**Ch.5.4 Notes** **Derivative of Exponential Function $e^x$**

$y = \ln x$ and $y = e^x$ are inverse functions (meaning $f(g(x)) = x$ and $g(f(x)) = x$ )

Example 1: Solve $7 = e^{x+1}$

Example 2: solve $\ln(2x - 3) = 5$

Reminder: exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ Reminder: e is a NUMBER: if $y = e^2$, then $y' = 0$

Exponential Function $e^x$ Derivative rule $\frac{d}{dx}e^u = e^u * u'$

Example 3: find y' for $y = \ln(2x - e^{-2x})$

Example 4: Find y' for $y = xe^{(x^2+2x+3)^3}$

Example 5: Find the equation of the tangent line to the graph at the given point:

$y = e^{-x} \ln x$ (1, 0)

Example 6: Find dy/dx

$xe^y - 10x + 3y = 0$

Example 7: Find the equation of the tangent line to the graph of the function at the given point.

$xe^y + ye^x = 1$ at (0, 1)

Example 8: Find the 2nd derivative of the function

$f(x) = (3 + 2x)e^{-3x}$

Ex. 9: find the extrema and points of inflection for $g(t) = 1 + (2 + t)e^{-t}$

Ex. 10: find the extrema and points of inflection for $f(x) = \frac{e^x - e^{-x}}{2}$ (use common denominators)

**Ch.5.4 Notes** **Derivative of Exponential Function $e^x$**

$y = \ln x$ and $y = e^x$ are inverse functions (meaning $f(g(x)) = x$ and $g(f(x)) = x$)

Example 1: Solve $7 = e^{x+1}$

$\ln 7 = \ln e^{x+1}$

$\ln 7 = (x+1)\cancel{\ln e}$

$\boxed{\ln 7 - 1 = x}$

Example 2: solve $\ln(2x - 3) = 5$

$\log_e(2x-3) = 5$

$e^5 = 2x - 3$

$\boxed{\frac{e^5+3}{2} = x}$

Reminder: exponent properties: $e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ Reminder: e is a NUMBER: if $y = e^2$, then y' = 0

Exponential Function $e^x$ Derivative rule $\frac{d}{dx}e^u = e^u * u'$

$\frac{d}{dx}e^x = e^x \cdot (1) = e^x$

Example 3: find y' for $y = \ln(2x - e^{-2x})$

$y' = \frac{2 - e^{-2x}(-2)}{2x - e^{-2x}} = \frac{2 + \frac{2}{e^{2x}}}{2x - \frac{1}{e^{2x}}}$

$\boxed{y' = \frac{2e^{2x} + 2}{2xe^{2x} - 1}}$

Example 4: Find y' for $y = xe^{(x^2+2x+3)^3}$

$y' = 1e^{(x^2+2x+3)^3} + x \cdot e^{(x^2+2x+3)^3} \cdot 3(x^2+2x+3)^2 \cdot (2x+2)$

$y' = e^{(x^2+2x+3)^3} + (6x^2+6x)(x^2+2x+3)^2 e^{(x^2+2x+3)^3}$

$y' = e^{(x^2+2x+3)^3}\left[1 + (6x^2+6x)(x^2+2x+3)^2\right]$

Example 5: Find the equation of the tangent line to the graph at the given point:
$y = e^{-x}\ln x$ (1, 0)

$y' = e^{-x}(-1)\ln x + e^{-x}\left(\frac{1}{x}\right)$

$= \frac{-\ln x}{e^x} + \frac{1}{xe^x}$

$y' = \frac{-x\ln x + 1}{xe^x}$

$y'(1) = \frac{-\ln(1) + 1}{(1)e^1} = \frac{1}{e}$

$y - 0 = \frac{1}{e}(x - 1)$

$\boxed{y = \frac{1}{e}(x-1)}$

Example 6: Find dy/dx

$xe^y - 10x + 3y = 0$

$1e^y + xe^y\left(\frac{dy}{dx}\right) - 10 + 3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(xe^y + 3) = 10 - e^y$

$\boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$

Example 7: Find the equation of the tangent line to the graph of the function at the given point.

