Calculus AB Ch. 3 Curve Sketching/Optimization Review WS #3

Definitions:

Extreme Value Theorem(EVT): If a function *f(x)* is ________________________ on a ________________________ interval *a, b* then

f(x) has both a ________________________ and ________________________ value on *a, b*

Mean Value Theorem(MVT): Let f be ________________________ on ________________________ interval a,b

and ________________________ on the ________________________ interval a,b

Then there is at least one point c in a,b where ________________________

Rolle's Theorem: Let f be ________________________ on ________________________ interval a,b

and ________________________ on the ________________________ interval a,b.

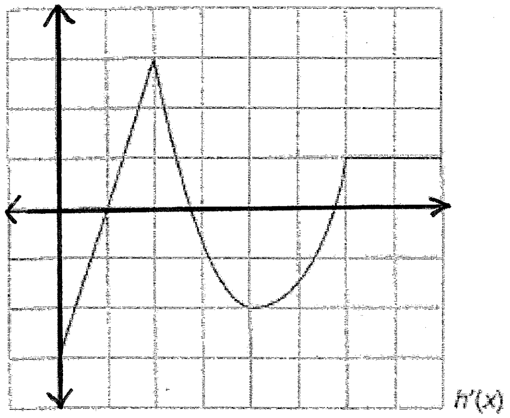
If ________________________ then there is at least one point c in a,b where ________________________

1. $f(x) = \frac{2x}{x^2+1}$ on the closed interval [-2, 2]. Determine the maximum and minimum values on the graph.

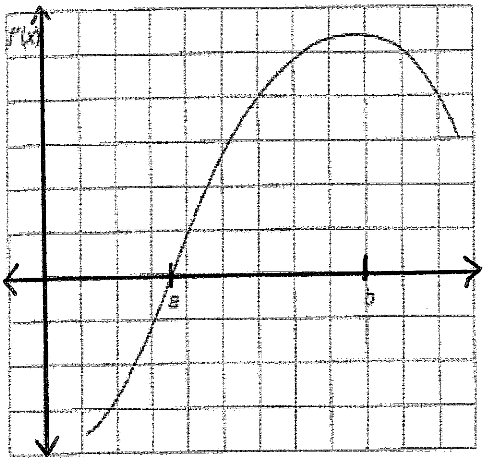
2. Determine if Rolle's Theorem is satisfied for $f(x) = (x - 3)(x + 2)^2$ on the closed interval [-2, 3]. (Step through conditions!) If so, find c.

3. Determine if MVT is satisfied for $f(x) = x(x^2 - x - 2)$ on the closed interval [-1,1]. (Step through conditions!) If so, find c.

4. The graph of the derivative of h(x) is given: Sketch a possible graph of h(x) (Create sign lines for f ‘(x) and f ‘’(x) first!)

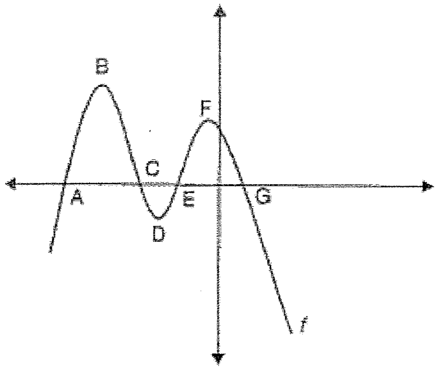


5. The derivative of f has a zero at $x = a$ and a relative maximum at $x = b$, as shown. Which of the following is not true?



A) f(x) has a relative minimum at x = a
B) f(x) has an absolute maximum at x = b
C) f(x) is increasing on (a, b)
D) f ''(x) is positive on (a, b)
E) f ''(x) has a zero at x = b

6. Which of the following statements are true of the graph of f below?



I. $f' \geq 0$ on the interval from D to F
II. $f'' = 0$ at points B, D, and F
III. $f'' > 0$ on the interval from A to B
IV. $f'' > 0$ on the interval from D to F

A. I and II
B. I and III
C. II and IV
D. II, III, and IV
E. I, II, III
F. Just I

7. Find the extrema of function f(x) = $\frac{1}{3}x^3 - 6x^2 + 35x - 1$

A) Absolute minimum at x = 0
B) Absolute maxima at x = 5, 7
C) Relative maximum at x = -5, relative minimum at x = -7
D) Relative maximum at x = 5, relative minimum at x = 7
E) Relative maximum at x = 7, relative minimum at x = 5

8. Find all critical points, c, for the function f(x) = $\frac{2}{3}x^3 + 5x^2 - 28x - 10$

A) c = 0, -7, -2
B) c = -7, 2
C) c = 0
D) c = -2,7
E) c = 10

9. Find the inflection point(s) for the function f(x) = $2x(x+4)^3$

A) (0,0)
B) (-4,0)
C) (0,0) , (-4,0)
D) (0,0) , (-4,0) , (4,0)
E) (-4, 8)
F) (-4,0), (-2, -32)

Calculus AB Ch. 3 Curve Sketching/Optimization Review WS #3 Solutions

Definitions:

Extreme Value Theorem(EVT): If a function *f(x)* is continuous on a closed interval [a, b] then *f(x)* has both a abs. max and abs. min value on [a, b]

Mean Value Theorem(MVT): Let f be continuous on closed interval [a,b]

and differentiable on the open interval (a,b)

Then there is at least one point c in (a,b) where $f'(x) = \frac{f(b)-f(a)}{b-a}$

Rolle's Theorem: Let f be Continuous on closed interval [a,b]

and differentiable on the open interval (a,b.)

