3.1-3.7 Test Review Packet

Calculus AB FRQ day **Curve Sketching**

1.

1994 AB 1

Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

a. Write an equation of the line tangent to the graph of f at the point (2, -28)

b. Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

c. Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

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Calculus AB FRQ day Curve Sketching

2.

1981 AB 3 BC 1

Let f be the function defined by $f(x) = 12x^{(2/3)} - 4x$.

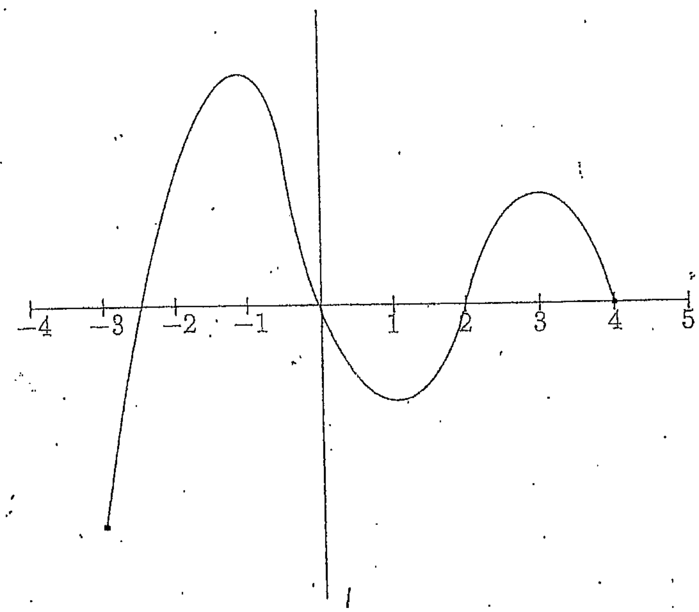
a. Find the intervals on which f is increasing.
b. Find the x- and y-coordinates of all relative maximum points.
c. Find the x- and y-coordinates of all relative minimum points.
d. Find the intervals on which f is concave downward.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

733. The figure below shows the graph of $g'(x)$, the derivative of a function g, with domain $[-3, 4]$.

a) Determine the values of x for which g has a relative minimum and a relative maximum. Justify your answer.

b) Determine the values of x for which g is concave down and concave up. Justify your answer.

c) Based on the information given and the fact that $g(-3) = 3$ and $g(4) = 6$, sketch a possible graph of g.



Calculus AB FRQ day **Curve Sketching** Solution Key

1.

1994 AB 1

Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

a. Write an equation of the line tangent to the graph of f at the point (2, -28)

b. Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

c. Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

a) $f'(x) = 12x^3 + 3x^2 - 42x$

$f'(2) = 12(2)^3 + 3(2)^2 - 42(2) = 24$

point: $(2, -28)$

slope: $m = 24$

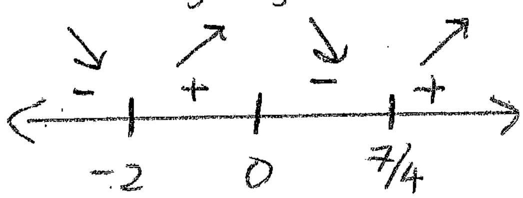
$\boxed{y + 28 = 24(x-2)}$

b) $f'(x) = 12x^3 + 3x^2 - 42x$

$0 = 3x(4x^2 + x - 14)$

$0 = 3x(4x-7)(x+2)$

$x = 0, 7/4, -2$



Abs. min must be either at $x = -2$ or $x = 7/4$ since $f'(x) < 0$ for all $x < -2$ and $f'(x) > 0$ for all $x > 7/4$

$f(-2) = -44$

$f(7/4) = -30.816$

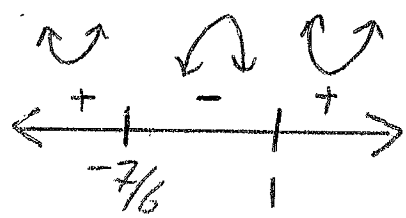
$\boxed{\text{Abs. min is } \underline{-44} \text{ at } x = -2}$

c) $f''(x) = 36x^2 + 6x - 42$

$= 6(6x^2 + x - 7)$

$0 = 6(6x+7)(x-1)$

$x = -7/6, x = 1$



$\boxed{\text{POI at } x = -7/6, x = 1 \text{ b/c } f''(x) \text{ change signs}}$

Calculus AB FRQ day **Curve Sketching**

2.

1981 AB 3 BC 1

Let f be the function defined by f(x) = 12x $^{(2/3)}$ -4x.

a. Find the intervals on which f is increasing.
b. Find the x- and y-coordinates of all relative maximum points.
c. Find the x- and y-coordinates of all relative minimum points.
d. Find the intervals on which f is concave downward.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

a) $f'(x) = 12 \cdot \frac{2}{3} x^{-1/3} - 4$

$f'(x) = \frac{8}{x^{1/3}} - 4 = \frac{8-4x^{1/3}}{x^{1/3}}$

$8 - 4x^{1/3} = 0$

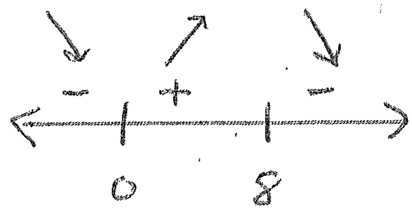
$4x^{1/3} = 8$

$x^{1/3} = 2$

$\boxed{x = 8}$

$x^{1/3} = 0$

$\boxed{x = 0}$



a) f(x) increasing (0, 8] b/c $f'(x) > 0$

b) Rel. max at (8, 16) b/c $f'(x)$ changes from + to −

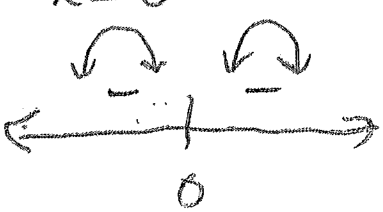
c) Rel. min at (0, 0) b/c $f'(x)$ changes from − to +

$f'(x) = 8x^{-1/3} - 4$

d) $f''(x) = 8 \cdot -\frac{1}{3} x^{-4/3} + 0$

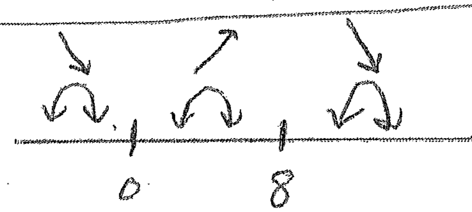
$0 = \frac{-8}{3x^{4/3}}$

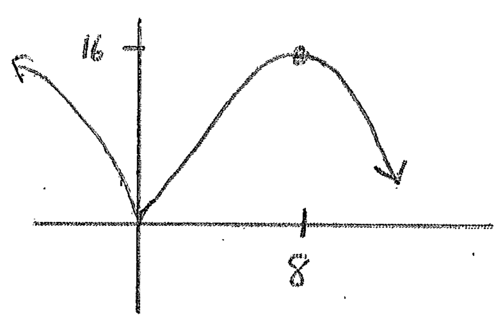
$x = 0$



f(x) concave down $(-\infty, 0) \cup (0, \infty)$ b/c $f''(x) < 0$

e)





