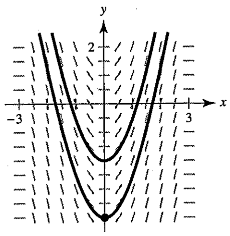
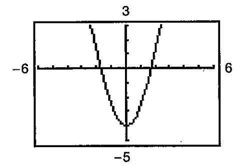
67. $\frac{1}{4}\sin^4 x + C$ **69.** $-2\sqrt{1 - \sin\theta} + C$

71. $\frac{1}{3\pi}(1 + \sec \pi x)^3 + C$

73. (a) Answers will vary.
Sample answer:



(b) $y = -\frac{1}{3}(9 - x^2)^{3/2} + 5$



75. $\frac{455}{2}$ **77.** 2 **79.** $28\pi/15$ **81.** 2 **83.** $\frac{468}{7}$

85. (a) $\frac{64}{5}$ (b) $\frac{32}{5}$ (c) $\frac{96}{5}$ (d) -32

87. Trapezoidal Rule: 0.285
Simpson's Rule: 0.284
Graphing Utility: 0.284

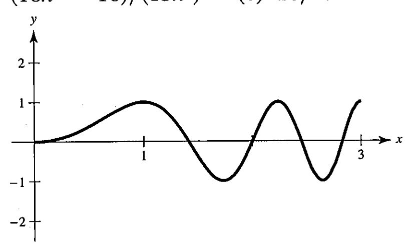
89. Trapezoidal Rule: 0.637
Simpson's Rule: 0.685
Graphing Utility: 0.704

P.S. Problem Solving *(page 315)*

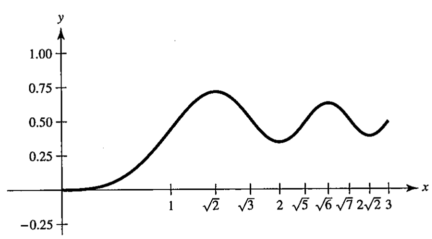
1. (a) $L(1) = 0$ (b) $L'(x) = 1/x,\ L'(1) = 1$
(c) $x \approx 2.718$ (d) Proof

3. (a) $\lim_{n\to\infty}\left[\frac{32}{n^5}\sum_{i=1}^{n} i^4 - \frac{64}{n^4}\sum_{i=1}^{n} i^3 + \frac{32}{n^3}\sum_{i=1}^{n} i^2\right]$
(b) $(16n^4 - 16)/(15n^4)$ (c) $16/15$

5. (a)

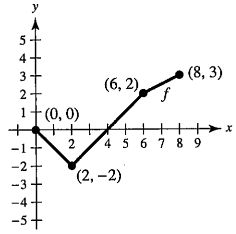


(b)



(c) Relative maxima at $x = \sqrt{2},\ \sqrt{6}$
Relative minima at $x = 2,\ 2\sqrt{2}$
(d) Points of inflection at $x = 1, \sqrt{3}, \sqrt{5}, \sqrt{7}$

7. (a)



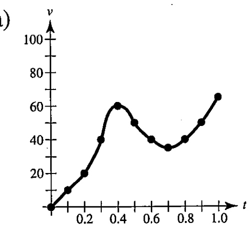
(b)

x	0	1	2	3	4	5	6	7	8
$F(x)$	0	$-\frac{1}{2}$	-2	$-\frac{7}{2}$	-4	$-\frac{7}{2}$	-2	$\frac{1}{4}$	3

(c) $x = 4, 8$ (d) $x = 2$

9. Proof **11.** $\frac{2}{3}$ **13.** $1 \le \int_0^1 \sqrt{1 + x^4}\,dx \le \sqrt{2}$

15. (a)



(b) (0, 0.4) and (0.7, 1.0) (c) 150 mi/h^2
(d) Total distance traveled in miles; 38.5 mi
(e) Sample answer: 100 mi/h^2

17. (a)–(c) Proofs

19. (a) $R(n), I, T(n), L(n)$
(b) $S(4) = \frac{1}{3}[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)] \approx 5.42$

Chapter 5

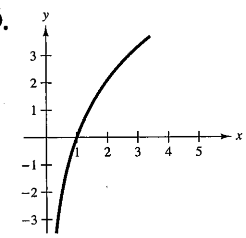
Section 5.1 *(page 325)*

1. (a) 3.8067 (b) $\ln 45 = \int_1^{45} \frac{1}{t}\,dt \approx 3.8067$

3. (a) -0.2231 (b) $\ln 0.8 = \int_1^{0.8} \frac{1}{t}\,dt \approx -0.2231$

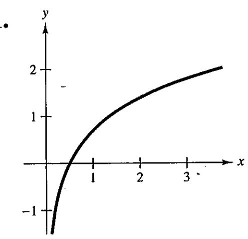
5. b **6.** d **7.** a **8.** c

9.



