AP Calculus Logs and Exponentials Test Review WS #1

Name______________________ Date______

1. Find the equation of the line tangent to $y = \frac{e^{2x}}{x^2}$ at $x = -1$

2. Let f be the function defined by $f(x) = \frac{x^2}{e^x}$

a. State the domain of $f(x)$.

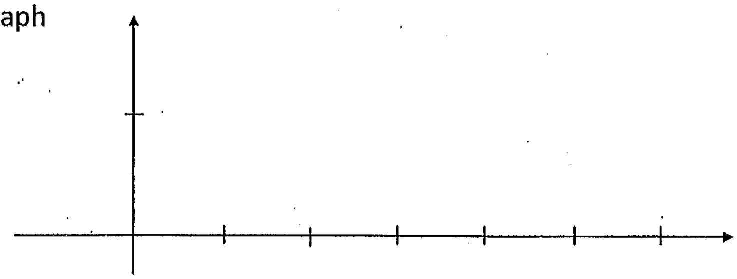
b. Find each relative maximum and relative minimum. Justify Answer

c. Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

d. State the range of f.

e. Find each point of inflection on the graph of f. Write your answer(s) as ordered pairs and justify each answer.

f. Sketch graph



②

3. 1994 #4: A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t\ln t - t$.

   a) Write an expression for the acceleration of the particle.
   b) For what values of t is the particle moving to the right?
   c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

4. Find dy/dx for $y \ln x + y^2 = 0$

5. Find dy/dx if $y = \sqrt[3]{(2 + 5x^3)^x}$

AP Calculus Logs and Exponentials Test Review WS #1 Name KEY Date_____

1. Find the equation of the line tangent to $y = \frac{e^{2x}}{x^2}$ at x = – 1

$$y' = \frac{(e^{2x})(2)x^2 - e^{2x}(2x)}{x^4}$$

$$y' = \frac{2xe^{2x}(x-1)}{x^4} = \frac{2e^{2x}(x-1)}{x^3}$$

$$y(-1) = \frac{e^{-2}}{(-1)^2} = \frac{1}{e^2}$$

point $(-1, \frac{1}{e^2})$

$$y'(-1) = \frac{2e^{-2}(-2)}{(-1)^3} = \frac{-4}{-e^2} = \frac{4}{e^2}$$

$m = \frac{4}{e^2}$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - \frac{1}{e^2} = \frac{4}{e^2}(x+1)}$$

2. Let f be the function defined by $f(x) = \frac{x^2}{e^x}$

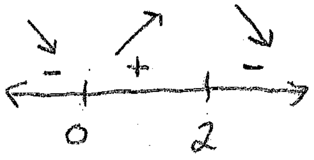
a. State the domain of $f(x)$.

$(-\infty, \infty)$

b. Find each relative maximum and relative minimum. Justify Answer

$f(x) = x^2e^{-x}$

$$f'(x) = 2xe^{-x} + x^2e^{-x}(-1) = e^{-x}(2x - x^2) = \frac{x(2-x)}{e^x}$$



Rel. min at (0, 0) b/c f'(x) changes from + to –

Rel. max at $(2, \frac{4}{e^2})$ b/c f'(x) changes from + to –

$$\boxed{f'(x) = \frac{2x - x^2}{e^x}}$$

c. Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

Comparative growth rate $L < R < P < E$

$$\lim_{x\to-\infty} \frac{x^2}{e^x} \rightarrow \frac{(-\infty)^2}{e^{-\infty}} = (-\infty)^2 e^{\infty} = \infty$$

$$\lim_{x\to\infty} \frac{x^2}{e^x} = 0$$

d. State the range of f.

$[0, \infty)$

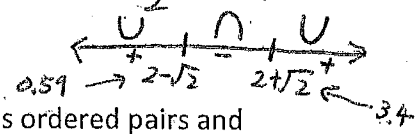
e) $f'(x) = \frac{2x - x^2}{e^x}$

$$f''(x) = \frac{(2-2x)e^x - (2x - x^2)e^x}{e^{2x}}$$

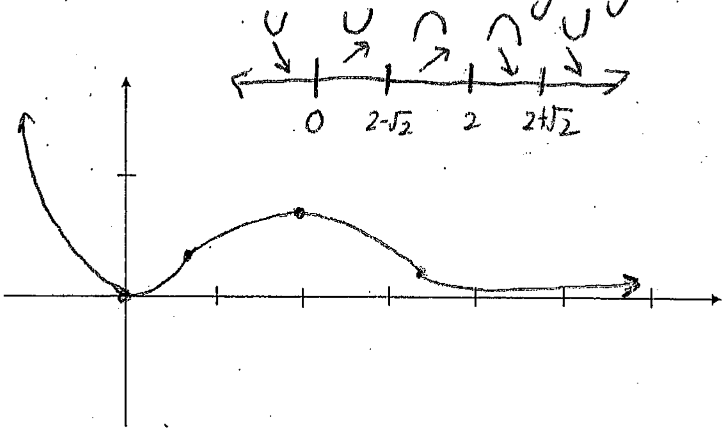
$$= \frac{e^x(2 - 2x - 2x + x^2)}{e^{2x}} = \boxed{\frac{x^2 - 4x + 2}{e^x}}$$

$$x^2 - 4x + 2 = 0$$

$$\frac{4 \pm \sqrt{16 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$



e. Find each point of inflection on the graph of f. Write your answer(s) as ordered pairs and justify each answer. POI at $x = 2 - \sqrt{2}$, $2 + \sqrt{2}$ b/c f''(x) change signs



f. Sketch graph

x-int: $\frac{x^2}{e^x} = 0$ $x = 0$ (0, 0)

POI: (0.59, 0.19)

(3.4, 0.39)

3. 1994 #4: A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t\ln t - t$.

a) Write an expression for the acceleration of the particle.
b) For what values of t is the particle moving to the right?
c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

a) $a(t) = (1)\ln t + t\left(\frac{1}{t}\right) - 1 = \ln t + 1 - 1 = \ln t$ $\boxed{a(t) = \ln t}$

b) Set $v(t)=0$, make sign line.

