Calculus Ch. 5.1b, 5.3 Notes

Warm-up Problem: If $g(2) = 3$ and $g'(2) = -4$, find $f'(2)$ if $f(x) = x^2\ln(g(x))$.

Logarithmic Differentiation : Simplifying non-log functions using log properties to expand before differentiating.

Log differentiation steps:

1. Take the **ln** (natural log) of both sides
2. Simplify and expand using log properties
3. Use implicit differentiation
4. Substitute for y

Example 1: Find the derivative of $y = \dfrac{(x-2)^2}{\sqrt{x^2+1}}$

Example 2: Find the derivative of $y = x^{2x+3}$

Absolute Value Rule: $\dfrac{d}{dx}\ln|u| = \dfrac{u'}{u}$

Example 3: Find $\frac{d}{dx}\ln|x^2-5|$

Example 4: Locate any relative extrema and inflection points for $y = \dfrac{x}{\ln x}$

Ch.5.3 Notes Derivatives of Inverse Function

A. Inverse Functions
1) x's and y's are swapped
2) Domains and ranges are swapped
3) Graphs are symmetric over the line y = x
4) F(x) must be one-to-one (pass the horizontal line test) for its inverse to be a function
5) A function that is **monotonic** (always increasing or always decreasing) will always have an inverse that is a function
6) If f and g are inverses, then f(g(x)) = x and g(f(x)) = x

Example 4: find the inverse of f(x) = 6x + 2

*ALWAYS restrict the domain of the inverse function to the range of the original function

Ex. 5 f(x) = $\sqrt{x-5}$. Find the domain of the inverse function

Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$

$f\,(b) = a$	$(f^{-1})(a) = b$
$f\,'(b) = n$	$(f^{-1})'(a) = \frac{1}{n}$

Example 6: f(x) = $x^3 + 4x + 2$ find $(f^{-1})'(-3)$

Example 7: f(x) = $\sqrt{x^3 - 7}$ find $(f^{-1})'(1)$

Example 8: If g(f(x)) = x , g(7) = 2, and g'(7) = 10, then f'(2) is

Example 9: If g(f(x)) = x , g(9) = 3, and g'(9) = -4, then f'(3) is

Calculus Ch. 5.1b, 5.3 Notes

Recall $\frac{d}{dx}\ln u = \frac{u'}{u}$

Warm-up Problem: If g(2) = 3 and g ' (2) = -4, find f ' (2) if f(x) = x^2ln(g(x)).

$$f'(x) = 2x \cdot \ln(g(x)) + 2^2 \cdot \frac{g'(x)}{g(x)}$$

$$f'(2) = 2(2) \cdot \ln[g(2)] + 2^2 \cdot \frac{g'(2)}{g(2)}$$

$$f'(2) = 4 \cdot \ln(3) + 2^2(-4/3)$$

$$\boxed{f'(2) = 4\ln 3 - 16/3}$$

Logarithmic Differentiation : Simplifying non-log functions using log properties to expand before differentiating.

Log differentiation steps:

1. Take the **ln** (natural log) of both sides
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4. Substitute for y

Example 1: Find the derivative of $y = \dfrac{(x-2)^2}{\sqrt{x^2+1}}$

$$\ln y = \ln\left[\frac{(x-2)^2}{(x^2+1)^{1/2}}\right]$$

$$\ln y = \ln(x-2)^2 - \ln(x^2+1)^{1/2}$$

$$\ln y = 2\ln(x-2) - \tfrac{1}{2}\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = 2\left(\frac{1}{x-2}\right) - \frac{1}{2}\left(\frac{2x}{x^2+1}\right)$$

$$\frac{dy}{dx} = y\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]$$

$$\frac{dy}{dx} = \frac{(x-2)^2}{\sqrt{x^2+1}}\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]$$