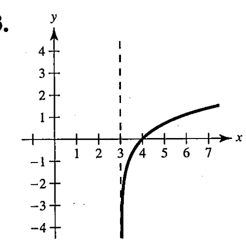
Domain: $x > 0$

11.



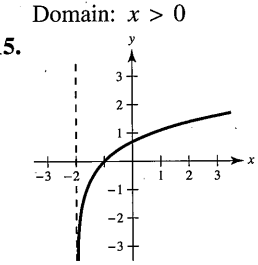
Domain: $x > 0$

13.



Domain: $x > 3$

15.



Domain: $x > -2$

17. (a) 1.7917 (b) -0.4055 (c) 4.3944 (d) 0.5493

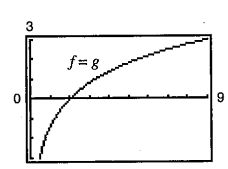
19. $\ln x - \ln 4$ **21.** $\ln x + \ln y - \ln z$

23. $\ln x + \frac{1}{2}\ln(x^2 + 5)$ **25.** $\frac{1}{2}[\ln(x - 1) - \ln x]$

27. $\ln z + 2\ln(z - 1)$ **29.** $\ln\frac{x - 2}{x + 2}$

31. $\ln\sqrt[3]{\frac{x(x + 3)^2}{x^2 - 1}}$ **33.** $\ln\frac{9}{\sqrt{x^2 + 1}}$

35. (a)



(b) $f(x) = \ln\frac{x^2}{4} = \ln x^2 - \ln 4$
$= 2\ln x - \ln 4$
$= g(x)$

37. $-\infty$ **39.** $\ln 4 \approx 1.3863$ **41.** $1/x$ **43.** $2/x$

45. $4(\ln x)^3/x$ **47.** $2/(t+1)$ **49.** $\dfrac{2x^2-1}{x(x^2-1)}$

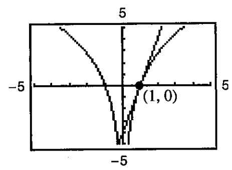
51. $\dfrac{1-x^2}{x(x^2+1)}$ **53.** $\dfrac{1-2\ln t}{t^3}$ **55.** $\dfrac{2}{x\ln x^2}=\dfrac{1}{x\ln x}$

57. $\dfrac{1}{1-x^2}$ **59.** $\dfrac{-4}{x(x^2+4)}$ **61.** $\cot x$

63. $-\tan x+\dfrac{\sin x}{\cos x-1}$

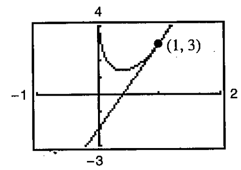
65. (a) $y=4x-4$

(b)



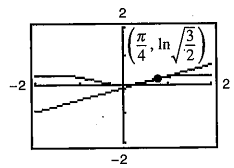
67. (a) $5x-y-2=0$

(b)



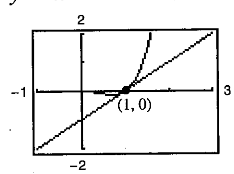
69. (a) $y=\frac{1}{3}x-\frac{1}{12}\pi+\frac{1}{2}\ln\left(\frac{3}{2}\right)$

(b)



71. (a) $y=x-1$

(b)



73. $\dfrac{2xy}{3-2y^2}$ **75.** $\dfrac{y(1-6x^2)}{1+y}$

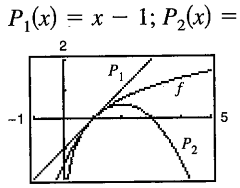
77. $xy''+y'=x(-2/x^2)+(2/x)=0$

79. Relative minimum: $\left(1,\frac{1}{2}\right)$

81. Relative minimum: $(e^{-1},-e^{-1})$

83. Relative minimum: (e,e); Point of inflection: $(e^2,e^2/2)$

85. $P_1(x)=x-1$; $P_2(x)=x-1-\frac{1}{2}(x-1)^2$



The values of f, P_1, and P_2 and their first derivatives agree at $x=1$.