$v(t) = t\ln t - 1$

$0 = t(\ln t - 1)$

$t=0$

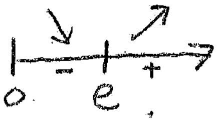
$\ln t - 1 = 0$

$\ln t = 1$

$\log_e t = 1$

$e^1 = t$

$\boxed{t=e}$



particle moving right (e,∞) b/c $v(t) > 0$

c) * minimum velocity occurs where $v'(t)$ changes from − to +, (POI)

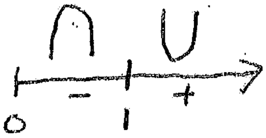
set $a(t) = 0$

$\ln t = 0$

$\log_e t = 0$

$e^0 = t$

$1 = t$



Minimum velocity at $t=1$ since $a(t)$ changes from − to +

4. Find $\frac{dy}{dx}$. $y\ln x + y^2 = 0$

$\frac{dy}{dx}\ln x + y\left(\frac{1}{x}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}(\ln x + 2y) = \frac{-y}{x}$

$\frac{dy}{dx} = \frac{\frac{-y}{x}}{\ln x + 2y} = \boxed{\frac{-y}{x(\ln x + 2y)}}$

5. Find dy/dx if y = $\sqrt[3]{(2 + 5x^3)^x}$

$y = (2+5x^3)^{x/3}$

$\ln y = \ln (2+5x^3)^{x/3}$

$\ln y = \left(\frac{x}{3}\right)\left[\ln(2+5x^3)\right]$

$\frac{1}{y}\frac{dy}{dx} = \left(\frac{1}{3}\right)\ln(2+5x^3) + \left(\frac{x}{3}\right)\left(\frac{1}{2+5x^3}\right)(15x^2)$

$\frac{dy}{dx} = y\left[\frac{1}{3}\ln(2+5x^3) + \frac{5x^3}{2+5x^3}\right] = \boxed{(2+5x^3)^{x/3}\left[\frac{1}{3}\ln(2+5x^3) + \frac{5x^3}{2+5x^3}\right]}$

Ch.5 Test Review Problem : Curve Sketching $e \approx 2.7$ ⑤

$f(x) = \frac{x}{\ln x}$ a) find domain c) POI e) $\lim_{x \to \infty} f(x)$

b) Rel. max/min d) $\lim_{x \to 1^+} f(x)$ f) sketch graph

⑥ Find $\frac{dy}{dx}$ for $\ln(xy) + xy = 50$

Ch.5 Test Review Problem: Curve Sketching

$e \approx 2.7$

$f(x) = \frac{x}{\ln x}$

a) find domain
b) Rel. max/min
c) POI
d) $\lim_{x \to 1^+} f(x)$
e) $\lim_{x \to \infty} f(x)$
f) sketch graph

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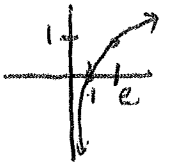
a) find domain: * consider individual restictions of family of functions.
* set denominator $= 0$

$y = \ln x$



$\rightarrow$ D: $(0, \infty)$

$\ln x = 0$
$\log_e x = 0$
$e^0 = x$

$x = 1$
$\boxed{x \neq 1}$
VA: $x = 1$

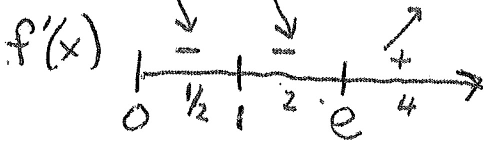
$\boxed{D: (0,1) \cup (1, \infty)}$

b) 1^{st} derivative test: $y = \frac{\overbrace{x}^{f}}{\underbrace{\ln x}_{g}}$ $y' = \frac{\overbrace{(1)}^{f'}\overbrace{(\ln x)}^{g} - \overbrace{(x)}^{f}\overbrace{(\frac{1}{x})}^{g'}}{\underbrace{(\ln x)^2}_{g^2}} = \boxed{\frac{\ln x - 1}{(\ln x)^2}}$

*critical points:

$\ln x - 1 = 0$
$\ln x = 1$
$\log_e x = 1$
$\underline{e^1 = x}$

$(\ln x)^2 = 0$
$\ln x = 0$
$\log_e x = 0$
$e^0 = x$
$\underline{x = 1}$



Rel. minimum at $(e, \underline{e})$ $\xleftarrow{} \frac{e}{\ln e}$ b/c $f'(x)$ changes from − to +

c) $y' = \frac{\overbrace{\ln x - 1}^{f}}{\underbrace{(\ln x)^2}_{g}}$ $y'' = \frac{\overbrace{(\frac{1}{x})}^{f'}\overbrace{(\ln x)^2}^{g} - \overbrace{(\ln x - 1)}^{f} \cdot \overbrace{2(\ln x)(\frac{1}{x})}^{g'}}{\underbrace{(\ln x)^4}_{g^2}}$ ← chain: out: $(\ )^2$ in: $\ln x$ $= \frac{\overbrace{\frac{1}{x}(\ln x)^2}^{} - \overbrace{2(\frac{1}{x})\ln x(\ln x - 1)}^{}}{(\ln x)^4}$

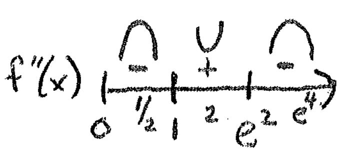
$y'' = \frac{\frac{1}{x}(\ln x)\left[\ln x - 2(\ln x - 1)\right]}{(\ln x)^4} = \frac{\frac{1}{x}\ln x\left[\ln x - 2\ln x + 2\right]}{(\ln x)^4} = \boxed{\frac{2 - \ln x}{x(\ln x)^3}}$

*critical pts:

$2 - \ln x = 0$
$2 = \ln x$
$\ln x = 2$
$\log_e x = 2$
$\underline{\underline{e^2 = x}}$

$x = 0$

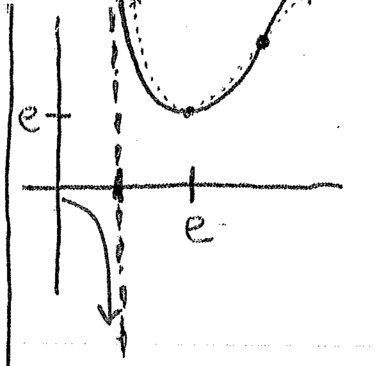
$(\ln x)^3 = 0$
$x = 1$



POI: $(e^2, \underline{\frac{e^2}{\ln e^2}})$
$(e^2, \underline{\frac{1}{2}e^2})$

