 and 7 **7.** 54 and 27

9. $l = w = 20$ m **11.** $l = w = 4\sqrt{2}$ ft **13.** (1, 1)

15. $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$

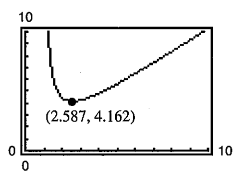
17. Dimensions of page: $(2 + \sqrt{30})$ in. $\times$ $(2 + \sqrt{30})$ in.

19. 700×350 m

21. Rectangular portion: $16/(\pi + 4) \times 32/(\pi + 4)$ ft

23. (a) $L = \sqrt{x^2 + 4 + \dfrac{8}{x-1} + \dfrac{4}{(x-1)^2}},\quad x > 1$

(b)

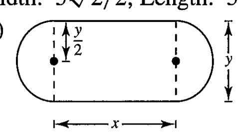


Minimum when $x \approx 2.587$

(c) (0, 0), (2, 0), (0, 4)

25. Width: $5\sqrt{2}/2$; Length: $5\sqrt{2}$

27. (a)



(b)

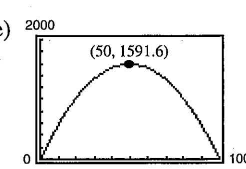
Length, x	Width, y	Area, xy
10	$2/\pi\,(100 - 10)$	$(10)(2/\pi)(100 - 10) \approx 573$
20	$2/\pi\,(100 - 20)$	$(20)(2/\pi)(100 - 20) \approx 1019$
30	$2/\pi\,(100 - 30)$	$(30)(2/\pi)(100 - 30) \approx 1337$
40	$2/\pi\,(100 - 40)$	$(40)(2/\pi)(100 - 40) \approx 1528$
50	$2/\pi\,(100 - 50)$	$(50)(2/\pi)(100 - 50) \approx 1592$
60	$2/\pi\,(100 - 60)$	$(60)(2/\pi)(100 - 60) \approx 1528$

The maximum area of the rectangle is approximately 1592 m^2.

(c) $A = 2/\pi(100x - x^2),\quad 0 < x < 100$

(d) $\dfrac{dA}{dx} = \dfrac{2}{\pi}(100 - 2x)$
$= 0$ when $x = 50$;
The maximum value is approximately 1592 when $x = 50$.

(e)



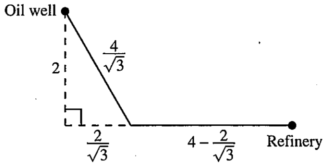
29. $18 \times 18 \times 36$ in.

31. No. The volume changes because the shape of the container changes when it is squeezed.

33. $r = \sqrt[3]{21/(2\pi)} \approx 1.50$ ($h = 0$, so the solid is a sphere.)

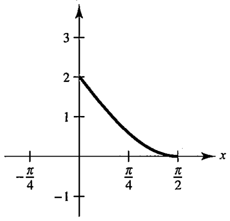
35. Side of square: $\dfrac{10\sqrt{3}}{9 + 4\sqrt{3}}$; Side of triangle: $\dfrac{30}{9 + 4\sqrt{3}}$

37. $w = \left(20\sqrt{3}\right)/3$ in., $h = \left(20\sqrt{6}\right)/3$ in.

39. 

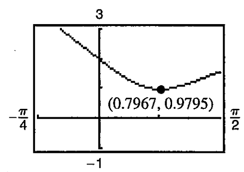
The path of the pipe should go underwater from the oil well to the coast following the hypotenuse of a right triangle with leg lengths of 2 miles and $2/\sqrt{3}$ miles for a distance of $4/\sqrt{3}$ miles. Then the pipe should go down the coast to the refinery for a distance of $(4 - 2/\sqrt{3})$ miles.

41. One mile from the nearest point on the coast

43. 

(a) Origin to y-intercept: 2;
Origin to x-intercept: $\pi/2$

(b) $d = \sqrt{x^2 + (2 - 2\sin x)^2}$



(c) Minimum distance is 0.9795 when $x \approx 0.7967$.

