# 5.1a Derivatives of Natural Logs

p.329-330 #7-15 odds, 37, 39, 47, 49, 53, 55, 59, 71, 77

47) $y = (\ln x)^4 \qquad y' = 4(\ln x)^3\left(\frac{1}{x}\right) = \frac{4(\ln x)^3}{x}$

49) $y = \ln x \sqrt{x^2-1}$

$y = \ln x + \ln(x^2-1)^{1/2} \qquad y = \ln x + \frac{1}{2}\ln(x^2-1)$

$y' = \frac{1}{x} + \frac{1}{2}\left(\frac{2x}{x^2-1}\right) = \boxed{\frac{1}{x} + \frac{x}{x^2-1}}$

53) $g(t) = \frac{\ln t}{t^2} \qquad g'(t) = \frac{\left(\frac{1}{t}\right)(t^2) - (\ln t)(2t)}{t^4}$

$g'(t) = \frac{t - 2t\ln t}{t^4} = \frac{t(1-2\ln t)}{t^4} = \boxed{\frac{1-2\ln t}{t^3}}$

55) $y = \ln(\ln x^2) = \ln(2\ln x)$

$y' = \frac{2\left(\frac{1}{x}\right)}{2\ln x} = \boxed{\frac{1}{x\ln x}}$

59) $f(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right) = \ln\sqrt{4+x^2} - \ln x$

$f(x) = \ln(4+x^2)^{1/2} - \ln x$

$f(x) = \frac{1}{2}\ln(4+x^2) - \ln x$

$f'(x) = \frac{1}{2}\left(\frac{2x}{4+x^2}\right) - \frac{1}{x}$

$= \boxed{\frac{x}{4+x^2} - \frac{1}{x}}$

71) Find equation of tangent line

$y = 3x^2 - \ln x$, $(1, 3)$

$\frac{dy}{dx} = 6x - \frac{1}{x}$

$\frac{dy}{dx}_{(1,3)} = 6(1) - \frac{1}{1} = 5$

point: $(1, 3)$

slope: $m = 5$

$\boxed{y - 3 = 5(x-1)}$

77) $x^2 - 3\ln y + y^2 = 10$ ~~*~~ implicit differentiation

$2x - 3\left(\frac{1}{y}\right)\frac{dy}{dx} + 2y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}\left(2y - \frac{3}{y}\right) = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y - \frac{3}{y}} \cdot \frac{(y)}{(y)} = \boxed{\frac{-2xy}{2y^2 - 3}}$