$\lim_{x \to \infty} \frac{x}{\ln x} = \infty$

$\lim_{x \to 1^+} \frac{x}{\ln x} = \infty$



⑧

Find $\frac{dy}{dx}$ for $\ln(xy) + xy = 50$

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* expand log expression

* implicit diff

* product rule

$$\ln x + \ln y + \overset{f}{x}\,\overset{g}{y} = 50$$

$$\frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) + \overset{f'}{1}\,\overset{g}{y} + \overset{f}{x}\overset{g'}{\left(\frac{dy}{dx}\right)} = 0$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) + x\left(\frac{dy}{dx}\right) = -\frac{1}{x} - y$$

$$\frac{dy}{dx}\left(\frac{1}{y} + x\right) = -\frac{1}{x} - y$$

$$\frac{dy}{dx} = \frac{-\frac{1}{x} - y}{\frac{1}{y} + x} \quad xy$$

$$\boxed{\frac{dy}{dx} = \frac{-y - xy^2}{x + x^2y}}$$

$$\frac{dy}{dx} = \frac{-y(1+xy)}{x(1+xy)} = \boxed{\frac{-y}{x}}$$

New Derivative Rules:

$\log_e x = \ln x$ $\log_{10} x = \log x$

1) $\frac{d}{dx}\ln u = \frac{u'}{u}$

2) $\frac{d}{dx}\log_a u = \frac{1}{\ln a}\cdot\frac{u'}{u}$

3) $\frac{d}{dx}e^u = e^u\cdot u'$

4) $\frac{d}{dx}a^u =$

$\ln a\cdot a^u\cdot u'$

log properties:

$\ln(ab) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

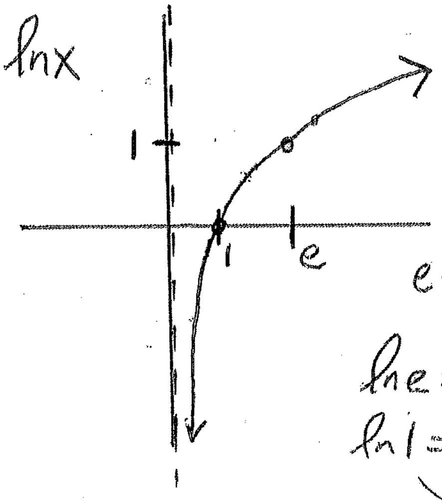
$\ln a^n = n\cdot\ln a$

$\log_a(cd) = \log_a c + \log_a d$

$\log_a\left(\frac{c}{d}\right) = \log_a c - \log_a d$

$\log_a c^n = n\cdot\log_a c$

$y = \ln x$



$e \approx 2.718$

$\ln\left(\frac{1}{2}\right) =$

$\ln(2) =$

$\ln(3) =$

$\ln e = 1$

$\ln 1 = 0$

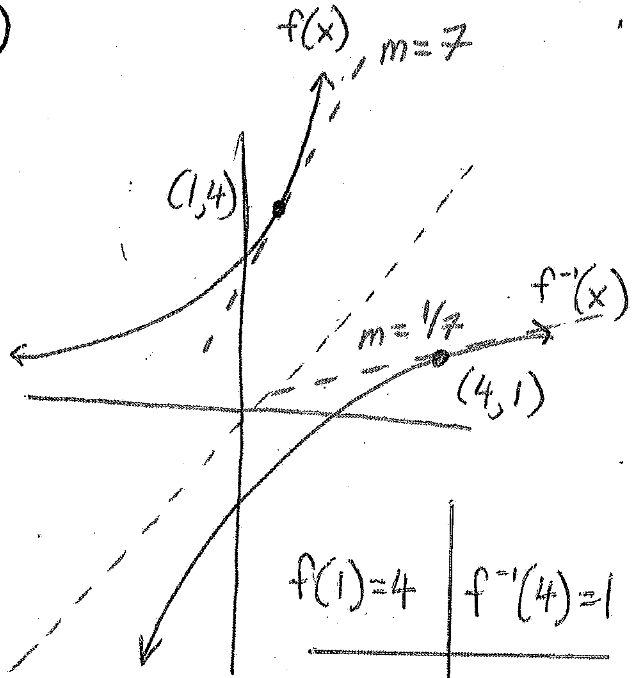
$\ln(ab) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln a^n = n\ln a$

$\cancel{\ln(a+b) \neq \ln a + \ln b}$

$\boxed{\frac{d}{dx}\ln u = \frac{u'}{u}}$



$f(1)=4$	$f^{-1}(4)=1$
$f'(1)=7$	$(f^{-1})'(4)=\frac{1}{7}$

Steps: Derivative of Inverse at a Point

1) The y-value of function is given
2) set function equal to y-value
3) Solve for x: guess and check
   * test values close to 0.
4) find derivative of function
5) Use the x-value to evaluate derivative to find slope
6) Derivative of the inverse at its corresponding point is the reciprocal of slope.

To determine Increasing/Decreasing Speed, compare signs between v(t) and a(t)

* Increasing speed if v(t) and a(t) have same signs
* Decreasing speed if v(t) and a(t) have opposite signs.

To determine increasing or decreasing velocity, look at sign of a(t)

* If $a(t) > 0$, velocity is increasing
* If $a(t) < 0$, velocity is decreasing

AP Calculus Logs and Exponentials Test Review WS #2 Name__________________Date______

1. The position of a particle moving on the x-axis is given by $x(t) = \frac{2t}{e^t}$

a) Show that $v(t) = \frac{2-2t}{e^t}$

b) Show that $a(t) = \frac{2t-4}{e^t}$

c) For what values of t is the particle at rest?

d) For the value found in part c, is the particle located to the right or the left of the origin? Justify your answer

e) For the same value from part c, is the velocity increasing or decreasing?

f) For the same value in part c, is the acceleration increasing or decreasing?

g) What is the velocity when the acceleration is zero?

h) At t = 3, is the velocity of the particle increasing or decreasing?

i) At t = 3,is the speed of the particle increasing or decreasing?

j) For what value(s) of t is the particle located at the origin?

