Key

A.P. Calculus AB 2.2-2.5 Review Session Problems (WS #4)

1) Consider the curve given by $x^2 - xy + y^2 = 4$.

a) Find the two points on the curve at $x = 0$

$(0)^2 - (0)y + y^2 = 4$

$y^2 = 4$

$\sqrt{y^2} = \pm\sqrt{4}$

$y = 2, -2$

$\boxed{\text{The 2 points are } (0,2) \text{ and } (0,-2)}$

b) Find $\frac{dy}{dx}$ by differentiating implicitly.

*product rule

$x^2 - (\underbrace{x}_{f}\underbrace{y}_{g}) + y^2 = 4$

$2x - \left(\underbrace{(1)}_{f'}\underbrace{(y)}_{g} + \underbrace{(x)}_{f}\underbrace{(1)\left(\frac{dy}{dx}\right)}_{g'}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$2x - y - x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$-x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = -2x + y$

$\frac{dy}{dx}(-x + 2y) = -2x + y$

$\boxed{\frac{dy}{dx} = \frac{-2x+y}{-x+2y}}$

c) Use $\frac{dy}{dx}$ to find the slope of the lines tangent to the curve at the points found in part a.

i) point: $(0,2)$

slope: $\left.\frac{dy}{dx}\right|_{(0,2)} = \frac{-2(0)+2}{-0+2(2)} = \frac{2}{4}$

$\boxed{\text{slope: } m = 1/2}$

$\boxed{y - 2 = \frac{1}{2}(x - 0)}$

ii) point: $(0,-2)$

slope: $\left.\frac{dy}{dx}\right|_{(0,-2)} = \frac{-2(0)-2}{-0+2(-2)} = \frac{-2}{-4}$

$\boxed{m = 1/2}$

$\boxed{y + 2 = \frac{1}{2}(x - 0)}$

d) Write the equation of the line tangent to the curve at the points above

$\boxed{y - 2 = \frac{1}{2}(x - 0)}$ $\boxed{y + 2 = \frac{1}{2}(x - 0)}$

e) Detail how you would set up (don't solve) in order to find vertical and horizontal tangent:

horizontal tangent: set numerator of $\frac{dy}{dx} = 0 \rightarrow \boxed{-2x + y = 0}$

vertical tangent: set denominator of $\frac{dy}{dx} = 0 \rightarrow \boxed{-x + 2y = 0}$

2) If g is differentiable everywhere and $g(x)=\begin{cases}6x^3-8x^2+8, & x<-2\\ ax+b, & x\ge -2\end{cases}$, find a and b

(Involve derivatives in your work)

continuous property: set equations equal (at $x=-2$)

$6x^3-8x^2+8=ax+b$

$6(-2)^3-8(-2)^2+8=a(-2)+b$

$-72=-2a+b$

$-72=-2(104)+b$

$\boxed{136=b}$

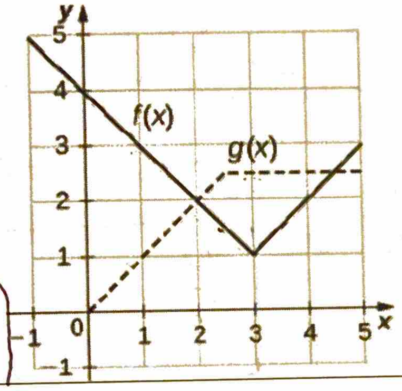
differentiable property: set derivatives equal (at $x=-2$)

$18x^2-16x=a$

$18(-2)^2-16(-2)=a$

$\boxed{a=104}$

3)