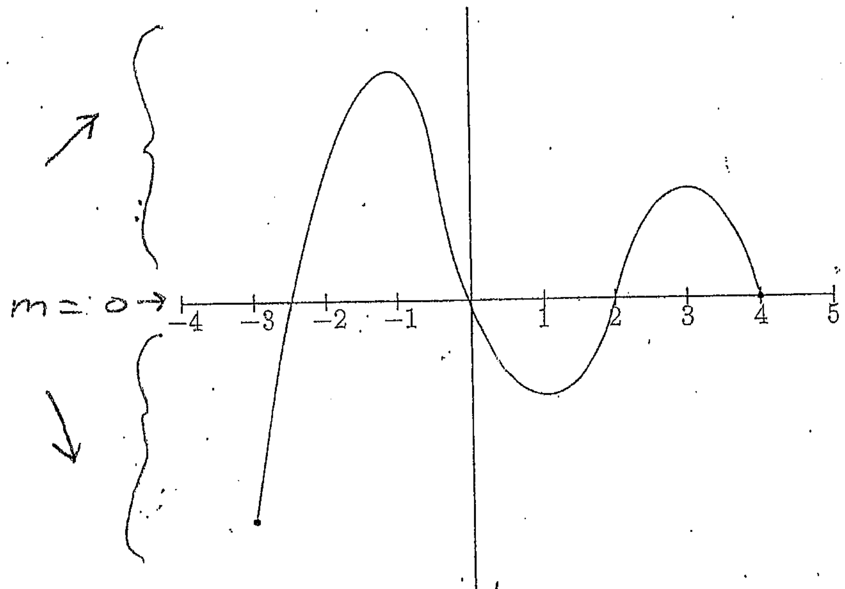
(6)

733. The figure below shows the graph of $g'(x)$, the derivative of a function g, with domain $[-3, 4]$.

a) Determine the values of x for which g has a relative minimum and a relative maximum. Justify your answer.

b) Determine the values of x for which g is concave down and concave up. Justify your answer.

c) Based on the information given and the fact that $g(-3) = 3$ and $g(4) = 6$, sketch a possible graph of g.



a) ↘ ↗ ↘ ↗

$-$ $+$ $-$ $+$

-3 $\;-2.5$ $\;0$ $\;2$ $\;4$

Rel. min at $x=-2.5, x=2$ b/c $f'(x)$ changes sign from $-$ to $+$

Rel. max at $x=0$ b/c $f'(x)$ changes sign from $+$ to $-$

b) ∪ ∩ ∪ ∩

$+$ $-$ $+$ $-$

-3 $\;-1$ $\;1$ $\;3$ $\;4$

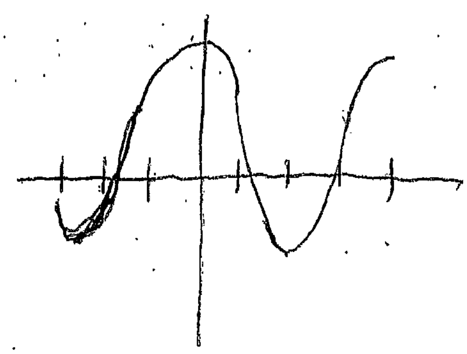
$f(x)$ is concave up $(-3,-1) \cup (1,3)$ b/c $f''(x) > 0$

$f(x)$ is concave down $(-1,1) \cup (3,4)$ b/c $f''(x) < 0$

c) ∪ ∪ ∩ ∩ ∪ ∪ ∩

↘ ↗ ↗ ↘ ↘ ↗ ↗

-3 $\;-2.5$ $\;-1$ $\;0$ $\;1$ $\;2$ $\;3$ $\;4$



Derivative Graph Practice Problem #2:

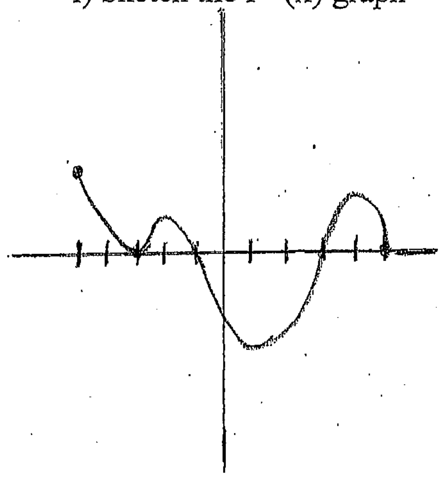
Given the f'(x) graph, find the characteristics of f(x) graph:

a) Relative minimum(s) b) Relative maximum(s) c) interval increasing

d) interval decreasing e) POI f) interval concave up g) interval concave down

h) Sketch f(x) graph given points (-5, -4) and (5, 3). The range is [-7, 5]

i) Sketch the f ''(x) graph



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First Derivative Test, Concavity Test Practice Problem #3:

Given that $f(x) = x^3 - 3x^2 + 3$, find the characteristics of f(x) graph:

a) Relative minimum(s) b) Relative maximum(s) c) interval increasing

d) interval decreasing e) POI f) interval concave up g) interval concave down

h) Sketch f(x) graph

/ Key 9

Derivative Graph Practice Problem #2:

Given the f'(x) graph, find the characteristics of f(x) graph:

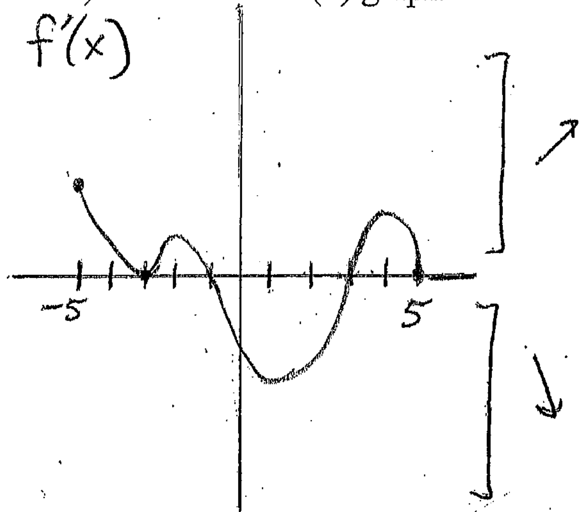
a) Relative minimum(s) b) Relative maximum(s) c) interval increasing

d) interval decreasing e) POI f) interval concave up g) interval concave down

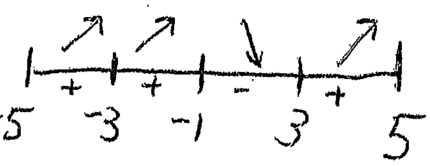
h) Sketch f(x) graph given points (-5, -4) and (5, 3). The range is [-7, 5]

i) Sketch the f ''(x) graph

f'(x)



f'(x)



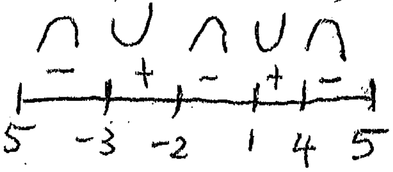
a) Rel. min at x = 3 b/c f'(x) changes from - to +

b) Rel. max at x = -1 b/c f'(x) changes from + to -

c) f(x) increasing (-5, -3), (-3, -1), (3, 5) b/c f'(x) > 0

d) f(x) decreasing (-1, 3) b/c f'(x) < 0

f''(x)

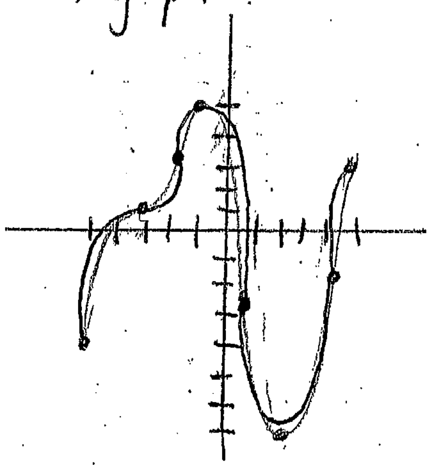


e) POI at x = -3, -2, 1, 4 b/c f''(x) change signs

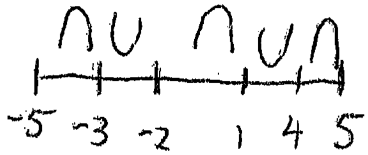
f) Concave up (-3, -2), (1, 4) b/c f''(x) > 0