87. $x\approx 0.567$ **89.** $(2x^2+1)/\sqrt{x^2+1}$

91. $\dfrac{3x^3+15x^2-8x}{2(x+1)^3\sqrt{3x-2}}$ **93.** $\dfrac{(2x^2+2x-1)\sqrt{x-1}}{(x+1)^{3/2}}$

95. The domain of the natural logarithmic function is $(0,\infty)$, and the range is $(-\infty,\infty)$. The function is continuous, increasing, and one-to-one, and its graph is concave downward. In addition, if a and b are positive numbers and n is rational, then $\ln(1)=0$, $\ln(a\cdot b)=\ln a+\ln b$, $\ln(a^n)=n\ln a$, and $\ln(a/b)=\ln a-\ln b$.

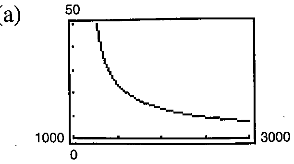
97. (a) Yes. If the graph of g is increasing, then $g'(x)>0$. Because $f(x)>0$, you know that $f'(x)=g'(x)f(x)$ and thus $f'(x)>0$. Therefore, the graph of f is increasing.

(b) No. Let $f(x)=x^2+1$ (positive and concave up), and let $g(x)=\ln(x^2+1)$ (not concave up).

99. False. $\ln x+\ln 25=\ln 25x$

101. False. π is a constant, so $\dfrac{d}{dx}[\ln\pi]=0$.

103. (a)



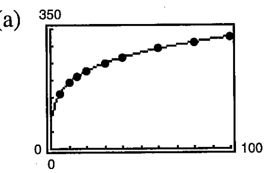
(b) 30 yr; $503,434.80

(c) 20 yr; $386,685.60

(d) When $x=1398.43$, $dt/dx\approx -0.0805$. When $x=1611.19$, $dt/dx\approx -0.0287$.

(e) Two benefits of a higher monthly payment are a shorter term and a lower total amount paid.

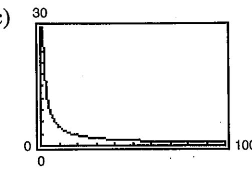
105. (a)



(b) $T'(10)\approx 4.75°/\text{lb/in.}^2$

$T'(70)\approx 0.97°/\text{lb/in.}^2$

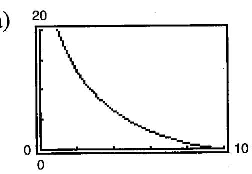
(c)



$\lim_{p\to\infty} T'(p)=0$

Answers will vary.

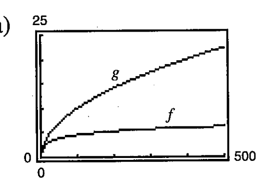
107. (a)



(b) When $x=5$, $dy/dx=-\sqrt{3}$. When $x=9$, $dy/dx=-\sqrt{19}/9$.

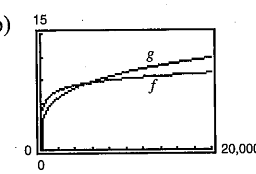
(c) $\lim_{x\to 10^-}\dfrac{dy}{dx}=0$

109. (a)



For $x>4$, $g'(x)>f'(x)$. g is increasing at a faster rate than f for large values of x.

(b)



For $x>256$, $g'(x)>f'(x)$. g is increasing at a faster rate than f for large values of x.

$f(x)=\ln x$ increases very slowly for large values of x.