Example 2: Find the derivative of y = x^{2x+3}

$$\ln y = \ln x^{2x+3}$$

$$\ln y = (2x+3)(\ln x)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = (2)(\ln x] + (2x+3)\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = y\left[2\ln x + \frac{2x+3}{x}\right]$$

$$\frac{dy}{dx} = x^{2x+3}\left[2\ln x + 2 + \frac{3}{x}\right]$$

Absolute Value Rule: $\dfrac{d}{dx}\ln|u| = \dfrac{u'}{u}$

Example 3: Find $\frac{d}{dx}\ln|x^2-5|$ $= \dfrac{2x}{x^2-5}$

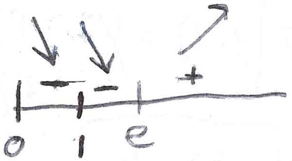
Example 4: Locate any relative extrema and inflection points for $y = \dfrac{x}{\ln x}$

$$y'(x) = \frac{1\ln x - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2} \rightarrow$$

set $\ln x - 1 = 0$

$\ln x = 1$

$e^1 = x$



Rel. min at $\left(e, \frac{e}{\ln e}\right)$

$= (e, e)$

$$y''(x) = \frac{\left(\frac{1}{x}\right)(\ln x)^2 - (\ln x - 1)2(\ln x)\left(\frac{1}{x}\right)}{(\ln x)^4}$$

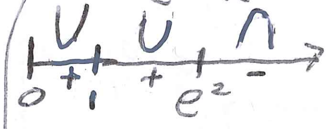
$$= \frac{\frac{1}{x}\ln x[\ln x - 2\ln x + 2]}{(\ln x)^4}$$

$$y''(x) = \frac{-\ln x + 2}{x(\ln x)^3}$$

$2 - \ln x = 0$

$2 = \ln x$

$e^2 = x$



POI at $\left(e^2, \frac{e^2}{2}\right)$ b/c $y''(x)$ change signs.

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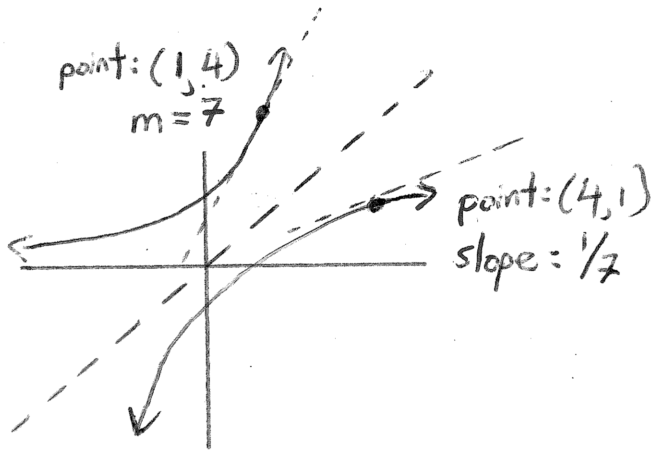

*At their corresponding points, the slopes of tangent line will be reciprocals of each other

Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$

$f\,(b) = a$	$(f^{-1})(a) = b$
$f\,'(b) = n\cdot$	$(f^{-1})'(a) = \frac{1}{n}$

Example 6: f(x) = x^3 + 4x + 2 find $(f^{-1})'(-3)$ $g'(-3)$

$f(-1)=-3$	$g(-3)=\underline{-1}$
$f'(-1)=7$	$g'(3)=1/7$

$-3=x^3+4x+2$
$x=-1$

$f'(x)=3x^2+4$
$f'(-1)=3(-1)^2+4$
$=3+4=7$

$\boxed{g'(3)=1/7}$

Example 7: f(x) = $\sqrt{x^3-7}$ find $(f^{-1})'(1)$ Find $g'(1)$

$f(2)=1$	$g(1)=\underline{2}$
$f'(2)=6$	$g'(1)=1/6$

$1=\sqrt{x^3-7}$
$1=x^3-7 \quad x^3=8, x=2$

$f(x)=(x^3-7)^{1/2}$
$f'(x)=\frac{1}{2}(x^3-7)^{-1/2}(3x^2)$
$=\frac{3x^2}{2\sqrt{x^3-7}}$

$f'(2)=\frac{12}{2(1)} \longleftarrow$

Example 8: If g(f(x)) = x , g(7) = 2, and g'(7) = 10, then f'(2) is

$g(7)=2$	$f(2)=7$
$g'(7)=10$	$f'(2)=\frac{1}{10}$

$\boxed{f'(2)=1/10}$

Example 9: If g(f(x)) = x , g(9) = 3, and g'(9) = -4, then f'(3) is

$g(9)=3$	$f(3)=9$
$g'(9)=-4$	$f'(3)=\frac{-1}{4}$

$\boxed{f'(3)=-1/4}$