2. Find dy/dx for $\ln(xy^2) = y^2 - 3x$

3. Given $f(x) = \ln(4 - x^2)$

a) Find Domain of f(x)

b) Find the Range of f(x)

c) Find intercepts for f(x)

d) Determine the asymptotes for f(x)

e) Find f′(x)

f) Find f ″(x)

g) Find intervals of increase/decrease and relative extrema. Justify your answer

h) Determine intervals of concave up/down and POI

Sketch Graph:

AP Calculus Logs and Exponentials Test Review WS #2 Name KEY Date______

1. The position of a particle moving on the x-axis is given by $x(t) = \frac{2t}{e^t}$

a) Show that $v(t) = \frac{2-2t}{e^t}$

$$v(t) = \frac{2(e^t) - 2t(e^t)}{e^{2t}} = \frac{e^t(2-2t)}{e^{2t}} = \boxed{\frac{2-2t}{e^t}}$$

b) Show that $a(t) = \frac{2t-4}{e^t}$

$$a(t) = \frac{(-2)(e^t) - (2-2t)(e^t)}{e^{2t}} = \frac{e^t(-2-2+2t)}{e^{2t}} = \boxed{\frac{2t-4}{e^t}}$$

c) For what values of t is the particle at rest?

$2-2t=0$ $\boxed{t=1}$

d) For the value found in part c, is the particle located to the right or the left of the origin? Justify your answer

$x(1) = \frac{2}{e}$ particle is located to the right of origin since $\frac{2}{e} > 0$

e) For the same value from part c, is the velocity increasing or decreasing?

$a(1) = \frac{2-4}{e} = \frac{-2}{e} < 0$

velocity is decreasing b/c $a(t) < 0$

f) For what value of t is acceleration $= 0$?

$0 = 2t-4$

$2t = 4$

$\boxed{t=2}$

g) What is the velocity when the acceleration is zero?

$v(2) = \frac{2-4}{e^2} = \boxed{\frac{-2}{e^2}}$

h) At t = 3, is the velocity of the particle increasing or decreasing?

$a(3) = \frac{6-4}{e^3} = \frac{2}{e^3} > 0$. Since $a(3) > 0$ velocity is increasing.

i) At t = 3, is the speed of the particle increasing or decreasing?

$v(3) = \frac{2-6}{e^3} = \frac{-4}{e^3} < 0$

Since $v(3)$ and $a(3)$ have opposite signs, speed is decreasing.

j) For what value(s) of t is the particle located at the origin?

$0 = \frac{2t}{e^t}$ $\boxed{t=0}$

2. Find dy/dx for $\ln(xy^2) = y^2 - 3x$

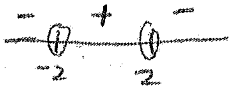
$\ln x + 2\ln y = y^2 - 3x$

$\frac{1}{x} + 2\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) = 2y\left(\frac{dy}{dx}\right) - 3$

$\frac{dy}{dx}\left(\frac{2}{y} - 2y\right) = -3 - \frac{1}{x}$

$\frac{dy}{dx} = \frac{-3-\frac{1}{x}}{\frac{2}{y}-2y} \cdot \frac{xy}{xy} = \boxed{\frac{-3xy-y}{2x-4xy^2}}$

3. Given $f(x) = \ln(4 - x^2)$



a) Find Domain of f(x)

$(-2, 2)$

b) Find the Range of f(x)

$(-\infty, \ln 4)$

c) Find intercepts for f(x)

$\ln(4-x^2)=0$

$e^0=4-x^2$

$1=4-x^2$

$-3=-x^2$

$x=\pm\sqrt{3}$

y-int at $(0, \ln 4)$

d) Determine the asymptotes for f(x)

$x=-2,\ x=2$

e) Find f'(x)

$\frac{1}{4-x^2}(-2x) = \boxed{\frac{-2x}{4-x^2}}$

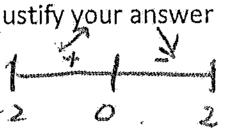
f) Find f''(x)

$\frac{(-2)(4-x^2)-(-2x)(-2x)}{(4-x^2)^2} = \frac{-8+2x^2-4x^2}{(4-x^2)^2}$

$\frac{-8-2x^2}{(4-x^2)^2} = \boxed{\frac{-2(4+x^2)}{(4-x^2)^2}}$

g) Find intervals of increase/decrease and relative extrema. Justify your answer

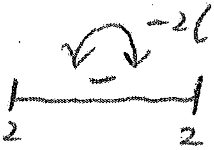
$-2x=0 \quad x=0$



Rel. max at $(0, \ln 4)$ b/c $f'(x)$ changes from + to −.

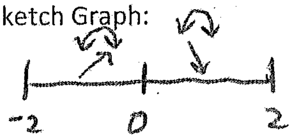
h) Determine intervals of concave up/down and POI

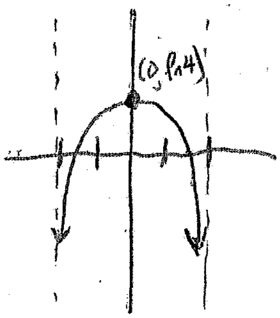
No critical values since $-2(4+x^2) \neq 0$



$f(x)$ concave down $(-2, 2)$ b/c $f''(x) < 0$

Sketch Graph:





AP Calculus Logs and Exponentials Test Review WS #3 Name________________ Date_____

1. Let *f* be the function defined by $f(x) = \frac{x^3}{e^x}$

a. State the domain of $f(x)$.

b. Find the range of *f*. (find this <u>after</u> sketching your curve)

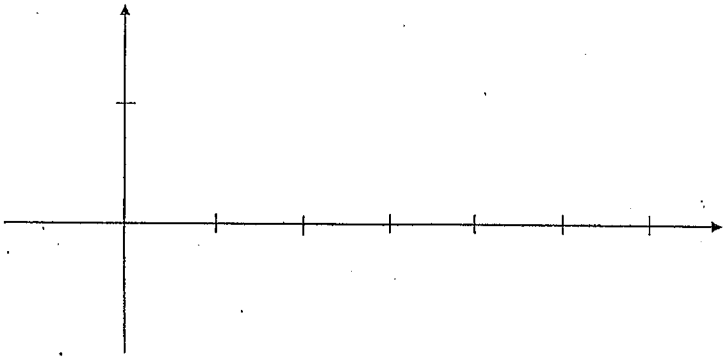
c. Find each relative maximum and relative minimum. (ordered pairs) Justify Answer

Find each point of inflection on graph of f. (Provide only x value) Justify answer.

d. Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

e. Find x-intercept(s)

f. Sketch graph.



