# Ch.5 Test Review (WS#4) KEY

1) Position function given. Particle moves along x-axis for all Real number values of $t$.

$x(t)=t^2e^{-t}$

a) find $v(t)$ and $a(t)$

b) Determine interval moving left

c) Is particle velocity inc or dec at $t=3$?

d) Is particle speed inc or dec at $t=3$?

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a) $v(t)=\overbrace{2t}^{f'}\cdot\overbrace{e^{-t}}^{g}+\overbrace{t^2}^{f}\cdot\overbrace{e^{-t}(-1)}^{g'}$

$v(t)=2te^{-t}-t^2e^{-t}$

$v(t)=te^{-t}(2-t)$

$a(t)=\overbrace{(2)}^{f'}\overbrace{e^{-t}}^{g}+\overbrace{2t}^{f}\cdot\overbrace{e^{-t}(-1)}^{g'}\overset{+}{-}\left(\overbrace{2t}^{f'}\cdot\overbrace{e^{-t}}^{g}+\overbrace{t^2}^{f}\cdot\overbrace{e^{-t}(-1)}^{g'}\right)$

$a(t)=2e^{-t}-2te^{-t}-2te^{-t}+t^2e^{-t}$

$a(t)=e^{-t}(2-2t-2t+t^2)\rightarrow\boxed{a(t)=e^{-t}(t^2-4t+2)}$

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c) $a(3)=e^{-3}(3^2-12+2)<0$

velocity is decreasing at $t=3$ since $a(t)<0$

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d) $v(3)=3e^{-3}(2-3)<0$. Since $v(3)$ and $a(3)$ have same signs, speed is increasing at $t=3$.

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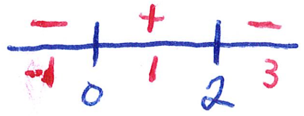
b) * create $v(t)$ sign line

* set $v(t)=0$

$0=te^{-t}(2-t)$

| | | |
|---|---|---|
| $t=0$ | $e^{-t}=0$ | $2-t=0$ |
| $t=0$ | none | $t=2$ |

$v(t)$


particle moving left $(-\infty,0)\cup(2,\infty)$ b/c $v(t)<0$

2) Find $\frac{dy}{dx}$ $\quad y=(3-5x)^{2^x}$

*log differentiation

$$\ln y = \ln(3-5x)^{2^x}$$

$$\ln y = 2^x \cdot \ln(3-5x)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \overbrace{(\ln 2)(2^x)(1)}^{f'} \cdot \overbrace{\ln(3-5x)}^{g} + \overbrace{2^x}^{f} \cdot \overbrace{\left(\frac{-5}{3-5x}\right)}^{g'}$$

$$\frac{dy}{dx} = y \cdot \left[(\ln 2)2^x \ln(3-5x) + \frac{2^x(-5)}{3-5x}\right]$$

$$\boxed{\frac{dy}{dx} = (3-5x)^{2^x}\left[\ln 2(2^x)\ln(3-5x) - \frac{2^x(5)}{3-5x}\right]}$$

$*\ \frac{d}{dx}\ln u = \frac{u'}{u}$

$\frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

3) Find $y'$ for $y = 3\log_7\left(\frac{x}{e^{2x}\sqrt{1-3x^2}}\right)$

*expand first

$$y = 3\log_7 x - 3\log_7 e^{2x} - 3\log_7(1-3x^2)^{1/2}$$

$$\leftarrow \boxed{* \frac{d}{dx}\log_a u = \frac{1}{\ln a}\left(\frac{u'}{u}\right)}$$

$$y = 3\log_7 x - 3\log_7 e^{2x} - 3\cdot\frac{1}{2}\log_7(1-3x^2)$$

$$y' = \frac{3}{\ln 7}\left(\frac{1}{x}\right) - 3\cdot\frac{1}{\ln 7}\left(\frac{\cancel{e^{2x}}(2)}{\cancel{e^{2x}}}\right) - \frac{3}{2}\cdot\frac{1}{\ln 7}\cdot\left(\frac{-6x}{1-3x^2}\right)$$

$$\frac{d}{dx}e^u = e^u \cdot u'$$

$$\boxed{y' = \frac{3}{x\ln 7} - \frac{6}{\ln 7} + \frac{9x}{\ln 7(1-3x^2)}}$$

4) $\ln\left(\frac{\sqrt[3]{y}}{x^5}\right) = 3x^2y - y + 5x - 3$

Find $\left(\frac{dy}{dx}\right)$.

$$\ln\sqrt[3]{y} - \ln x^5 = 3x^2y - y + 5x - 3$$

$$\ln y^{1/3} - \ln x^5 = 3x^2y - y + 5x - 3$$

* Implicit differentiatio

* expansion property

* product rule

$$\frac{1}{3}\ln y - 5\ln x = \overbrace{3x^2}^{f}\overbrace{y}^{g} - y + 5x - 3$$

$$\frac{1}{3}\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) - 5\left(\frac{1}{x}\right) = \overbrace{6x}^{f'}\cdot\overbrace{y}^{g} \overset{+}{+} \overbrace{3x^2}^{f}\cdot\overbrace{\left(\frac{dy}{dx}\right)}^{g'} - 1\left(\frac{dy}{dx}\right) + 5$$

$$\frac{1}{3y}\left(\frac{dy}{dx}\right) - 3x^2\left(\frac{dy}{dx}\right) + 1\left(\frac{dy}{dx}\right) = 6xy + \frac{5}{x} + 5$$

$$\frac{dy}{dx}\left(\frac{1}{3y} - 3x^2 + 1\right) = 6xy + \frac{5}{x} + 5$$

$$\boxed{\frac{dy}{dx} = \frac{6xy + \frac{5}{x} + 5}{\frac{1}{3y} - 3x^2 + 1}}$$

5) Find tangent line equation for $f(x) = e^{-x}\ln x$ at $(1,0)$

$\star \frac{d}{dx}e^{u} = e^{u} \cdot u'$

$\frac{d}{dx}\ln u = \frac{u'}{u}$

$$f'(x) = \overbrace{e^{-x}(-1)}^{f'} \cdot \overbrace{\ln x}^{g} \overset{+}{} + \overbrace{e^{-x}}^{f} \cdot \overbrace{\left(\frac{1}{x}\right)}^{g'} = \frac{-\ln x}{e^{x}} + \frac{1}{xe^{x}}$$

$$f'(1) = \frac{-\ln 1}{e^{1}} + \frac{1}{1(e^{1})} = 0 + \frac{1}{1(e)} = \boxed{\frac{1}{e}}$$

point: $(1,0)$

slope: $m = \frac{1}{e}$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 0 = \frac{1}{e}(x-1)}$$