45. About 1.153 radians or 66° **47.** 8%

49. $y = \frac{64}{141}x$; $S \approx 6.1$ mi **51.** $y = \frac{3}{10}x$; $S_3 \approx 4.50$ mi

53. Putnam Problem A1, 1986

Section 3.8 *(page 229)*

In the answers for Exercises 1 and 3, the values in the tables have been rounded for convenience. Because a calculator and a computer program calculates internally using more digits than they display, you may produce slightly different values from those shown in the tables.

1.

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	2.2000	−0.1600	4.4000	−0.0364	2.2364
2	2.2364	0.0015	4.4728	0.0003	2.2361

3.

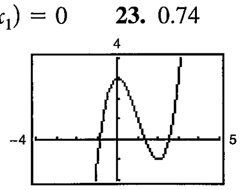
n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	1.6	−0.0292	−0.9996	0.0292	1.5708
2	1.5708	0	−1	0	1.5708

5. −1.587 **7.** 0.682 **9.** 1.250, 5.000

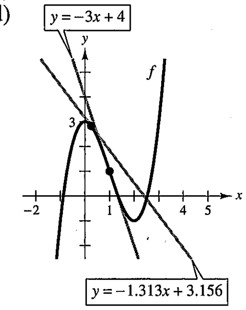
11. 0.900, 1.100, 1.900 **13.** 1.935 **15.** 0.569

17. 4.493 **19.** (a) Proof (b) $\sqrt{5} \approx 2.236$; $\sqrt{7} \approx 2.646$

21. $f'(x_1) = 0$ **23.** 0.74 **25.** Proof

27. (a) 

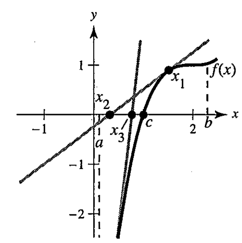
(b) 1.347 (c) 2.532

(d) 

x-intercept of $y = -3x + 4$ is $\frac{4}{3}$.
x-intercept of
$y = -1.313x + 3.156$
is approximately 2.404.

(e) If the initial estimate $x = x_1$ is not sufficiently close to the desired zero of a function, then the x-intercept of the corresponding tangent line to the function may approximate a second zero of the function.

29. Answers will vary. Sample answer:
If f is a function continuous on $[a, b]$ and differentiable on (a, b), where $c \in [a, b]$ and $f(c) = 0$, then Newton's Method uses tangent lines to approximate c. First, estimate an initial x_1 close to c. (See graph.) Then determine x_2 using $x_2 = x_1 - f(x_1)/f'(x_1)$. Calculate a third estimate x_3 using $x_3 = x_2 - f(x_2)/f'(x_2)$. Continue this process until $|x_n - x_{n+1}|$ is within the desired accuracy, and let x_{n+1} be the final approximation of c.



31. (1.939, 0.240) **33.** $x \approx 1.563$ mi

35. False; let $f(x) = \dfrac{x^2 - 1}{x - 1}$. **37.** True **39.** 0.217

Section 3.9 *(page 236)*

1. $T(x) = 4x - 4$

x	1.9	1.99	2	2.01	2.1
$f(x)$	3.610	3.960	4	4.040	4.410
$T(x)$	3.600	3.960	4	4.040	4.400

3. $T(x) = 80x - 128$

x	1.9	1.99	2	2.01	2.1
$f(x)$	24.761	31.208	32	32.808	40.841
$T(x)$	24.000	31.200	32	32.800	40.000

5. $T(x) = (\cos 2)(x - 2) + \sin 2$

x	1.9	1.99	2	2.01	2.1
$f(x)$	0.946	0.913	0.909	0.905	0.863
$T(x)$	0.951	0.913	0.909	0.905	0.868

7. $\Delta y = 0.331$; $dy = 0.3$ **9.** $\Delta y = -0.039$; $dy = -0.040$

11. $6x\,dx$ **13.** $(x \sec^2 x + \tan x)\,dx$

15. $-\dfrac{13}{(2x - 1)^2}\,dx$ **17.** $\dfrac{-x}{\sqrt{9 - x^2}}\,dx$ **19.** $(3 - \sin 2x)\,dx$