2. If y = $(e^{-x})(\ln x)$, then dy/dx when x = 1 is

3. The position of a particle moving on the x-axis is given by x(t) = $\frac{4}{3}e^{3t} - 8t$

a) Write an expression for the velocity at any given time t

b) Write an expression for the acceleration at any given time t

c) For what values of t is the particle at rest?

d) Find the velocity when t = 3

e) Find the acceleration when t = 3

f) At t = 3, is the velocity of the particle increasing or decreasing?

g) At t = 3,is the speed of the particle increasing or decreasing?

4. Find dy/dx if $\ln(yx) = y^2 - x^3$ – e

AP Calculus Logs and Exponentials Test Review WS #3

Name KEY Date

1. Let f be the function defined by $f(x) = \frac{x^3}{e^x}$

a. State the domain of $f(x)$.

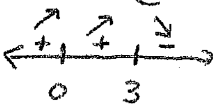
$(-\infty, \infty)$

b. Find the range of f. (find this after sketching your curve)

$\left(-\infty, \frac{27}{e^3}\right)$

c. Find each relative maximum and relative minimum. (ordered pairs) Justify Answer

$f'(x) = \frac{3x^2e^x - x^3e^x}{e^{2x}} = \frac{3x^2 - x^3}{e^x} = \boxed{\frac{x^2(3-x)}{e^x}}$ $x = 0, 3$

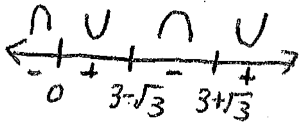


Rel. max at $\left(3, \frac{27}{e^3}\right)$
b/c $f'(x)$ changes from + to −

Find each point of inflection on graph of f. (Provide only x value) Justify answer.

$f''(x) = \frac{(6x - 3x^2)e^x - (3x^2 - x^3)e^x}{e^{2x}} = \frac{6x - 3x^2 - 3x^2 + x^3}{e^x}$

$f''(x) = \frac{x^3 - 6x^2 + 6x}{e^x} = \frac{x(x^2 - 6x + 6)}{e^x}$ $x = 0, 3+\sqrt{3}, 3-\sqrt{3}$



POI at $x = 0, 3 \pm \sqrt{3}$
b/c $f''(x)$ change signs.

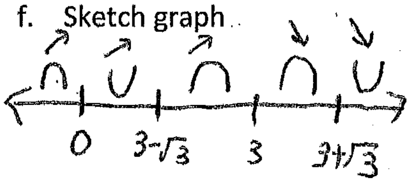
d. Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$

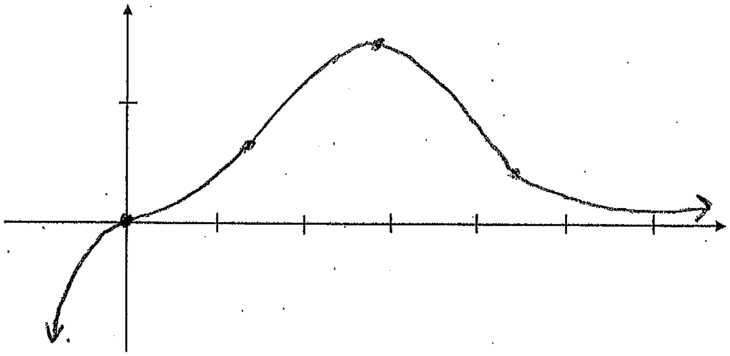
Comparative growth rate

$\lim\limits_{x\to\infty} \frac{x^3}{e^x} = 0$ | $\lim\limits_{x\to-\infty} \frac{x^3}{e^x} \Rightarrow \frac{(-\infty)^3}{e^{-\infty}} = (-\infty)^3 e^{\infty} = \boxed{-\infty}$

e. Find x-intercept(s) $(0, 0)$

f. Sketch graph





2. If $y = (e^{-x})(\ln x)$, then dy/dx when x = 1 is

$y' = (e^{-x})(-1)\ln x + e^{-x}\left(\frac{1}{x}\right)$

$y' = \frac{-\ln x}{e^x} + \frac{1}{xe^x}$

$y'(1) = \frac{-\ln 1}{e^1} + \frac{1}{e^1} = \boxed{\frac{1}{e}}$

3. The position of a particle moving on the x-axis is given by x(t) = $\frac{4}{3}e^{3t} - 8t$

a) Write an expression for the velocity at any given time t

$v(t) = \frac{4}{3}e^{3t} \cdot (3) - 8$

$v(t) = 4e^{3t} - 8$

b) Write an expression for the acceleration at any given time t

$a(t) = 4e^{3t} \cdot 3 - 0$

$a(t) = 12e^{3t}$

c) For what values of t is the particle at rest?

$0 = 4e^{3t} - 8$

$4e^{3t} = 8$

$e^{3t} = 2$

$\ln e^{3t} = \ln 2$

$3t = \ln 2$

$\boxed{t = \frac{\ln 2}{3}}$

d) Find the velocity when t = 3

$V(3) = 4e^{3(3)} - 8 = 4e^{9} - 8$

e) Find the acceleration when t = 3

$a(3) = 12e^{9}$

f) At t = 3, is the velocity of the particle increasing or decreasing?

velocity is increasing at $t = 3$ b/c $a(t) > 0$

g) At t = 3, is the speed of the particle increasing or decreasing?

At $t = 3$, speed is increasing b/c $v(t)$ and $a(t)$ have same signs.

