1) $\displaystyle\int \frac{x^2+\sqrt{x^5}+1}{3\sqrt{x}}\,dx$

$\displaystyle\int \frac{(x^2+\sqrt{x^5}+1)}{3}x^{-1/2}$

Ch. 4 Test Review WS#2 Key

$$\int \frac{x^2}{3\sqrt{x}}+\frac{\sqrt{x^5}}{3\sqrt{x}}+\frac{1}{3\sqrt{x}}\,dx$$

$\sqrt{x^5} \to \cancel{x^{1/5}} \to x^{5/2}$

$\sqrt{x} = x^{1/2}$

$\sqrt[2]{x^1} = x^{1/2}$

$$\int \frac{x^2}{3x^{1/2}}+\frac{x^{5/2}}{3x^{1/2}}+\frac{1}{3x^{1/2}}\,dx$$

$$\int \frac{x^2x^{-1/2}}{3}+\frac{x^{5/2}x^{-1/2}}{3}+\frac{x^{-1/2}}{3}\,dx$$

$$\int \frac{1}{3}x^{3/2}+\frac{1}{3}x^2+\frac{1}{3}x^{-1/2}\,dx$$

$$\frac{1}{3}\cdot\frac{x^{5/2}}{5/2}+\frac{1}{3}\cdot\frac{x^3}{3}+\frac{1}{3}\cdot\frac{x^{1/2}}{1/2}+C$$

$$\boxed{\frac{2}{15}x^{5/2}+\frac{1}{9}x^3+\frac{2}{3}x^{1/2}+C}$$

2) $\int 2x\sqrt{1-3x^2}\,dx$

$\int 2x\left(\underline{1-3x^2}\right)^{1/2}\underline{dx}$

* u-sub

1) parentheses

$u = 1-3x^2$

$\frac{du}{dx} = -6x$

$du = -6x\,dx$

$\frac{du}{-6x} = dx$

$\int 2\cancel{x} \cdot u^{1/2} \cdot \frac{du}{-6\cancel{x}}$

$-\frac{1}{3}\int u^{1/2}\,du$

$-\frac{1}{3} \cdot \frac{u^{3/2}}{3/2} + C$

$-\frac{1}{3} \cdot \frac{2}{3}u^{3/2} + C$

$-\frac{2}{9}u^{3/2} + C$

$\boxed{-\frac{2}{9}\left(1-3x^2\right)^{3/2} + C}$

3) $\int 5\sqrt{x}(4-3x^2)\,dx$

*expand, using power rule

$\int 5x^{1/2}(4-3x^2)\,dx$

$\int 20x^{1/2} - 15x^{5/2}\,dx$

$20 \cdot \frac{x^{3/2}}{3/2} - 15 \cdot \frac{x^{7/2}}{7/2} + C$

$\boxed{\frac{40}{3}x^{3/2} - \frac{30}{7}x^{7/2} + C}$

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4) $\int 5x\sec^2(3x^2)\,dx$

$u = 3x^2$

$\frac{du}{dx} = 6x$

$du = 6x\,dx$

$\frac{du}{6x} = dx$

$\int 5x\sec^2 u \cdot \frac{du}{6x}$

$\frac{5}{6}\int \sec^2 u\,du$

$\frac{5}{6}\tan u + C$

$\boxed{\frac{5}{6}\tan(3x^2) + C}$

5) $\int x^2\sqrt{7-x}\,dx$

$$\int x^2(7-x)^{1/2}\,dx$$

$u=7-x$

$\frac{du}{dx}=-1$

$du=-1dx$

$-du=dx$

$$\int x^2\cdot u^{1/2}\cdot -du$$

solve for x: $x=\boxed{7-u}$

$$\int (7-u)^2 u^{1/2}\cdot -du$$

$$\int -u^{1/2}(7-u)^2\,du$$

$(7-u)(7-u)$

$$\int -u^{1/2}(49-14u+u^2)\,du$$

$$\int -49u^{1/2}+14u^{3/2}-u^{5/2}\,du$$

$$-\frac{49u^{3/2}}{3/2}+\frac{14u^{5/2}}{5/2}-\frac{u^{7/2}}{7/2}+C$$

$$-\frac{98}{3}u^{3/2}+\frac{28}{5}u^{5/2}-\frac{2}{7}u^{7/2}+C$$

$$\boxed{-\frac{98}{3}(7-x)^{3/2}+\frac{28}{5}(7-x)^{5/2}-\frac{2}{7}(7-x)^{7/2}+C}$$

6) $\int_1^2 x(1-2x^2)^3\,dx$

$\int x(1-2x^2)^3\,dx$

$u = 1-2x^2$

$\frac{du}{dx} = -4x$

$du = -4x\,dx$

$\frac{du}{-4x} = dx$

$\int \cancel{x}\cdot u^3 \cdot \frac{du}{-4\cancel{x}}$

$-\frac{1}{4}\int u^3\,du$

$-\frac{1}{4}\cdot\frac{u^4}{4} \rightarrow -\frac{1}{16}u^4$

Method 1

convert bounds:

If $x=1$, $u=1-2x^2 \rightarrow u=-1$

If $x=2$, $u=1-2x^2 \rightarrow u=-7$

$$-\frac{1}{16}u^4\Big]_{-1}^{-7} = -\frac{1}{16}(-7)^4 - -\frac{1}{16}(-1)^4 = \boxed{-150}$$

Method 2:

$$-\frac{1}{16}(1-2x^2)^4\Big]_1^2 = -\frac{1}{16}(1-8)^4 - -\frac{1}{16}(1-2)^4 = \boxed{-150}$$

7) $\int \frac{\sin x}{\cos^5 x}dx$

u-sub

$\int \frac{\sin x}{(\cos x)^5}dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

$\frac{du}{-\sin x} = dx$

$-\int \frac{1}{u^5}du$

$\int \frac{\cancel{\sin x}}{u^5} \cdot \frac{du}{-\cancel{\sin x}}$

$-\int u^{-5}du$

$-\frac{u^{-4}}{-4} + C$

$\frac{1}{4(u^4)} + C$

$\boxed{\frac{1}{4(\cos x)^4} + C}$