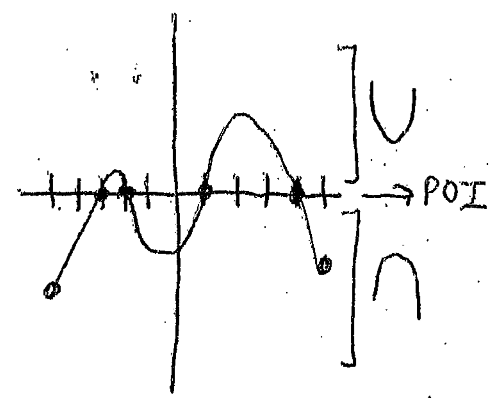
g) concave down (-5, -3), (-2, 1), (4, 5) b/c f''(x) < 0

h) f(x) graph



i) f''(x) graph





First Derivative Test, Concavity Test Practice Problem #3:

Given that $f(x) = x^3 - 3x^2 + 3$, find the characteristics of f(x) graph:

a) Relative minimum(s) b) Relative maximum(s) c) interval increasing

d) interval decreasing e) POI f) interval concave up g) interval concave down

h) Sketch f(x) graph

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0, 2$

$+ \quad - \quad +$

$-1 \quad 0 \quad 1 \quad 2 \quad 3$

a) Rel. min $(2, -1)$ b/c $f'(x)$ changes from − to +

b) Rel. max $(0, 3)$ b/c $f'(x)$ changes from + to −

c) $f(x)$ increasing $(-\infty, 0), (2, \infty)$ b/c $f'(x) > 0$

d) $f(x)$ decreasing $(0, 2)$ b/c $f'(x) < 0$

$f''(x) = 6x - 6$

$0 = 6(x-1)$

$x = 1$

$- \quad +$

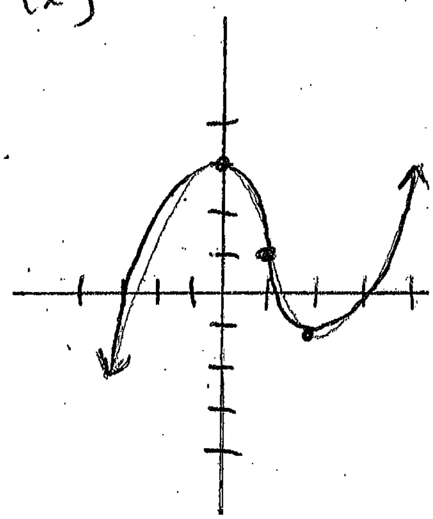
$0 \quad 1 \quad 2$

e) POI at $(1, 1)$ b/c $f''(x)$ change signs

f) concave up $(1, \infty)$ b/c $f''(x) > 0$

g) concave down $(-\infty, 1)$ b/c $f''(x) < 0$

h) $f(x)$



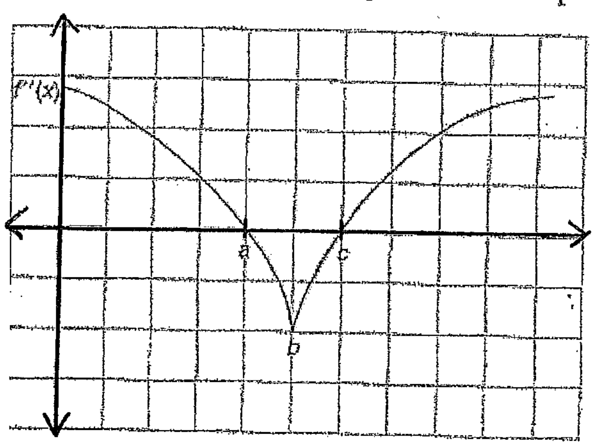
Calculus AB Ch. 3 Test Review WS #1

1. A t-shirt maker estimates that the weekly cost of making x shirts is $C(x) = 50 + 2x + \frac{x^2}{20}$
The weekly revenue from selling x shirts is given by the function $R(x) = 20x + \frac{x^2}{200}$

   a) What is the profit if all the shirts made are sold? (Profit = Revenue – Cost)

   b) What is the maximum weekly profit?

2. The second derivative of $f(x)$ has zeros at $x = a$ and $x = c$ and a minimum at $x = b$ as shown. The function $f(x)$ is concave up



(A) when $0x < a$
(B) when $0x < b$
(C) when $x > b$
(D) when $0x < a$ and $x > c$
(E) nowhere

3. Verify whether $f(x) = 3x^2 - 12x + 1$ satisfies Rolle's theorem on the interval [0, 4] and find all numbers c that satisfy $f'(c) = 0$
   A) c = 0
   B) c = 1
   C) c = 2
   D) c = 4
   E) f(x) does not satisfy Rolle's theorem on interval [0, 4] .

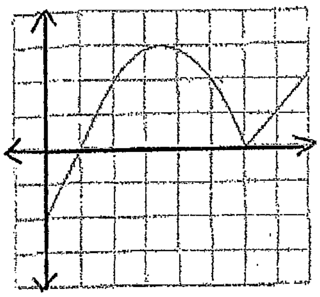
4. Which of the following statements is true of the function $f(x) = x^{2/3}$

I. There is a critical point at (0, 0
II. $f'(0)$ and $f''(0)$ are undefined
III. The curve is concave up over the interval $(0, \infty)$
IV. The curve is concave down over interval $(-\infty, 0)$

A. I and III only
B. I, II, IV only
C. I, II, III
D. I, III, and IV
E. I, II, III, and IV

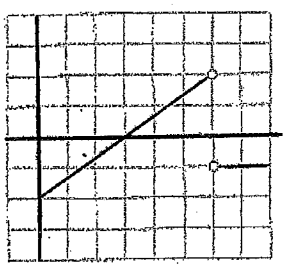
12

The graph of a function $g(x)$ is given.

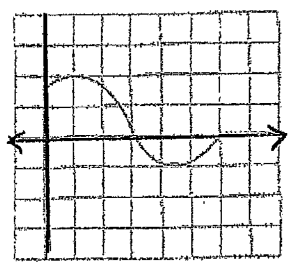


Which of the following could be the graph of $g'(x)$?

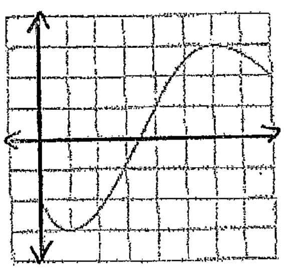
(A)



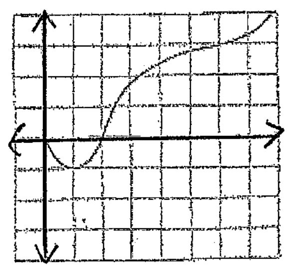
(B)



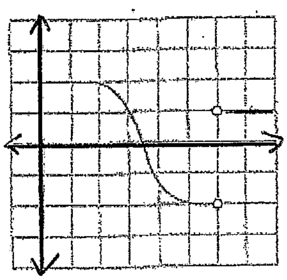
(C)



(D)



(E)



6. The height of an object t seconds after it is dropped from a height of 500 meters is $x(t) = -4.9t^2 + 500$

a) Find the avg velocity of the object during the first 4 seconds (Think avg slope)

b) Use the Mean Value Theorem to verify that, at some point during the first 4 seconds of the fall, the instantaneous velocity equals the avg velocity. Find that time and height.

7. Max plans to build two side-by-side identical rectangular pens for his pigs that will enclose a total area of 216 ft^2. What is the minimum length of fencing he will need?

8. A manufacturer wants to design an open-top box having a square base and a surface area of 80 square inches.
   a. What dimensions will provide a box with maximum volume?
   b. Find maximum volume

Calculus AB Ch. 3 Test Review WS #1

Solutions

13

1. A t-shirt maker estimates that the weekly cost of making x shirts is $C(x) = 50 + 2x + \frac{x^2}{20}$

The weekly revenue from selling x shirts is given by the function $R(x) = 20x + \frac{x^2}{200}$

a) What is the profit if all the shirts made are sold? (Profit = Revenue – Cost)

$P(x) = 20x + \frac{x^2}{200} - \left(50 + 2x + \frac{x^2}{20}\right)$

$= 18x - 50 - \frac{9x^2}{200}$

b) What is the maximum weekly profit?