4. Find dy/dx if $\ln(yx) = y^2 - x^3 - e$

$\ln y + \ln x = y^2 - x^3 - e$

$\frac{1}{y}\frac{dy}{dx} + \frac{1}{x} = 2y\left(\frac{dy}{dx}\right) - 3x^2 - 0$

$\frac{dy}{dx}\left[\frac{1}{y} - 2y\right] = -3x^2 - \frac{1}{x}$

$\frac{dy}{dx} = \frac{-3x^2 - \frac{1}{x}}{\frac{1}{y} - 2y} \cdot \frac{(xy)}{(xy)} = \boxed{\frac{-3x^3y - y}{x - 2xy^2}}$

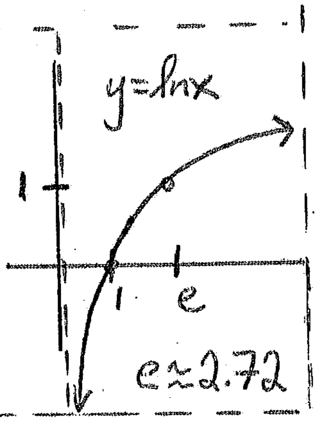
Ex. 4 Find Relative extrema, POI's for $y = \frac{x}{\ln x}$

$\boxed{\text{Ex.4}}$ Find Relative extrema, POI's for $y = \frac{x}{\ln x}$

*Domain: $(0,1) \cup (1,\infty)$

$\ln x = 0$ | $x \neq 1$
$\log_e x = 0$
$e^0 = 1$

*Recall:
y = ln x graph
$\ln e = 1$
$\ln 1 = 0$
$\ln 0.5 < 0$
$\ln 2 \approx 0.5$
$\ln 3 \approx 1.2$

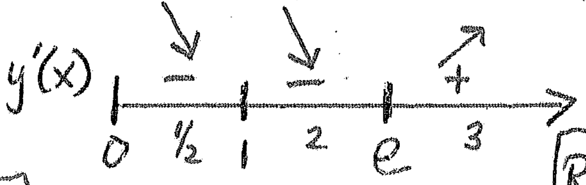


$$y'(x) = \frac{(1)(\ln x) - x\left(\frac{1}{x}\right)}{(\ln x)^2}$$

$$\boxed{y'(x) = \frac{\ln x - 1}{(\ln x)^2}}$$

*find critical points

$\ln x - 1 = 0$ | $(\ln x)^2 = 0$
$\ln x = 1$ | $\ln x = 0$
$\log_e x = 1$ | $\log_e x = 0$
$e^1 = x$ | $e^0 = x$
$\underline{\underline{x = e}}$ | $\underline{\underline{x = 1}}$



$\boxed{\text{Rel. min at } x = e}$

$$y''(x) = \frac{\overbrace{\left(\frac{1}{x}\right)}^{f'}\overbrace{(\ln x)^2}^{g} - \overbrace{(\ln x - 1)}^{f}\cdot\overbrace{2(\ln x)\left(\frac{1}{x}\right)}^{g'}}{\underbrace{(\ln x)^4}_{g^2}}$$ ← *chain rule out: $(\ )^2$ in.: $\ln x$

$$y''(x) = \frac{\frac{1}{x}(\ln x)^2 - \frac{2}{x}\ln x(\ln x - 1)}{(\ln x)^4}$$

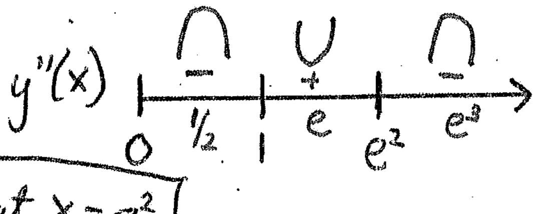
$$y''(x) = \frac{\frac{1}{x}\ln x\left(\ln x - 2(\ln x - 1)\right)}{(\ln x)^4}$$

$$y''(x) = \frac{\frac{1}{x}\ln x(\ln x - 2\ln x + 2)}{(\ln x)^4}$$

$$y''(x) = \frac{\frac{1}{x}(2 - \ln x)}{(\ln x)^3} = \boxed{\frac{2 - \ln x}{x(\ln x)^3}} = y''(x)$$

*critical pts:

$2 - \ln x = 0$ | $x = 0$ | $(\ln x)^3 = 0$
$2 = \ln x$ | $\underline{\underline{x = 0}}$ | $\ln x = 0$
$2 = \log_e x$ | | $e^0 = x$, $\underline{x = 1}$
$\underline{\underline{e^2 = x}}$



$\boxed{\text{POI at } x = e^2}$

5) find $\frac{dy}{dx}$ $y = \sqrt[7]{(x^2 - \ln x)^x}$

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6) Find $\frac{d}{dx}f^{-1}(2)$ given $f(x) = x^3 + 2x - 1$

7) Find $\frac{dy}{dx}$ $y = \log_5\left(\frac{4}{x^2\sqrt{1-x}}\right)$

5) find $\frac{dy}{dx}$ $y=\sqrt[7]{(x^2-\ln x)^x}$

$$y=(x^2-\ln x)^{x/7}$$

$$y=(x^2-\ln x)^{\frac{1}{7}x}$$

$$\ln y=\ln(x^2-\ln x)^{\frac{1}{7}x}$$

$$\ln y=\overbrace{\tfrac{1}{7}x}^{f}\,\overbrace{\ln(x^2-\ln x)}^{g}$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right)=\overbrace{\tfrac{1}{7}}^{f'}\cdot\overbrace{\ln(x^2-\ln x)}^{g}+\overbrace{\tfrac{1}{7}x}^{f}\cdot\overbrace{\frac{2x-\frac{1}{x}}{x^2-\ln x}}^{g'}$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right)=\frac{1}{7}\ln(x^2-\ln x)+\frac{2x^2-1}{7(x^2-\ln x)}$$

$$\frac{dy}{dx}=y\left[\frac{1}{7}\ln(x^2-\ln x)+\frac{2x^2-1}{7(x^2-\ln x)}\right]$$

$$\boxed{\frac{dy}{dx}=\sqrt[7]{(x^2-\ln x)^x}\left[\frac{1}{7}\ln(x^2-\ln x)+\frac{2x^2-1}{7(x^2-\ln x)}\right]}$$