Section 5.2 *(page 334)*

1. $5\ln|x|+C$ **3.** $\ln|x+1|+C$ **5.** $\frac{1}{2}\ln|2x+5|+C$

7. $\frac{1}{2}\ln|x^2-3|+C$ **9.** $\ln|x^4+3x|+C$

11. $x^2/2-\ln(x^4)+C$ **13.** $\frac{1}{3}\ln|x^3+3x^2+9x|+C$

15. $\frac{1}{2}x^2-4x+6\ln|x+1|+C$ **17.** $\frac{1}{3}x^3+5\ln|x-3|+C$

19. $\frac{1}{3}x^3-2x+\ln\sqrt{x^2+2}+C$ **21.** $\frac{1}{3}(\ln x)^3+C$

23. $-\frac{2}{3}\ln|1-3\sqrt{x}|+C$

25. $2\ln|x-1|-2/(x-1)+C$

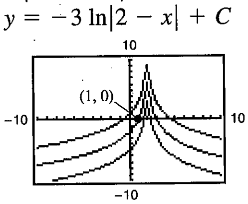
27. $\sqrt{2x}-\ln|1+\sqrt{2x}|+C$

29. $x+6\sqrt{x}+18\ln|\sqrt{x}-3|+C$ **31.** $3\ln\left|\sin\dfrac{\theta}{3}\right|+C$

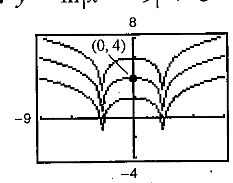
33. $-\frac{1}{2}\ln|\csc 2x+\cot 2x|+C$ **35.** $\frac{1}{3}\sin 3\theta-\theta+C$

37. $\ln|1+\sin t|+C$ **39.** $\ln|\sec x-1|+C$

41. $y=-3\ln|2-x|+C$

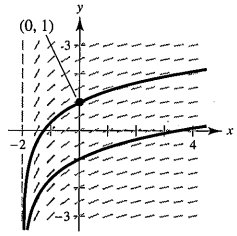


43. $y=\ln|x^2-9|+C$

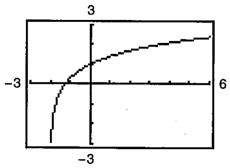


45. $f(x)=-2\ln x+3x-2$

47. (a)



(b) $y = \ln\left(\frac{x+2}{2}\right) + 1$



49. $\frac{5}{3}\ln 13 \approx 4.275$ **51.** $\frac{7}{3}$ **53.** $-\ln 3 \approx -1.099$

55. $\ln\left|\frac{2 - \sin 2}{1 - \sin 1}\right| \approx 1.929$ **57.** $2[\sqrt{x} - \ln(1 + \sqrt{x})] + C$

59. $\ln\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + 2\sqrt{x} + C$ **61.** $\ln(\sqrt{2} + 1) - \frac{\sqrt{2}}{2} \approx 0.174$

63. $1/x$ **65.** $1/x$ **67.** $6 \ln 3$ **69.** $\frac{1}{2}\ln 2$

71. $\frac{15}{2} + 8 \ln 2 \approx 13.045$ **73.** $(12/\pi)\ln(2 + \sqrt{3}) \approx 5.03$

75. Trapezoidal Rule: 20.2
Simpson's Rule: 19.4667

77. Trapezoidal Rule: 5.3368
Simpson's Rule: 5.3632

79. Power Rule **81.** Log Rule **83.** d **85.** $x = 2$

87. Proof

89. $-\ln|\cos x| + C = \ln|1/\cos x| + C = \ln|\sec x| + C$

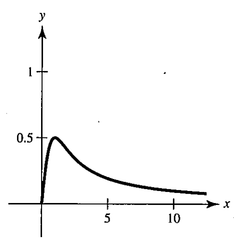
91. $\ln|\sec x + \tan x| + C = \ln\left|\frac{\sec^2 x - \tan^2 x}{\sec x - \tan x}\right| + C$
$= -\ln|\sec x - \tan x| + C$

93. 1 **95.** $1/(e - 1) \approx 0.582$

97. $P(t) = 1000(12 \ln|1 + 0.25t| + 1)$; $P(3) \approx 7715$

99. About 4.15 min

101.