$P'(x) = 18 - 2\left(\frac{9}{200}\right)x$

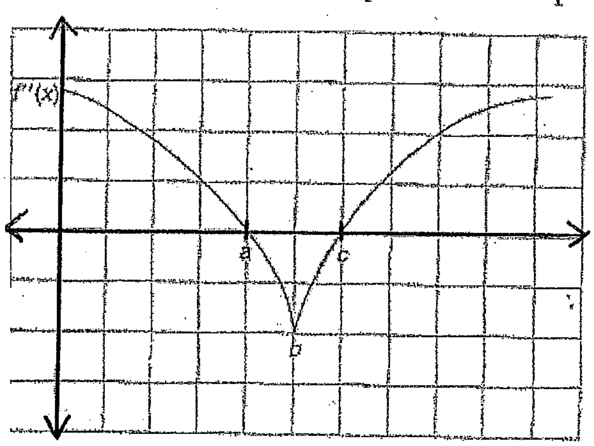
$0 = 18 - \frac{18}{200}x$

$18 = 0.09x$

$\boxed{x = 200 \text{ shirts}}$

$\boxed{P(200) = \$1750}$

2. The second derivative of $f(x)$ has zeros at $x = a$ and $x = c$ and a minimum at $x = b$ as shown. The function $f(x)$ is concave up



$f''(x)$: + | − | + (a, c)

(A) when $x < a$
(B) when $x < b$
(C) when $x > b$
$\boxed{\text{(D) when } x < a \text{ and } x > c}$
(E) nowhere

3. Verify whether $f(x) = 3x^2 - 12x + 1$ satisfies Rolle's theorem on the interval [0, 4] and find all numbers c that satisfy $f'(c) = 0$

A) c = 0
B) c = 1
$\boxed{\text{C) } c = 2}$
D) c = 4
E) f(x) does not satisfy Rolle's theorem on interval [0, 4]

f(x) continuous on [0,4] differentiable on (0,4)

$f(0) = 1$, $f(4) = 1$] $f(0) = f(4) = 1$

*set $f'(x) = 0$

$f'(x) = 6x - 12$

$6x - 12 = 0$

$x = 2$

$\boxed{c = 2}$

4. Which of the following statements is true of the function $f(x) = x^{2/3}$

I. There is a critical point at (0, 0) True
II. $f'(0)$ and $f''(0)$ are undefined True
~~III~~. The curve is concave up over the interval $(0, \infty)$ False
IV. The curve is concave down over interval $(-\infty, 0)$ True

A. I and III only
$\boxed{\text{B. I, II, IV only}}$
C. I, II, III
D. I, III, and IV
E. I, II, III, and IV

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

− | + (0)

$f''(x) = \frac{2}{3}\left(-\frac{1}{3}\right)x^{-4/3}$

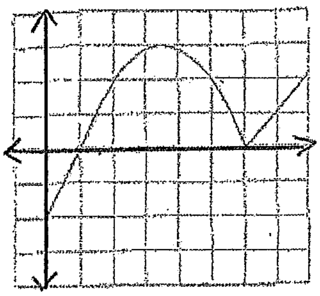
$= \frac{-2}{9x^{4/3}}$

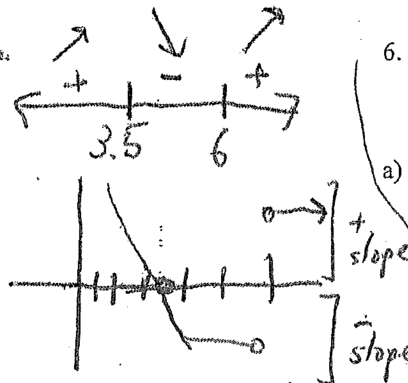
− | − (0)

$\boxed{B}$

(14)

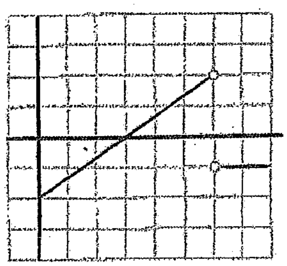
5) The graph of a function $g(x)$ is given.



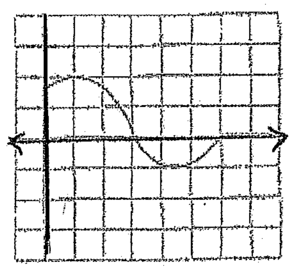


Which of the following could be the graph of $g'(x)$? $*f'(6)$ undefined

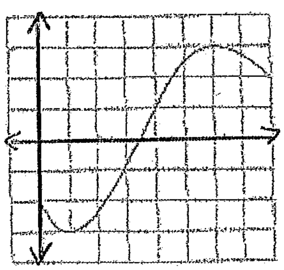
(A)



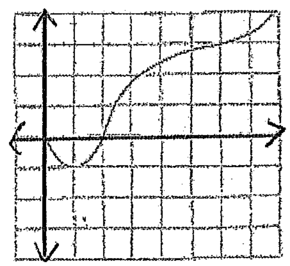
(B)



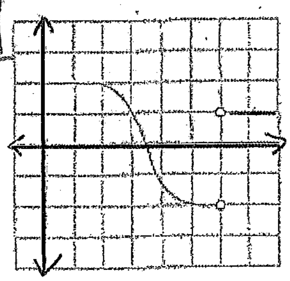
(C)



(D)



(E)



6. The height of an object t seconds after it is dropped from a height of 500 meters is $x(t) = -4.9t^2 + 500$

a) Find the avg velocity of the object during the first 4 seconds (Think avg slope)

$s(0) = 500$

$s(4) = 421.6$

$m_{Avg} = \frac{500-421.6}{0-4}$

$= -19.6\, m/s$

b) Use the Mean Value Theorem to verify that, at some point during the first 4 seconds of the fall, the instantaneous velocity equals the avg velocity. Find that time and height.

$*$set $x'(t) = m_{Avg}$

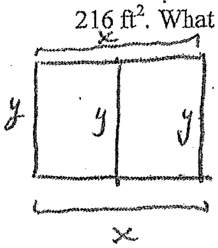
$x'(t) = -9.8t$

$-9.8t = -19.6$

$\boxed{t = 2 \text{ seconds}}$

$\boxed{x(2) = 480.4 \text{ ft}}$

7. Max plans to build two side-by-side identical rectangular pens for his pigs that will enclose a total area of $216\ \text{ft}^2$. What is the minimum length of fencing he will need?



$*P = 2x + 3y$

$A = xy$

$216 = xy \quad y = \frac{216}{x}$

$P = 2x + 3\left(\frac{216}{x}\right)$

$P = 2x + 648x^{-1}$

$P'(x) = 2 - 648x^{-2}$

$0 = 2 - \frac{648}{x^2}$

$x^2 = 324$

$x = 18$ ft.

$216 = xy$

$216 = 18y$

$y = 12$ ft.

$P = 2x + 3y$

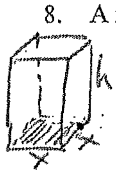
$2(18) + 3(12)$

$\boxed{P_{min} = 72 \text{ ft. of fencing}}$

8. A manufacturer wants to design an open-top box having a square base and a surface area of 80 square inches.

a. What dimensions will provide a box with maximum volume?

b. Find maximum volume



$S = x^2 + 4xh$

$80 = x^2 + 4xh$

$\frac{80 - x^2}{4x} = h$

$V = x^2 h$

$V = x^2\left[\frac{80 - x^2}{4x}\right]$

$V = 20x - \frac{1}{4}x^3$

$V'(x) = 20 - \frac{3}{4}x^2$

$0 = 20 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 20$

$x^2 = \frac{80}{3}$

$x = \sqrt{80/3} \approx 5.164$ in.