6) Find $\frac{d}{dx}f^{-1}(2)$ given $f(x)=x^3+2x-1$

$f(\ )=2$ | $(f^{-1})(2)=$_

$(f^{-1})'(2)=$_

$$2=x^3+2x-1$$

$$0=x^3+2x-3$$

test x-values $x=0,1,-1,2,-2,\ldots$

$$x=1$$

$$f'(x)=3x^2+2$$

$$f'(1)=3(1)^2+2$$

$$f'(1)=5$$

$$\boxed{(f^{-1})'(2)=\frac{1}{5}}$$

7) Find $\frac{dy}{dx}$ $y = \log_5\left(\frac{4}{x^2\sqrt{1-x}}\right)$

$$y = \log_5 4 - \log_5 x^2 - \log_5 (1-x)^{1/2}$$

$$y = \log_5 4 - 2\log_5 x - \frac{1}{2}\log_5(1-x)$$

$\frac{d}{dx}\log_a u = \frac{1}{\ln a}\cdot\frac{u'}{u}$

$$y' = 0 - 2\cdot\frac{1}{\ln 5}\cdot\frac{1}{x} - \frac{1}{2}\cdot\frac{1}{\ln 5}\cdot\frac{-1}{1-x}$$

$$\boxed{\frac{dy}{dx} = \frac{-2}{x\ln 5} + \frac{1}{2\ln 5(1-x)}}$$

Top 5 Student Mistakes: 5.1/5.3 Quiz

1) Improper expansion of log rules. Follow the correct expansion rules:

   ln(ab) = ln(a) + ln(b) . Remember, there are no expansion rules for ln(a+b)

   $\ln(a+b) \neq \ln a + \ln b$ (example: **$\ln(5+4x) \neq \ln(5) + \ln(4x)$**)

2) Forgetting to expand log problem before finding derivative.
3) Log differentiation mistakes
   a. Incorrectly applying log differentiation to a problem that already has logs in the problem.
   b. Not knowing when to apply log differentiation: ($y = x^{3x+5}$)
   c. Forgetting to apply derivative product rule for the above log differentiation problem.
4) Forgetting Domain/Range properties of parent graphs ($y = \ln(x)$ and $y = \sqrt{x}$)
5) Not knowing how to establish relationships between ordered pairs and slopes regarding a function and its inverse function

   Basically, know how to use the below table/chart to find $(f^{-1})'(a)$

   Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$

$f(b) = a$	$(f^{-1})(a) = b$
$f'(b) = n$	$(f^{-1})'(a) = \frac{1}{n}$

Top 5 5.4-5.5 Quiz Mistakes

1. Students confusing rules between $\frac{d}{dx}e^u$, $\frac{d}{dx}\ln u$, $\frac{d}{dx}\log_a u$, and $\frac{d}{dx}a^u$
   a. For example $\frac{d}{dx}e^{3x-2x^4} \neq e^{3x-2x^4} * \left(\frac{3-8x}{3x-2x^4}\right)$

2. Students expanding problem that is not log expression

3. Incorrectly applying derivative power rule with log power property

   a. $\frac{d}{dx}\ln(3-5x^3)^{\frac{4}{3}} \neq \frac{4}{3}\ln(3-5x^3)^{\frac{1}{3}} * (-15x^2)$

4. Careless mistakes forgetting (ln a) for $\frac{d}{dx}\log_a u$, and $\frac{d}{dx}a^u$ derivative rules

5. Forgetting location of (**ln a**) for $\frac{d}{dx}\log_a u = \frac{1}{\ln a} * \frac{u'}{u}$, and $\frac{d}{dx}a^u = \ln a * a^u * u'$ derivative rules

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Top 5 Mistakes: Ch. 5 Exponential/Logs Test

1. Not knowing how to interpret (given time t) determining velocity increase/decrease vs. speed increasing/decreasing

2. Mistakes throughout the curve sketching problem

    a. Forgetting how to approximate values for ln x . Memorize the **y = ln x** parent graph (For instance: ln(0)= ___ , ln(-3)= ___ , ln(1)= ___
    ln(2)≈ ___ , ln(e)= ___ , ln(3)≈ ___, ln(1/3)≈ ___
    b. Following through with all the steps for first derivative test, concavity test, justification
    c. Applying sign lines
    d. Sketch graph according to the sign lines.

3. Forgetting to apply log expansion for log expressions before finding derivative

4. Applying implicit differentiation appropriately, correctly

5. Not knowing when to apply log differentiation and/or making mistakes within log differentiation

Calculus Chapter 5 Morning Test Review (WS #4) - Logs/Exponential Functions and Derivatives

1) The position function is given. The particle moves along the x-axis for all Real number values of t. $x(t) = t^2e^{-t}$

a) Find v(t) and a(t)
b) Determine the interval that the particle is moving to the left
c) Is the particle's velocity increasing or decreasing at t = 3? (Justify with because statement)
d) Is the particle's speed increasing or decreasing at t = 3? (Justify with because statement)

2) $find\ \frac{dy}{dx}$ $\quad y = (3-5x)^{2^x}$

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3) *find* y' $y = 3log_7\left(\frac{x}{(e^{2x})\sqrt{1-3x^2}}\right)$

4) *find* $\frac{dy}{dx}$ $ln\left(\frac{\sqrt[3]{y}}{x^5}\right) = 3x^2y - y + 5x - 3$

5) Find the tangent line equation for the function $f(x) = e^{-x}(\ln x)$ at (1, 0)

Ch.5 Test Review (WS#4) KEY

1) Position function given. Particle moves along x-axis for all Real number values of t.