If $f(a)=f(b)$, then there is at least one point c in (a,b) where $f'(c)=0$

1. $f(x) = \frac{2x}{x^2+1}$ on the closed interval [-2, 2]. Determine the maximum and minimum values on the graph.

f(x) continuous on [-2,2]

$f'(x) = \frac{2(x^2+1)-2x(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$f'(x) = \frac{2-2x^2}{(x^2+1)^2} = \frac{2(1-x^2)}{(x^2+1)^2}$

$x=1,-1$

$f(2)=0.8$

$f(-2)=-0.8$

$f(1)=1$

$f(-1)=-1$

Abs max is 1 at $x=1$

Abs. min is -1 at $x=-1$

2. Determine if Rolle's Theorem is satisfied for f(x) = (x – 3)(x + 2)^2 on the closed interval [-2, 3]. (Step through conditions!) If so, find c.

f(x) continuous on [-2,3], differentiable on (-2,3)

$f(-2)=0$

$f(3)=0$

] Rolle's theorem applies

$f'(x)=(1)(x+2)^2+(x-3)(2)(x+2)$

$=(x+2)[x+2+2(x-3)]$

$=(x+2)(x+2+2x-6)$

$0=(x+2)(3x-4)$

$x=-2, 4/3$

~~c=-2~~

$\boxed{c=4/3}$

3. Determine if MVT is satisfied for f(x) = x(x^2 – x – 2) on the closed interval [-1,1]. (Step through conditions!) If so, find c.

f(x) continuous on [-1,1], differentiable on (-1,1)