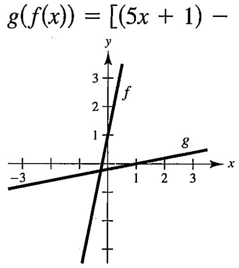
(a) $A = \frac{1}{2}\ln 2 - \frac{1}{4}$
(b) $0 < m < 1$
(c) $A = \frac{1}{2}(m - \ln m - 1)$

103. False. $\frac{1}{2}\ln x = \ln x^{1/2}$ **105.** True **107.** Proof

Section 5.3 *(page 343)*

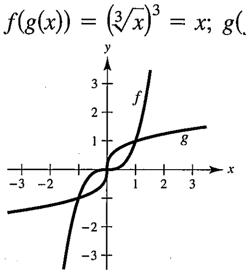
1. (a) $f(g(x)) = 5[(x - 1)/5] + 1 = x$;
$g(f(x)) = [(5x + 1) - 1]/5 = x$

(b)



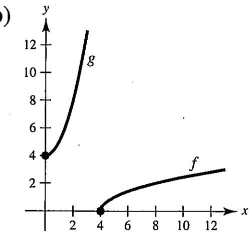
3. (a) $f(g(x)) = (\sqrt[3]{x})^3 = x$; $g(f(x)) = \sqrt[3]{x^3} = x$

(b)



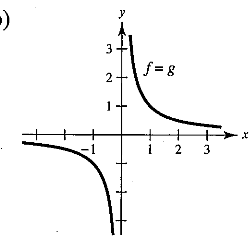
5. (a) $f(g(x)) = \sqrt{x^2 + 4 - 4} = x$;
$g(f(x)) = (\sqrt{x - 4})^2 + 4 = x$

(b)



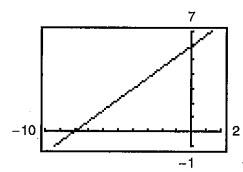
7. (a) $f(g(x)) = \frac{1}{1/x} = x$; $g(f(x)) = \frac{1}{1/x} = x$

(b)



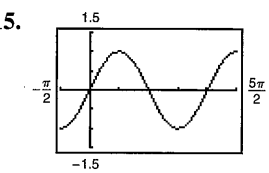
9. c **10.** b **11.** a **12.** d

13.



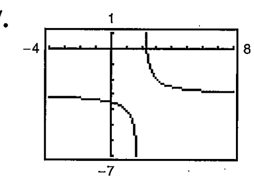
One-to-one, inverse exists.

15.



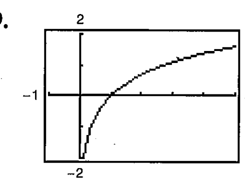
Not one-to-one, inverse does not exist.

17.



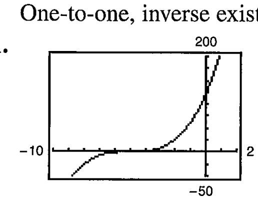
One-to-one, inverse exists.

19.



One-to-one, inverse exists.

21.



One-to-one, inverse exists.

23. Inverse exists. **25.** Inverse does not exist.

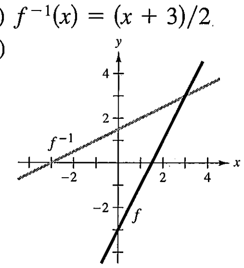
27. Inverse exists. **29.** $f'(x) = 2(x - 4) > 0$ on $(4, \infty)$

31. $f'(x) = -8/x^3 < 0$ on $(0, \infty)$

33. $f'(x) = -\sin x < 0$ on $(0, \pi)$

35. (a) $f^{-1}(x) = (x + 3)/2$

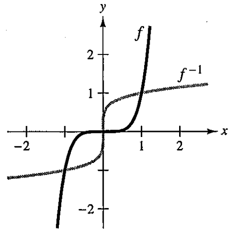
(b)



(c) f and f^{-1} are symmetric about $y = x$.
(d) Domain of f and f^{-1}: all real numbers
Range of f and f^{-1}: all real numbers

37. (a) $f^{-1}(x) = x^{1/5}$

(b)



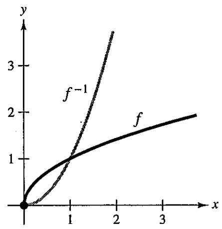
(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: all real numbers

Range of f and f^{-1}: all real numbers

39. (a) $f^{-1}(x) = x^2, \quad x \geq 0$

(b)



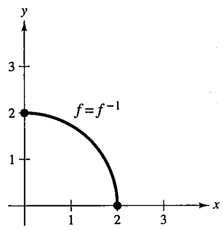
(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: $x \geq 0$

Range of f and f^{-1}: $y \geq 0$

41. (a) $f^{-1}(x) = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$

(b)