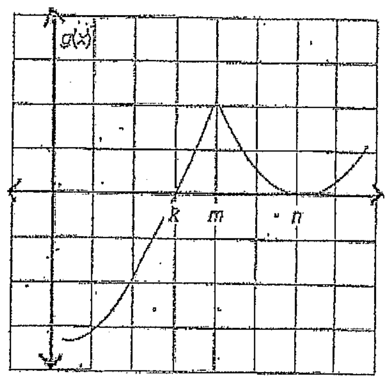
$h = \frac{80 - (5.164)^2}{4(5.164)}$

$h = 2.582$ in.

$\boxed{V_{max} = 68.853 \text{ in}^3}$

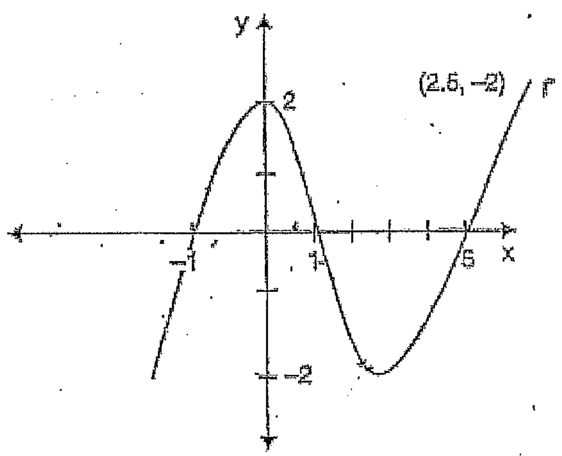
Calculus AB Ch. 3 Test Review WS #2

1. The graph of g(x) has zeros at x = k, x = n, and a relative maximum at m as shown. Based on the graph, which of the following is true?



a) g'(x) has a relative maximum at x = k
b) g'(x) has a zero at x = m
c) g''(x) has a zero at x = n
d) g'(x) is continuous everywhere
e) g''(x) is never negative

2. Given the graph of f', find the following properties of the function f:



a) The intervals on which f is increasing or decreasing

b) The location of the relative maxima and minima

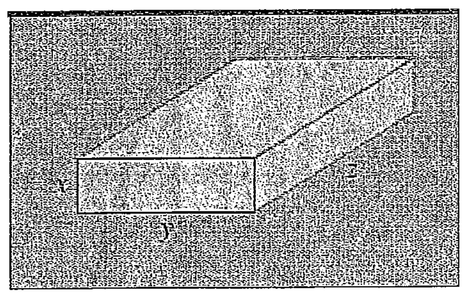
c) The points of inflection and concavity of f

d) Draw a sketch of f, given that f(-1) = −5
f(1) = 5, f(0) = 0, and f(5) = −5

3. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$30/foot and on the other 3 sides by a metal fence costing \$10/foot. If the area of the garden is 1000 square feet, find the dimensions of the garden that minimze cost. Round dimensions to 3 decimal places.

4. A jewel box is to be constructed of material that costs \$1 per square inch for the bottom, \$2 per square inch for the sides, and \$5 per square inch for the top. If the total volume is to be 96 in.^{3} what dimensions will minimize the total cost of construction?

OMIT



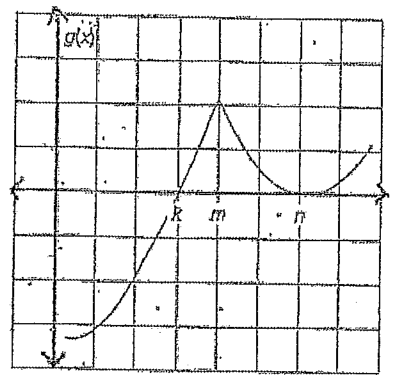
5. A 300-room hotel in Las Vegas is filled to capacity every night at \$80 a room. For each \$1 increase in rent, 3 fewer rooms are rented. If each rented room costs \$10 to service per day, how much should the management charge for each room to maximize gross profit? What is the maximum gross profit?

Calculus AB Ch. 3 Test Review WS #2 **Solutions**

1. The graph of g(x) has zeros at x = k, x = n, and a relative maximum at m as shown. Based on the graph, which of the following is true?

$g'(x)$: ↗ on (0, m), ↘ on (m, n), ↗ on (n, 8)

$g''(x)$: + on (0, m), + on (m, 8)



a) g'(x) has a relative maximum at x = k false

b) g'(x) has a zero at x = m slope undefined, false

c) g''(x) has a zero at x = n. concave up, $g''>0$, false

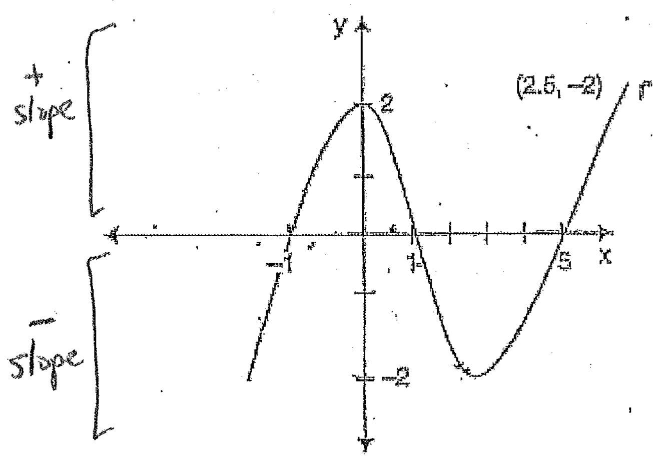
d) g'(x) is continuous everywhere false, $g'(m)$ undefined

e) g''(x) is never negative

True, $f''>0$, except when f'' undefined at $x=m$

2. Given the graph of f', find the following properties of the function f:

f': − on $(-\infty,-1)$, + on $(-1,1)$, − on $(1,5)$, + on $(5,\infty)$



a) The intervals on which f is increasing or decreasing

$f(x)$ increasing $(-1,1)\cup(5,\infty)$ b/c $f'(x)>0$

$f(x)$ decreasing $(-\infty,-1)\cup(1,5)$ b/c $f'(x)<0$

b) The location of the relative maxima and minima

Rel. max at $x=1$ b/c $f'(x)$ changes from + to −

Rel. min at $x=-1, x=5$ b/c $f'(x)$ changes from − to +.

c) The points of inflection and concavity of f

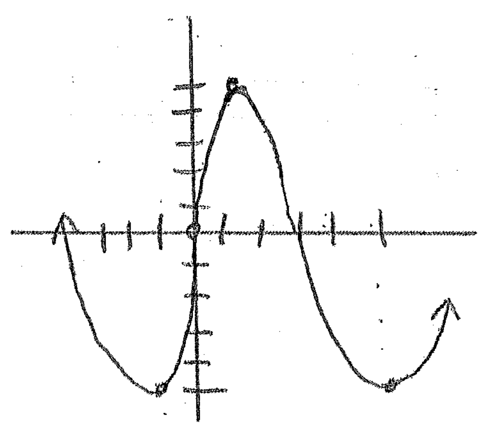
f'': + on $(-\infty,0)$, − on $(0,3)$, + on $(3,\infty)$

Concave up $(-\infty,0)\cup(3,\infty)$ b/c $f''(x)>0$

Concave down $(0,3)$ b/c $f''(x)<0$

POI at $x=0, 3$ b/c $f''(x)$ change signs.

d) Draw a sketch of f, given that f(-1) = – 5, f(1) = 5, f(0) = 0, and f(5) = –5



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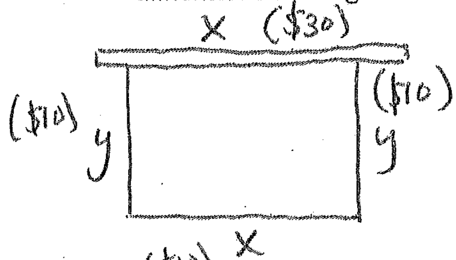
3. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$30/foot and on the other 3 sides by a metal fence costing \$10/foot. If the area of the garden is 1000 square feet, find the dimensions of the garden that minimze cost. Round dimensions to 3 decimal places.