$f(-1)=0$

$f(1)=-2$

$m_{Avg} = \frac{-2-0}{1--1} = \frac{-2}{2} = \boxed{-1}$

$f(x)=x^3-x^2-2x$

$f'(x)=3x^2-2x-2$

*set $f'(x)=m_{Avg}$

$3x^2-2x-2=-1$

$3x^2-2x-1=0$

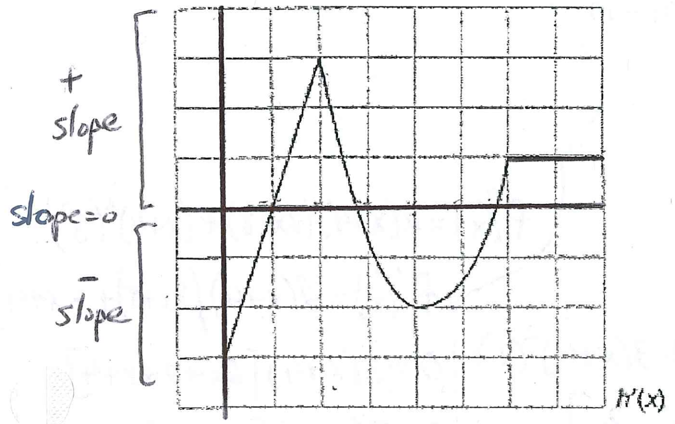
$(3x+1)(x-1)=0$

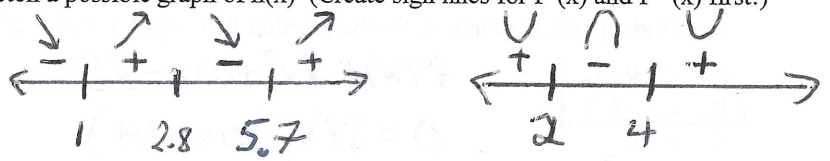
$x=-1/3, x=1$

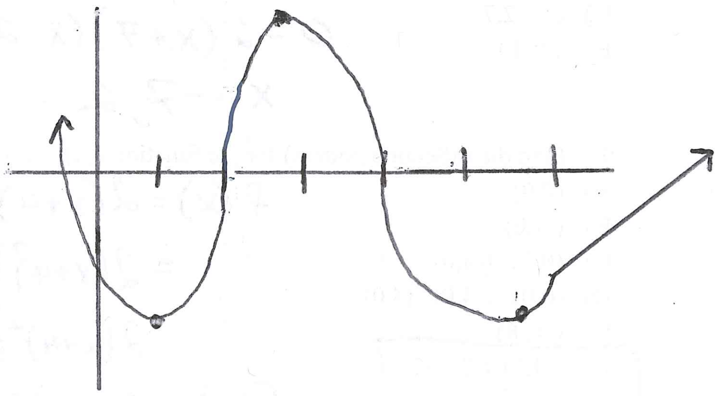
~~c=1~~

$\boxed{c=-1/3}$

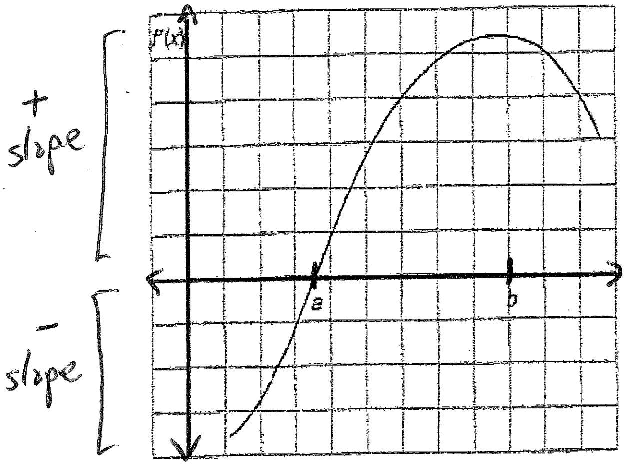
4. The graph of the derivative of h(x) is given: Sketch a possible graph of h(x) (Create sign lines for f '(x) and f ''(x) first!)

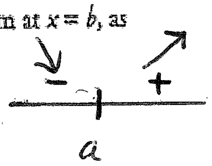


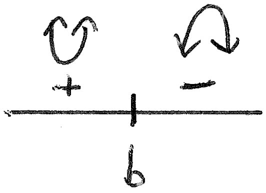




5. The derivative of f has a zero at $x = a$ and a relative maximum at $x = b$, as shown. Which of the following is not true?







A) f(x) has a relative minimum at x = a true

[✓ B)] f(x) has an absolute maximum at x = b false

C) f(x) is increasing on (a, b) true

D) f ''(x) is positive on (a, b) true

E) f ''(x) has a zero at x = b true

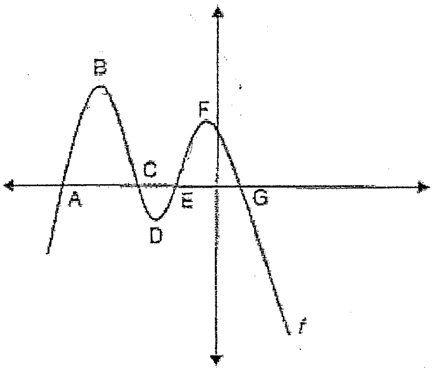
6. Which of the following statements are true of the graph of f below?

I. $f' \geq 0$ on the interval from D to F true

II. $f'' = 0$ at points B, D, and F false

III. $f'' > 0$ on the interval from A to B false

IV. $f'' > 0$ on the interval from D to F false



A. I and II

B. I and III

C. II and IV

D. II, III, and IV

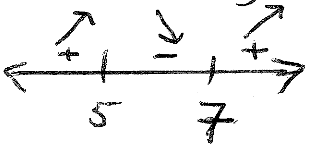
E. I, II, III

[✓ F. Just I]

7. Find the extrema of function f(x) = $\frac{1}{3}x^3 - 6x^2 + 35x - 1$

$f'(x) = x^2 - 12x + 35$

$0 = (x-7)(x-5) \quad x = 5, 7$



A) Absolute minimum at x = 0

B) Absolute maxima at x = 5, 7

C) Relative maximum at x = -5, relative minimum at x = -7

[✓ D)] Relative maximum at x = 5, relative minimum at x = 7

E) Relative maximum at x = 7, relative minimum at x = 5

8. Find all critical points, c, for the function f(x) = $\frac{2}{3}x^3 + 5x^2 - 28x - 10$

$f'(x) = 2x^2 + 10x - 28$

$0 = 2(x^2 + 5x - 14)$

$0 = 2(x+7)(x-2)$

$x = -7, 2$

A) c = 0, -7, -2

[✓ B) c = -7, 2]

C) c = 0

D) c = -2,7

E) c = 10

9. Find the inflection point(s) for the function f(x) = 2x(x+4)^3

$f'(x) = 2(x+4)^3 + (2x)\cdot 3(x+4)^2(1)$

$= 2(x+4)^2[(x+4) + 3x]$

$2(x+4)^2(4x+4)$

$f'(x) = (x+4)^2(8x+8)$

$f''(x) = 2(x+4)(8x+8) + (x+4)^2(8)$

$f''(x) = 8(x+4)[2(x+1) + x+4]$

$0 = 8(x+4)[2x+2+x+4]$

$0 = 8(x+4)[3x+6]$

$x = -4, -2$

$f(-4) = 0, \ f(-2) = -32$

A) (0,0)

B) (-4,0)

C) (0,0) , (-4,0)

D) (0,0) , (-4,0) , (4,0)

E) (-4, 8)

[✓ F) (-4,0), (-2, -32)]