Ch. 5.5 Notes **Derivative of Logs of other bases**

Change of Base: $log_a x = \frac{lnx}{lna}$

Ex. 1 solve for x: $3^x = 1/81$

Ex. 2 solve: $\log_2 x = -4$

Derivative Rule for logs of other bases : $\frac{d}{dx}\log_a u = \frac{u'}{(\ln a)\,u'}$

Ex. 3 Find f '(x) for $f(x) = log_5 \sqrt[3]{(2x^2+7)}$

Ex. 4 Find f '(x) for $f(x) = log \frac{5x^3}{(x^2-3x)^3}$

Derivative Rule for Exponential functions of base a^x : $\frac{d}{dx}a^u = (\ln a)a^u * u'$

Ex. 5 Find f '(x) for $f(x) = 5^{x^2-2x}$

Ex. 6 Find f '(x) for $f(x) = x(4^{-x})$

Ex. 7:

1994 #4: A particle moves along the *x*-axis so that at any time $t > 0$ its velocity is given by $v(t) = t\ln t - t$.

a) Write an expression for the acceleration of the particle.

b) For what values of *t* is the particle moving to the right?

c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

Ch. 5.5

~~Ch. 5.4~~ **Notes** **Derivative of Logs of other bases**

<u>Change of Base</u>: $log_a x = \frac{ln x}{ln a}$

Ex. 1 solve for x: $3^x = 1/81$

$\log_3 3^x = \log_3(1/81)$

$x = \log_3(3)^{-4}$

$x = -4$

Ex. 2 solve: $\log_2 x = -4$

$2^{\log_2 x} = 2^{-4}$

$x = 2^{-4} = \boxed{\frac{1}{16}}$

Derivative Rule for logs of other bases : $\frac{d}{dx}\log_a u = \frac{u'}{(\ln a)u'}$ $= \left(\frac{1}{\ln a}\right)\left(\frac{u'}{u}\right)$

$\frac{d}{dx}\log_a x = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \left(\frac{1}{\ln a}\right)\cdot\left(\frac{1}{x}\right) = \frac{1}{\boxed{\ln a}\, x}$

Recall: $\frac{d}{dx}\ln u = \frac{u'}{u}$

$\frac{d}{dx}\log_a u = \frac{d}{dx}\left(\frac{\ln u}{\ln a}\right) = \left(\frac{1}{\ln a}\right)\cdot\frac{d}{dx}(\ln u) = \left(\frac{1}{\ln a}\right)\left(\frac{u'}{u}\right)$

Ex. 3 Find f '(x) for f(x) = $log_5 \sqrt[3]{(2x^2+7)}$

$f(x) = \log_5(2x^2+7)^{1/3}$

$= \frac{1}{3}\log_5(2x^2+7)$

$f'(x) = \frac{1}{3}\left(\frac{1}{\ln 5}\right)\left(\frac{4x}{2x^2+7}\right) = \boxed{\frac{4x}{3\ln 5(2x^2+7)}}$

Ex. 4 Find f '(x) for f(x) = $log \frac{5x^3}{(x^2-3x)^3}$

$f(x) = \log 5x^3 - \log(x^2-3x)^3$

$= \log 5x^3 - 3\log(x^2-3x)$

$f'(x) = \left(\frac{1}{\ln 10}\right)\left(\frac{15x^2}{5x^3}\right) - 3\left(\frac{1}{\ln 10}\right)\left(\frac{2x-3}{x^2-3x}\right)$

$= \frac{3}{x\ln 10} - \frac{3(2x-3)}{(\ln 10)(x^2-3x)}$

$* \; e^{\ln x} = x$

Derivative Rule for Exponential functions of base a^x : $\frac{d}{dx} a^u = (\ln a) a^u * u'$

$$\frac{d}{dx} a^x = e^{(\ln a)x} \rightarrow \frac{d}{dx} e^{(\ln a)x} = e^{(\ln a)(x)} \cdot \ln a = (\ln a) e^{(\ln a)x} = (\ln a) a^x$$

Recall: $\frac{d}{dx} e^u = e^u \cdot u'$

$\frac{d}{dx} a^u = \ln a \cdot a^u \cdot u'$

Ex. 5 Find f '(x) for $f(x) = 5^{x^2-2x}$

$$f'(x) = (\ln 5)(5^{x^2-2x})(2x-2)$$

Ex. 6 Find f '(x) for $f(x) = x(4^{-x})$

$$f'(x) = (1)(4^{-x}) + x \cdot (\ln 4)(4^{-x})(-1)$$

$$= \frac{1}{4^x} - \frac{x \ln 4}{4^x} = \boxed{\frac{1 - x\ln 4}{4^x}}$$

Ex. 7

$v(t) = t\ln t - t$

a) $a(t) = v'(t) = \ln t + t\left(\frac{1}{t}\right) - 1 = \ln t + 1 - 1 = \ln t$

b) Find critical points : set $v(t) = 0$

$v(t) = t\ln t - t$

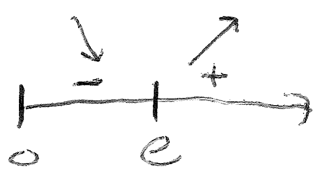
$0 = t(\ln t - 1)$

$t = 0, \quad t = e$

$\ln t - 1 = 0$

$\ln t = 1$

$e^1 = t$



particle moving right (e, ∞) b/c $v(t) > 0$

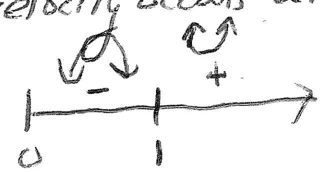
c) * minimum velocity occurs where $f''(x)$ changes from − to +

$a(t) = \ln t$

$0 = \ln t$

$e^0 = t$

$t = 1$



$v(1) = 1\ln(1) - 1$

$\boxed{v(1) = -1}$

Minimum velocity is −1 b/c $a(t) < 0$ for t in $(0, 1)$ and $a(t) > 0$ for all $t > 1$