$A = xy$

$1000 = xy$

$y = \frac{1000}{x}$

(Optimize perimeter)

$P = 2x + 2y$

$C(x) = 30x + 10x + 10y + 10y$

$C(x) = 40x + 20y$

$C(x) = 40x + 20\left(\frac{1000}{x}\right)$

$C(x) = 40x + 20000x^{-1}$

$C'(x) = 40 - 20000x^{-2}$

$0 = 40 - \frac{20000}{x^2}$

$40 = \frac{20000}{x^2}$

$x = \sqrt{500} = 10\sqrt{5} \approx \underline{22.361}$

$1000 = xy$

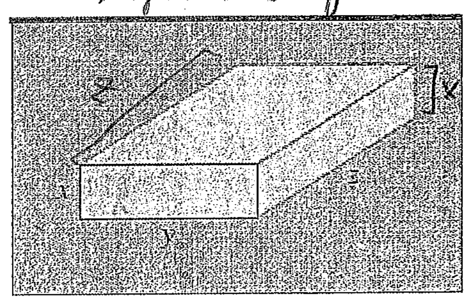
$1000 = 10\sqrt{5}(y)$

$y \approx \underline{44.721 \text{ ft.}}$

$\boxed{22.361 \text{ ft by } 44.721 \text{ ft.}}$

4. A jewel box is to be constructed of material that costs \$1 per square inch for the bottom, \$2 per square inch for the sides, and \$5 per square inch for the top. If the total volume is to be 96 in.^{3} what dimensions will minimize the total cost of construction?

*optimize surface area



$C(x) = \$5(yz) + \$1(yz) + \$2(xy + xy + xz + xz)$

$= 5yz + yz + 4xy + 4xz$

$C(x) = 6yz + 4xy + 4xz$

$V = xyz$

5. A 300-room hotel in Las Vegas is filled to capacity every night at \$80 a room. For each \$1 increase in rent, 3 fewer rooms are rented. ~~If each rented room costs \$10 to service per day,~~ how much should the management charge for each room to maximize gross profit? What is the maximum gross profit?

Rate = (change in rent)(change in rooms rented)

$R(x) = (80 + 1x)(300 - 3x)$

$= 24000 + 300x - 240x - 3x^2$

$= 24000 + 60x - 3x^2$

$R'(x) = 60 - 6x$

$0 = 60 - 6x$

$6x = 60$

$x = 10$ rent increases

Rent $= 80 + 1x$

$= 80 + 1(10)$

$= \boxed{\$90}$

$\boxed{\text{Max profit } R(10) = \$24{,}300}$

Optimization Review Problem (Involving Cost)

1)

A rectangular storage container with an open top is to have a Volume of 10 m^3. The length of its base is twice its length. Material for the base costs \$10/ m^2. Material for the sides cost \$6/m^2. Find the cost of material for the cheapest container. (Hint: Minimize surface area)

2)

The manager of a department store wants to build a 600 square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per running foot. The forth side will be built of cement blocks, at a cost of \$28 per running foot. What dimensions will minimize the total cost of the building materials? What will this minimum cost be?

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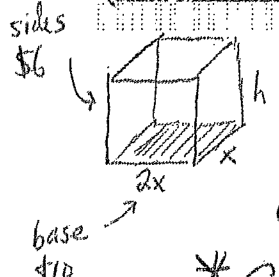
Key

Optimization Review Problem (Involving Cost)

1)

A rectangular storage container with an open top is to have a Volume of 10 m³. The length of its base is twice its length. Material for the base costs \$10/ m². Material for the sides cost \$6/m². Find the cost of material for the cheapest container. (Hint: Minimize surface area)



$S = 2x^2 + xh + xh + 2xh + 2xh$

$S = 2x^2 + 6xh$

$Cost = 10(2x^2) + 6(6xh)$

$* \; Cost = 20x^2 + 36xh$

Volume $= (2x)(x)(h)$

$10 = 2x^2 h$

$\frac{10}{2x^2} = h$

$\boxed{\frac{5}{x^2} = h}$

$Cost = 20x^2 + 36x\left(\frac{5}{x^2}\right)$

$C(x) = 20x^2 + \frac{180}{x}$

$C(x) = 20x^2 + 180x^{-1}$

$C'(x) = 40x - 180x^{-2}$

$0 = 40x - \frac{180}{x^2}$

$\frac{180}{x^2} = \frac{40x}{1}$

$40x^3 = 180$

$x^3 = \frac{180}{40} = \frac{9}{2}$

$x^3 = 9/2$

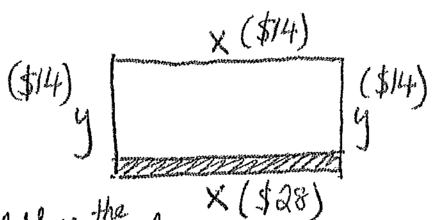
$x = \sqrt[3]{9/2} \approx 1.651$ meters

$C(1.651) = 20(1.651)^2 + \frac{180}{1.651}$

$= \boxed{\$163.54}$

2)

The manager of a department store wants to build a 600 square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per running foot. The forth side will be built of cement blocks, at a cost of \$28 per running foot. What dimensions will minimize the total cost of the building materials? What will this minimum cost be? (perimeter)



Add up the costs associated with each side

$* \; Cost = 28x + 14x + 14y + 14y$

$C = 42x + 28y$

Area $= xy$

$600 = xy$

$\boxed{\frac{600}{x} = y}$

$C(x) = 42x + 28\left(\frac{600}{x}\right)$

$C(x) = 42x + 16800x^{-1}$

$C'(x) = 42 - 16800x^{-2}$

$0 = 42 - \frac{16800}{x^2}$

$\frac{16800}{x^2} = \frac{42}{1}$

$42x^2 = 16800$

$x^2 = \frac{16800}{42}$

$x^2 = 400$

$x = 20$ ft

$y = \frac{600}{x} \to \frac{600}{20} = 30$ ft

$Cost = 42(20) + 28(30)$

$\boxed{Cost = \$1680}$

Calculus AB Ch. 3 Curve Sketching/Optimization Review WS #3 (Morning Review)

Definitions:

Extreme Value Theorem(EVT): If a function $f(x)$ is ______________ on a ______________ interval a, b then $f(x)$ has both a ______________ and ______________ value on a, b

Mean Value Theorem(MVT): Let f be ______________ on ______________ interval a,b

and ______________ on the ______________ interval a,b

Then there is at least one point c in a,b where ______________

Rolle's Theorem: Let f be ______________ on ______________ interval a,b

and ______________ on the ______________ interval a,b.

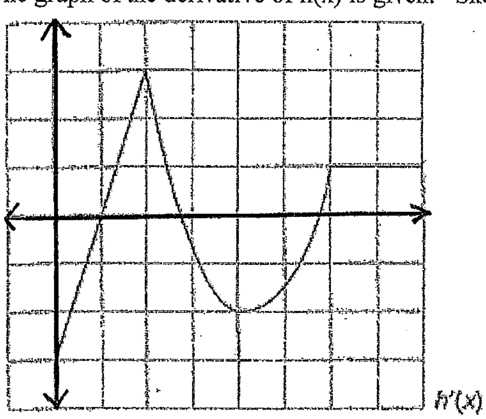
If ______________ then there is at least one point c in a,b where ______________

1. $f(x) = \dfrac{2x}{x^2+1}$ on the closed interval [-2, 2]. Determine the maximum and minimum values on the graph.

2. Determine if Rolle's Theorem is satisfied for $f(x) = (x-3)(x+2)^2$ on the closed interval [-2, 3]. (Step through conditions!) If so, find c.

3. Determine if MVT is satisfied for $f(x) = x(x^2 - x - 2)$ on the closed interval [-1,1]. (Step through conditions!) If so, find c.

4. The graph of the derivative of h(x) is given: Sketch a possible graph of h(x) (Create sign lines for f '(x) and f ''(x) first!)