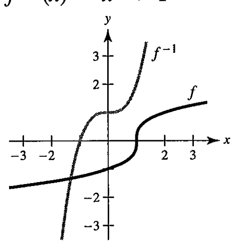
(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: $0 \leq x \leq 2$

Range of f and f^{-1}: $0 \leq y \leq 2$

43. (a) $f^{-1}(x) = x^3 + 1$

(b)



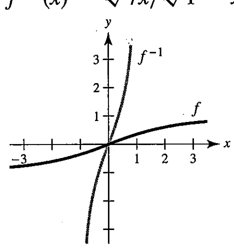
(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: all real numbers

Range of f and f^{-1}: all real numbers

45. (a) $f^{-1}(x) = \sqrt{7}x/\sqrt{1 - x^2}, \quad -1 < x < 1$

(b)



(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f: all real numbers

Domain of f^{-1}: $-1 < x < 1$

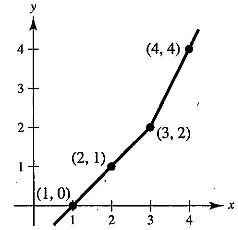
Range of f: $-1 < y < 1$

Range of f^{-1}: all real numbers

47.

x	0	1	2	4
$f(x)$	1	2	3	4

x	1	2	3	4
$f^{-1}(x)$	0	1	2	4



49. (a) Proof

(b) $y = \frac{20}{7}(80 - x)$

x: total cost

y: number of pounds of the less expensive commodity

(c) $[62.5, 80]$ (d) 20 lb

51. One-to-one

$f^{-1}(x) = x^2 + 2, \quad x \geq 0$

53. One-to-one

$f^{-1}(x) = 2 - x, \quad x \geq 0$

55. Sample answer: $f^{-1}(x) = \sqrt{x} + 3, \quad x \geq 0$

57. Sample answer: $f^{-1}(x) = x - 3, \quad x \geq 0$

59. Inverse exists. Volume is an increasing function, and therefore is one-to-one. The inverse function gives the time t corresponding to the volume V.

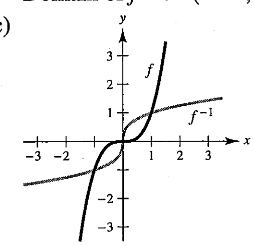
61. Inverse does not exist. **63.** $-1/6$ **65.** $1/17$

67. $2\sqrt{3}/3$ **69.** -2

71. (a) Domain of f: $(-\infty, \infty)$ (b) Range of f: $(-\infty, \infty)$

Domain of f^{-1}: $(-\infty, \infty)$ Range of f^{-1}: $(-\infty, \infty)$

(c)

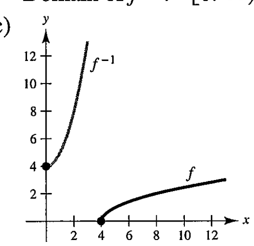


(d) $f'\left(\frac{1}{2}\right) = \frac{3}{4}, (f^{-1})'\left(\frac{1}{8}\right) = \frac{4}{3}$

73. (a) Domain of f: $[4, \infty)$ (b) Range of f: $[0, \infty)$

Domain of f^{-1}: $[0. \infty)$ Range of f^{-1}: $[4, \infty)$

(c)



(d) $f'(5) = \frac{1}{2}, (f^{-1})'(1) = 2$

75. 32 **77.** 600 **79.** $(g^{-1} \circ f^{-1})(x) = (x + 1)/2$

81. $(f \circ g)^{-1}(x) = (x + 1)/2$

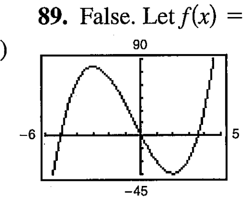
83. Let $y = f(x)$ be one-to-one. Solve for x as a function of y. Interchange x and y to get $y = f^{-1}(x)$. Let the domain of f^{-1} be the range of f. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Sample answer: $f(x) = x^3$; $y = x^3$; $x = \sqrt[3]{y}$; $y = \sqrt[3]{x}$; $f^{-1}(x) = \sqrt[3]{x}$

85. Many x-values yield the same y-value. For example, $f(\pi) = 0 = f(0)$. The graph is not continuous at $[(2n - 1)\pi]/2$, where n is an integer.