a) If $h(x)=f(x)\cdot g(x)$, find $h'(2)$ * product rule

$h'(x)=f'(x)g(x)+f(x)g'(x)$

$h'(2)=f'(2)g(2)+f(2)g'(2)$

$h'(2)=(-1)(2)+(2)(1)$

$\boxed{h'(2)=0}$

b) If $w(x)=f(g(x))$, find $w'(3)$ * chain rule

$w'(x)=f'[g(x)]\cdot g'(x)$

$w'(3)=f'[g(3)]\cdot g'(3)$

$w'(3)=f'[2.5]\cdot g'(3)$

$w'(3)=(-1)(0)$

$\boxed{w'(3)=0}$

* chain rule

out: $2[\ \ ]^3$

in: $f(x)$

c) If $z(x)=2[f(x)]^3$ find $z'(0)$

$z'(x)=6(\ \ )^2\cdot f'(x)$

$z'(x)=6[f(x)]^2\cdot f'(x)$

$z'(0)=6[f(0)]^2\cdot f'(0)$

$z'(0)=6[4]^2\cdot(-1)$

$\boxed{z'(0)=-96}$

d) If $k(x)=f(f(x))$, find $k'(4)$ ← chain rule

$k'(x)=f'[f(x)]\cdot f'(x)$

$k'(4)=f'[f(4)]\cdot f'(4)$

$k'(4)=f'[2]\cdot f'(4)$

$k'(4)=(-1)(1)$

$\boxed{k'(4)=-1}$

e) If $p(x)=\frac{g(x)}{f(x)}$ find $p'(1)$ * quotient rule

$p'(x)=\dfrac{g'(x)f(x)-g(x)f'(x)}{[f(x)]^2}$

$p'(1)=\dfrac{g'(1)f(1)-g(1)f'(1)}{[f(1)]^2}$

$p'(1)=\dfrac{(1)(3)-(1)(-1)}{3^2}$

$p'(1)=\dfrac{3+1}{9}=\boxed{\dfrac{4}{9}}$

4) Find $\frac{dy}{dx}$ for $y=2\left(\frac{5x^3-2}{3-x^2}\right)^7$ (Write derivative as a simplified rational expression)

$45x^2-15x^4+10x^4-4x$

* i) chain rule

ii) quotient rule

out: $2[\ \ ]^7$

in: $\dfrac{5x^3-2}{3-x^2}$

quotient rule

$y'=14\left[\dfrac{5x^3-2}{3-x^2}\right]^6\cdot\left[\dfrac{\underbrace{15x^2}_{f'}\underbrace{(3-x^2)}_{g}-\underbrace{(5x^3-2)}_{f}\underbrace{(-2x)}_{g'}}{\underbrace{(3-x^2)^2}_{g^2}}\right]$

$y'=14\left[\dfrac{5x^3-2}{3-x^2}\right]^6\left[\dfrac{-5x^4+45x^2-4x}{(3-x^2)^2}\right]$

$\boxed{y'=\dfrac{14(5x^3-2)^6(-5x^4+45x^2-4x)}{(3-x^2)^8}}$

5) A particle moves along a straight line according to the given equation: $x(t) = \frac{t^4}{4} - t^3 - 2t^2 + 1$, for all real numbers in **meters per minute**

a) Find the velocity and acceleration function

$v(t) = t^3 - 3t^2 - 4t$

$a(t) = 3t^2 - 6t - 4$

b.) Find when the particle changes direction (justify with because statement)

$0 = t(t^2 - 3t - 4)$

$0 = t(t-4)(t+1)$

$t = 0, -1, 4$

$v(t)$

$-$ $+$ $-$ $+$

-2 -1 $-1/2$ 0 1 4 5

Direction change at $t = -1, 0, 4$ b/c $v(t)$ change signs

c) Determine interval when particle is moving left (justify with because statement)

particle moves left $(-\infty, -1) \cup (0, 4)$ b/c $v(t) < 0$

d) Determine interval when particle is moving left (justify with because statement)

particle moves right $(-1, 0) \cup (4, \infty)$ b/c $v(t) > 0$

e) Find the average velocity of particle in interval [1, 3] (show work, include units)

Avg. velocity $= \frac{\text{change in position}}{\text{change in time}} \rightarrow \frac{x(3) - x(1)}{3 - 1}$

$x(3) = -23.75$

$x(1) = -1.75$

$\frac{-23.75 - (-1.75)}{3 - 1} = \boxed{-11 \text{ meters/min}}$

f) Find the average acceleration of particle in interval [1, 3] (show work, include units)

Avg. acceleration $= \frac{\text{change in velocity}}{\text{change in time}} \rightarrow \frac{v(3) - v(1)}{3 - 1}$

$v(3) = -12$

$v(1) = -6$

$\frac{-12 - (-6)}{3 - 1} = \boxed{-3 \text{ meters/min}^2}$

g) Find particle's displacement from t = 1 to t = 6 (Show your work)

displacement = final position − initial position

displacement $= x(6) - x(1)$

$= 37 - (-1.75)$

$x(6) = 37$

$x(1) = -1.75$

$\boxed{\text{displacement is } 38.75\text{m}}$

h) Find particle's distance from t = 1 to t = 6 (Show your work) *count the distance of endpoints to locations of direction change

(*direction change at $t = 4$)

$x(1) = -1.75$

$x(4) = -31$

$x(6) = 37$

$x(t)$

-31 -1.75 37

29.25

68

$\boxed{\text{Total distance is } 29.25 + 68 = 97.25\text{m}}$

i) At t = 2, is the speed increasing or decreasing? Provide justification for your answer.

$v(2) = -12$ $a(2) = -4$

$\boxed{\text{Speed is increasing since } v(t) \text{ and } a(t) \text{ have same signs at } t = 2}$

j) At t = 5, is the velocity increasing or decreasing? Provide justification for your answer.

$a(5) = 41$

*This phrasing is describing acceleration, not velocity

$\boxed{\text{Velocity is increasing at } t = 5 \text{ since acceleration is positive}}$