$x(t) = t^2e^{-t}$

a) find v(t) and a(t)

b) Determine interval moving left

c) Is particle velocity inc or dec at $t=3$?

d) Is particle speed inc or dec at $t=3$?

a) $v(t) = \overbrace{2t}^{f'}\cdot\overbrace{e^{-t}}^{g} + \overbrace{t^2}^{f}\cdot\overbrace{e^{-t}(-1)}^{g'}$

$v(t) = 2te^{-t} - t^2e^{-t}$

$v(t) = te^{-t}(2-t)$

$a(t) = \overbrace{(2)}^{f'}\overbrace{e^{-t}}^{g} + \overbrace{2t}^{f}\cdot\overbrace{e^{-t}(-1)}^{g'} \pm \left(\overbrace{2t}^{f'}\cdot\overbrace{e^{-t}}^{g} + \overbrace{t^2}^{f}\cdot\overbrace{e^{-t}(-1)}^{g'}\right)$

$a(t) = 2e^{-t} - 2te^{-t} - 2te^{-t} + t^2e^{-t}$

$a(t) = e^{-t}(2-2t-2t+t^2) \rightarrow \boxed{a(t) = e^{-t}(t^2-4t+2)}$

c) $a(3) = e^{-3}(3^2-12+2) < 0$

Velocity is decreasing at $t=3$ since $a(t)<0$

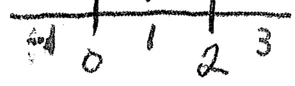
d) $V(3) = 3e^{-3}(2-3) < 0$. Since $V(3)$ and $a(3)$ have same signs, speed is increasing at $t=3$.

b) * create v(t) sign line

* set $v(t) = 0$

$0 = te^{-t}(2-t)$

$t=0$	$e^{-t}=0$	$2-t=0$
$t=0$	none	$t=2$



particle moving left $(-\infty, 0) \cup (2, \infty)$ b/c $v(t) < 0$

2) Find $\frac{dy}{dx}$ $\quad y=(3-5x)^{2^x}$

*log differentiation

$$\ln y = \ln(3-5x)^{2^x}$$

$$\ln y = 2^x \cdot \ln(3-5x)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \overbrace{(\ln 2)(2^x)(1)}^{f'} \cdot \overbrace{\ln(3-5x)}^{g} + \overbrace{2^x}^{f} \cdot \overbrace{\left(\frac{-5}{3-5x}\right)}^{g'}$$

$$\frac{dy}{dx} = y \cdot \left[(\ln 2)2^x \ln(3-5x) + \frac{2^x(-5)}{3-5x}\right]$$

$$\boxed{\frac{dy}{dx} = (3-5x)^{2^x}\left[\ln 2(2^x)\ln(3-5x) - \frac{2^x(5)}{3-5x}\right]}$$

$*\ \frac{d}{dx}\ln u = \frac{u'}{u}$

$\frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

3) Find y' for $y = 3\log_7\left(\frac{x}{e^{2x}\sqrt{1-3x^2}}\right)$

*expand first

$$y = 3\log_7 x - 3\log_7 e^{2x} - 3\log_7(1-3x^2)^{1/2}$$

$$y = 3\log_7 x - 3\log_7 e^{2x} - 3\cdot\frac{1}{2}\log_7(1-3x^2) \longleftarrow \boxed{*\frac{d}{dx}\log_a u = \frac{1}{\ln a}\left(\frac{u'}{u}\right)}$$

$$y' = \frac{3}{\ln 7}\left(\frac{1}{x}\right) - 3\cdot\frac{1}{\ln 7}\left(\frac{\cancel{e^{2x}}(2)}{\cancel{e^{2x}}}\right) - \frac{3}{2}\cdot\frac{1}{\ln 7}\cdot\left(\frac{-6x}{1-3x^2}\right)$$

$$\frac{d}{dx}e^u = e^u\cdot u'$$

$$\boxed{y' = \frac{3}{x\ln 7} - \frac{6}{\ln 7} + \frac{9x}{\ln 7(1-3x^2)}}$$

4) $\ln\left(\frac{\sqrt[3]{y}}{x^5}\right) = 3x^2y - y + 5x - 3$

Find $\left(\frac{dy}{dx}\right)$.

$$\ln\sqrt[3]{y} - \ln x^5 = 3x^2y - y + 5x - 3$$

$$\ln y^{1/3} - \ln x^5 = 3x^2y - y + 5x - 3$$

$$\frac{1}{3}\ln y - 5\ln x = \overbrace{3x^2}^{f}\overbrace{y}^{g} - y + 5x - 3$$

$$\frac{1}{3}\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) - 5\left(\frac{1}{x}\right) = \overbrace{6x}^{f'} \cdot \overbrace{y}^{g} + \overbrace{3x^2}^{f} \cdot \overbrace{\left(\frac{dy}{dx}\right)}^{g'} - 1\left(\frac{dy}{dx}\right) + 5$$

$$\frac{1}{3y}\left(\frac{dy}{dx}\right) - 3x^2\left(\frac{dy}{dx}\right) + 1\left(\frac{dy}{dx}\right) = 6xy + \frac{5}{x} + 5$$

$$\frac{dy}{dx}\left(\frac{1}{3y} - 3x^2 + 1\right) = 6xy + \frac{5}{x} + 5$$

$$\boxed{\frac{dy}{dx} = \frac{6xy + \frac{5}{x} + 5}{\frac{1}{3y} - 3x^2 + 1}}$$

* Implicit differentiation

* expansion property

* product rule

5) Find tangent line equation for $f(x)=e^{-x}\ln x$ at $(1,0)$

$$*\frac{d}{dx}e^{u}=e^{u}\cdot u'$$

$$\frac{d}{dx}\ln u=\frac{u'}{u}$$

$$f'(x)=\overbrace{e^{-x}(-1)}^{f'}\cdot\overbrace{\ln x}^{g}\overset{+}{+}\overbrace{e^{-x}}^{f}\cdot\overbrace{\left(\frac{1}{x}\right)}^{g'}=\frac{-\ln x}{e^{x}}+\frac{1}{xe^{x}}$$

$$f'(1)=\frac{-\ln 1}{e^{1}}+\frac{1}{1(e^{1})}=0+\frac{1}{1(e)}=\boxed{\frac{1}{e}}$$

point: $(1,0)$

slope: $m=\frac{1}{e}$

$$y-y_1=m(x-x_1)$$

$$\boxed{y-0=\frac{1}{e}(x-1)}$$

AP Calculus Ch. 5 Test Topics - Logarithms/Exponentials (7 problems)

1) Given function f(x), find relative min/max/inc/dec/POI/concave up/concave down

2) Particle Motion Problem- Given Position Function, find the following:
    - v(t), a(t), intervals moving left/right, inc/dec speed, inc/dec velocity

3 & 4) (**2 problems**) Find derivatives involving:

    a) Expanding log terms
    b) logs and exponentials of base e and base a
    c) involving product/quotient/chain rule
    d) Find tangent line equation

5) Implicit Differentiation
    - Involving log expansion, product/quotient rule

6) Log differentiation

7) Find derivative of inverse at a point