$xe^y + ye^x = 1$ at (0, 1)

$1e^y + xe^y\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^x + ye^x = 0$

$1e^1 + 0e^1\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^0 + 1e^0 = 0$

$e + \frac{dy}{dx} + 1 = 0$

$\left.\frac{dy}{dx}\right|_{(0,1)} = -e - 1$

$\boxed{y - 1 = (-e-1)[x - 0]}$

Example 8: Find the $2^{nd}$ derivative of the function

$f(x) = (3 + 2x)e^{-3x}$

$f' = (2)e^{-3x} + (3+2x)e^{-3x}(-3) = \frac{2 - 9 - 6x}{e^{3x}}$

$= \frac{-7 - 6x}{e^{3x}}$

$f'' = \frac{(-6)e^{3x} - (-7-6x)e^{3x}(3)}{(e^{3x})^2}$

$\frac{-6e^{3x} + 21e^{3x} + 18xe^{3x}}{(e^{3x})^2} = \boxed{\frac{15 + 18x}{e^{3x}}}$

$$g(t) = 1 + \frac{2+t}{e^t}$$

Ex. 9: find the extrema and points of inflection for $g(t) = 1 + (2+t)e^{-t}$

$$g' = 0 + 1(e^{-t}) + (2+t)e^{-t}(-1) = \frac{1-2-t}{e^t} = \frac{-1-t}{e^t} \quad \boxed{t=-1}$$

$\nearrow$ + | $\searrow$ − ; −1

Rel. max at $(-1, \underline{1+e})$ b/c $f'(x)$ changes from + to −.

$$g(-1) = 1 + (2-1)e^{1}$$
$$= 1 + e$$

$$g''(t) = \frac{(-1)e^t - (-1-t)e^t}{e^{2t}}$$

$$g''(t) = \frac{-e^t + e^t + te^t}{e^{2t}} = \frac{t}{e^t}$$

$$0 = \frac{t}{e^t}$$
$$t = 0$$

$\frown$ − | $\smile$ + ; 0

$$f(0) = 1 + (2+0)e^0$$
$$= 1 + 2 = 3$$

POI at $(0, 3)$ b/c $f''(x)$ changes signs

Ex. 10: find the extrema and points of inflection for $f(x) = \frac{e^x - e^{-x}}{2}$ (use common denominators)

$$f(x) = \frac{e^x - \frac{1}{e^x}}{2} = \frac{\frac{e^{2x}-1}{e^x}}{2} = \frac{e^{2x}-1}{2e^x}$$

$$f'(x) = \frac{e^{2x}(2)[2e^x] - (e^{2x}-1)(2e^x)}{4e^{2x}}$$

$$= \frac{4e^{3x} - 2e^{3x} + 2e^x}{4e^{2x}} = \frac{2e^{3x} + 2e^x}{4e^{2x}} = \frac{2e^x(e^{2x}+1)}{4e^{2x}} = \frac{e^{2x}+1}{2e^x} > 0$$

$$e^{2x} + 1 = 0$$
$$e^{2x} = -1$$

$\nearrow$

$\boxed{\text{f(x) always increasing, no relative extrema}}$

$$f''(x) = \frac{e^{2x}(2)[2e^x] - [e^{2x}+1]2e^x}{(2e^x)^2}$$

$$= \frac{4e^{3x} - 2e^{3x} - 2e^x}{4e^{2x}}$$

$$= \frac{2e^{3x} - 2e^x}{4e^{2x}}$$

$$= \frac{2e^x(e^{2x}-1)}{4e^{2x}} = \frac{e^{2x}-1}{2e^x} = 0$$

$$e^{2x} - 1 = 0$$
$$e^{2x} = 1$$
$$\ln e^{2x} = \ln 1$$
$$2x = 0$$
$$x = 0$$

$$f(0) = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0$$

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$\boxed{\text{POI at (0,0) b/c f''(x) changes sign}}$