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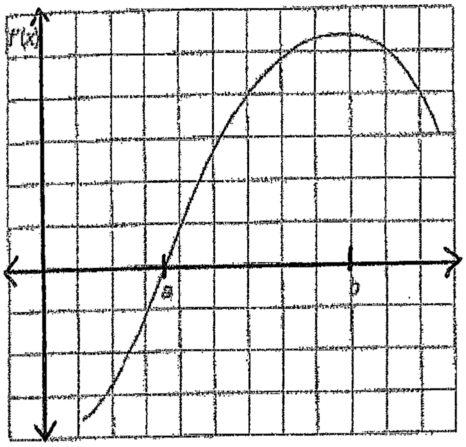
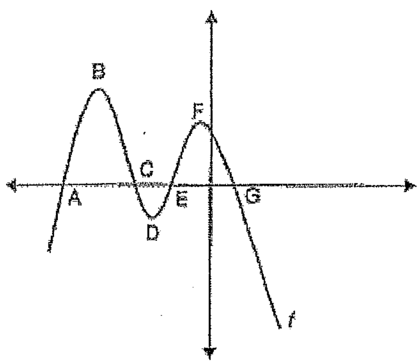
5. The derivative of f has a zero at $x = a$ and a relative maximum at $x = b$, as shown. Which of the following is not true?



A) f(x) has a relative minimum at x = a
B) f(x) has an absolute maximum at x = b
C) f(x) is increasing on (a, b)
D) f ''(x) is positive on (a, b)
E) f ''(x) has a zero at x = b

6. Which of the following statements are true of the graph of f below?



I. $f' \geq 0$ on the interval from D to F
II. $f'' = 0$ at points B, D, and F
III. $f'' > 0$ on the interval from A to B
IV. $f'' > 0$ on the interval from D to F

A. I and II
B. I and III
C. II and IV
D. II, III, and IV
E. I, II, III
F. Just I

7. Find the extrema of function $f(x) = \frac{1}{3}x^3 - 6x^2 + 35x - 1$

A) Absolute minimum at x = 0
B) Absolute maxima at x = 5, 7
C) Relative maximum at x = -5, relative minimum at x = -7
D) Relative maximum at x = 5, relative minimum at x = 7
E) Relative maximum at x = 7, relative minimum at x = 5

8. Find all critical points, c, for the function $f(x) = \frac{2}{3}x^3 + 5x^2 - 28x - 10$

A) c = 0, -7, -2
B) c = -7, 2
C) c = 0
D) c = -2,7
E) c = 10

9. Find the inflection point(s) for the function $f(x) = 2x(x+4)^3$

A) (0,0)
B) (-4,0)
C) (0,0) , (-4,0)
D) (0,0) , (-4,0) , (4,0)
E) (-4, 8)
F) (-4,0), (-2, -32)

Calculus AB Ch. 3 Curve Sketching/Optimization Review WS #3 $\boxed{\text{Solutions}}$

Definitions:

Extreme Value Theorem(EVT): If a function $f(x)$ is continuous on a closed interval $[a, b]$ then $f(x)$ has both a abs. max and abs. min value on $[a, b]$

Mean Value Theorem(MVT): Let f be continuous on closed interval $[a,b]$ and differentiable on the open interval (a,b)

Then there is at least one point c in (a,b) where $f'(x) = \frac{f(b)-f(a)}{b-a}$

Rolle's Theorem: Let f be continuous on closed interval $[a,b]$ and differentiable on the open interval $(a,b.)$

If $f(a)=f(b)$, then there is at least one point c in (a,b) where $f'(c) = 0$.

1. $f(x) = \dfrac{2x}{x^2+1}$ on the closed interval [-2, 2]. Determine the maximum and minimum values on the graph.

f(x) continuous on $[-2,2]$

$f'(x) = \dfrac{2(x^2+1)-2x(2x)}{(x^2+1)^2} = \dfrac{2x^2+2-4x^2}{(x^2+1)^2}$

$f'(x) = \dfrac{2-2x^2}{(x^2+1)^2} = \dfrac{2(1-x^2)}{(x^2+1)^2}$

$x = 1, -1$

$f(2) = 0.8$

$f(-2) = -0.8$

$f(1) = 1$

$f(-1) = -1$

Abs max is 1 at $x = 1$

Abs. min is -1 at $x = -1$

2. Determine if Rolle's Theorem is satisfied for $f(x) = (x-3)(x+2)^2$ on the closed interval [-2, 3]. (Step through conditions!) If so, find c.

f(x) continuous on $[-2,3]$, differentiable on $(-2,3)$

$f(-2) = 0$

$f(3) = 0$

] Rolle's theorem applies

$f'(x) = (1)(x+2)^2 + (x-3)(2)(x+2)$

$= (x+2)[x+2+2(x-3)]$

$= (x+2)(x+2+2x-6)$

$0 = (x+2)(3x-4)$

$x = -2, 4/3$

$\cancel{c=-2}$

$\boxed{c = 4/3}$

3. Determine if MVT is satisfied for $f(x) = x(x^2-x-2)$ on the closed interval [-1,1]. (Step through conditions!) If so, find c.

f(x) continuous on $[-1,1]$, differentiable on $(-1,1)$

$f(-1) = 0$

$f(1) = -2$

$m_{Avg} = \dfrac{-2-0}{1--1} = \dfrac{-2}{2} = \boxed{-1}$

$f(x) = x^3-x^2-2x$

$f'(x) = 3x^2-2x-2$

*set $f'(x) = m_{Avg}$

$3x^2-2x-2 = -1$

$3x^2-2x-1 = 0$

$(3x+1)(x-1) = 0$

$x = -1/3, x = 1$

$\cancel{c=1}$

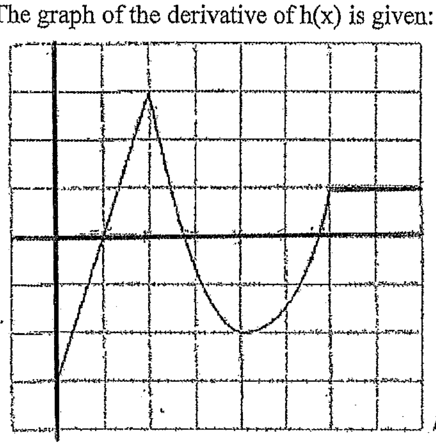
$\boxed{c = -1/3}$

4. The graph of the derivative of h(x) is given: Sketch a possible graph of h(x) (Create sign lines for f '(x) and f ''(x) first!)

\+ slope

slope=0

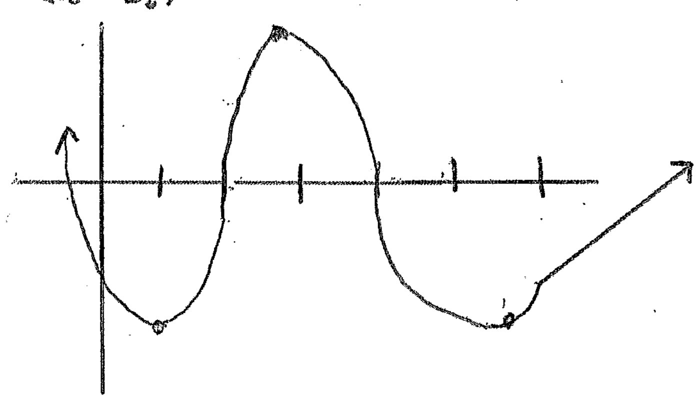
− slope



h'(x)

↘ − 1 ↗ + 2.8 ↘ − 5.7 ↗ +

∪ + 2 ∩ − 4 ∪ +



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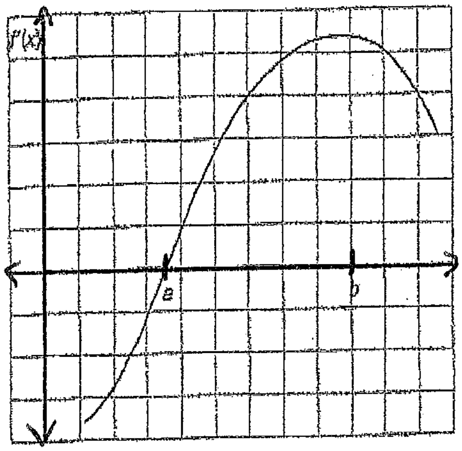
5. The derivative of f has a zero at $x = a$ and a relative maximum at $x = b$, as shown. Which of the following is not true?

+ slope

− slope

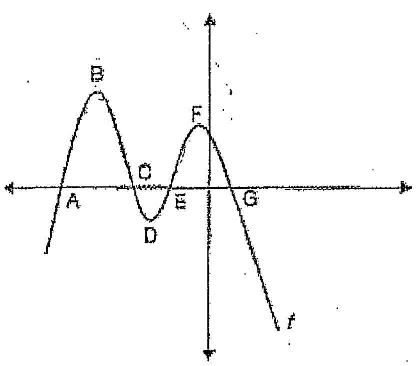


− +
a

+ −
b

A) f(x) has a relative minimum at x = a true
$\boxed{\text{B)}}$ f(x) has an absolute maximum at x = b false
C) f(x) is increasing on (a, b) true
D) f''(x) is positive on (a, b) true
E) f''(x) has a zero at x = b true

6. Which of the following statements are true of the graph of f below?



I. $f' \geq 0$ on the interval from D to F true
II. $f'' = 0$ at points B, D, and F false
III. $f'' > 0$ on the interval from A to B false
IV. $f'' > 0$ on the interval from D to F false

A. I and II
B. I and III
C. II and IV
D. II, III, and IV
E. I, II, III
$\boxed{\text{F. Just I}}$

7. Find the extrema of function $f(x) = \frac{1}{3}x^3 - 6x^2 + 35x - 1$

A) Absolute minimum at x = 0
B) Absolute maxima at x = 5, 7
C) Relative maximum at x = -5, relative minimum at x = -7
$\boxed{\text{D)}}$ Relative maximum at x = 5, relative minimum at x = 7
E) Relative maximum at x = 7, relative minimum at x = 5

$f'(x) = x^2 - 12x + 35$
$0 = (x-7)(x-5)$ $x = 5, 7$

+ − +
5 7

8. Find all critical points, c, for the function $f(x) = \frac{2}{3}x^3 + 5x^2 - 28x - 10$

A) c = 0, -7, -2
$\boxed{\text{B) c = -7, 2}}$
C) c = 0
D) c = -2,7
E) c = 10

$f'(x) = 2x^2 + 10x - 28$
$0 = 2(x^2 + 5x - 14)$
$0 = 2(x+7)(x-2)$
$x = -7, 2$

9. Find the inflection point(s) for the function $f(x) = 2x(x+4)^3$

A) (0,0)
B) (-4,0)
C) (0,0) , (-4,0)
D) (0,0) , (-4,0) , (4,0)
E) (-4, 8)
$\boxed{\text{F) (-4,0), (-2, -32)}}$

$f'(x) = 2(x+4)^3 + (2x)\cdot 3(x+4)^2(1)$
$= 2(x+4)^2[(x+4) + 3x]$
$2(x+4)^2(4x+4)$
$f'(x) = (x+4)^2(8x+8)$

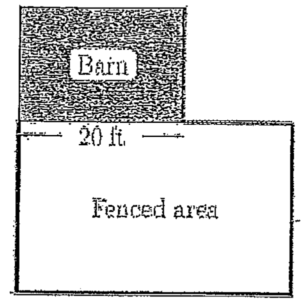
$f''(x) = 2(x+4)(8x+8) + (x+4)^2(8)$
$f''(x) = 8(x+4)[2(x+1) + x+4]$
$0 = 8(x+4)[2x+2+x+4]$
$0 = 8(x+4)[3x+6]$
$x = -4, -2$

− + +
-4 -2

$f(-4) = 0$, $f(-2) = -32$

Optimization Test Review Questions WS

10) A farmer wishes to erect a fence enclosing a rectangular area adjacent to a barn which is 20 feet long. The diagram illustrates his plan for the fenced area. Find the largest area that can be enclosed if 96 feet of fencing material is available. Justify your answer.



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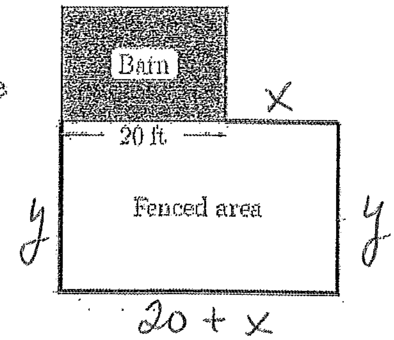
11) We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10 per square foot and the material used to build the sides cost \$6 per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box.

12) There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

Key (27)

10) A farmer wishes to erect a fence enclosing a rectangular area adjacent to a barn which is 20 feet long. The diagram illustrates his plan for the fenced area. Find the largest area that can be enclosed if 96 feet of fencing material is available. Justify your answer.



$* A = y(20+x)$

$P = x + 2y + 20 + x$

$P = 2x + 2y + 20$

$96 = 2x + 2y + 20$

$76 - 2x = 2y$

$\frac{76-2x}{2} = y$

$38 - x = y$

$A = (38-x)(20+x)$

$A = 760 - 20x + 38x - x^2$

$A = -x^2 + 18x + 760$

$A'(x) = -2x + 18$

$0 = -2x + 18$ $\quad 2x = 18$ $\quad \underline{x = 9}$ ft.

$y = 38 - x$

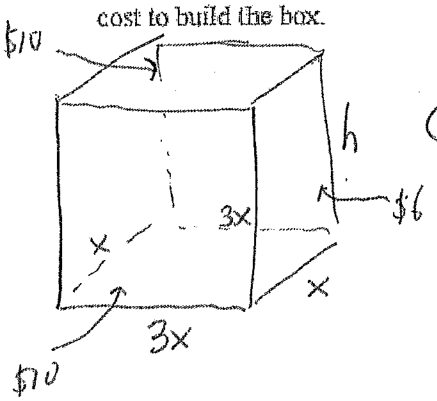
$y = 38 - 9 = 29$ ft

$Area_{(max)} = y(20+x) = 29(20+9)$

$\boxed{Area_{(max)} = 841 \text{ ft}^2}$

(28)

11) We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost $10 per square foot and the material used to build the sides cost $6 per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box.



$$C(x) = \underbrace{10(3x^2) + 10(3x^2)}_{\text{top and bottom}} + \underbrace{6xh + 6xh}_{\text{smaller vertical sides}} + \underbrace{6(3xh) + 6(3xh)}_{\text{larger vertical sides}}$$

$$C(x) = 60x^2 + 12xh + 36xh$$

$$C(x) = 60x^2 + 48xh$$

Volume $= 3x \cdot x \cdot h$

$V = 3x^2h$

$50 = 3x^2h$

$\frac{50}{3x^2} = h$

$$C(x) = 60x^2 + 48x\left(\frac{50}{3x^2}\right) = 60x^2 + 800x^{-1}$$

$$C'(x) = 120x - 800x^{-2}$$

$$0 = 120x - \frac{800}{x^2}$$

$$\frac{800}{x^2} = \frac{120x}{1} \qquad 120x^3 = 800 \qquad x^3 = \frac{20}{3}$$

$$x \approx 1.882$$

$$h \approx \frac{50}{3(1.882)^2} = 4.706$$

$$\boxed{\begin{array}{l}\text{Dimensions: } 5.646\text{ ft} \times 1.882\text{ ft} \times 4.706\text{ ft.} \\ \text{Minimum cost: } \$637.64\end{array}}$$

12) There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

$P = (\#\text{ of trees})(\text{apple output per tree})$

$P = (50 + x)(800 - 10x)$

$P = 40{,}000 + 300x - 10x^2$

$P'(x) = 300 - 2x$

$0 = 300 - 2x$

$0 = 20(15 - x)$

$x = 15$ trees

15 trees should be added to total 65 trees