87. $\frac{1}{4}$ **89.** False. Let $f(x) = x^2$. **91.** True

93. (a)



(b) $c = 2$

f does not pass the horizontal line test.

95–97. Proofs **99.** Proof; concave upward

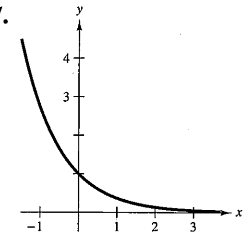
101. Proof; $\sqrt{5}/5$

103. (a) Proof (b) $f^{-1}(x) = \dfrac{b - dx}{cx - a}$
(c) $a = -d$, or $b = c = 0$, $a = d$

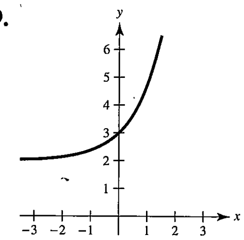
Section 5.4 *(page 352)*

1. $x = 4$ **3.** $x \approx 2.485$ **5.** $x = 0$ **7.** $x \approx 0.511$
9. $x \approx 8.862$ **11.** $x \approx 7.389$ **13.** $x \approx 10.389$
15. $x \approx 5.389$

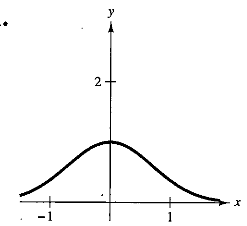
17.



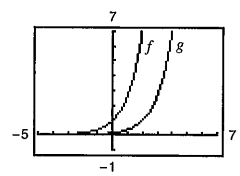
19.



21.

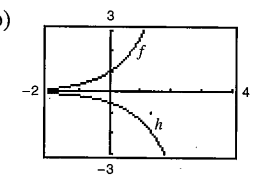


23. (a)



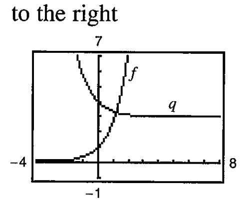
Translation two units to the right

(b)



Reflection in the x-axis and a vertical shrink

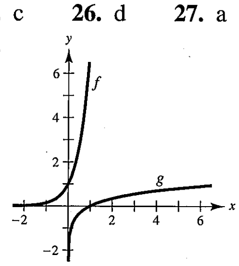
(c)



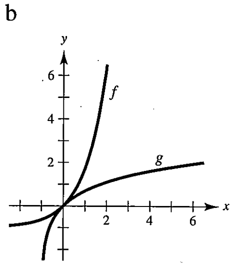
Reflection in the y-axis and a translation three units upward

25. c **26.** d **27.** a **28.** b

29.

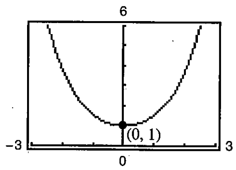


31.

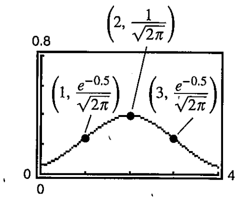


33. $2e^{2x}$ **35.** $e^{\sqrt{x}}/(2\sqrt{x})$ **37.** e^{x-4} **39.** $e^x\left(\dfrac{1}{x} + \ln x\right)$
41. $e^x(x^3 + 3x^2)$ **43.** $3(e^{-t} + e^t)^2(e^t - e^{-t})$
45. $2e^{2x}/(1 + e^{2x})$ **47.** $-2(e^x - e^{-x})/(e^x + e^{-x})^2$
49. $-2e^x/(e^x - 1)^2$ **51.** $2e^x \cos x$ **53.** $\cos(x)/x$
55. $y = 3x + 1$ **57.** $y = -x + 2$ **59.** $y = (1/e)x - 1/e$
61. $y = ex$ **63.** $\dfrac{10 - e^y}{xe^y + 3}$ **65.** $y = (-e - 1)x + 1$
67. $3(6x + 5)e^{-3x}$
69. $y'' - y = 0$
$4e^{-x} - 4e^{-x} = 0$

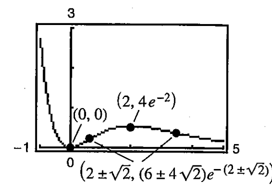
71. Relative minimum: $(0, 1)$



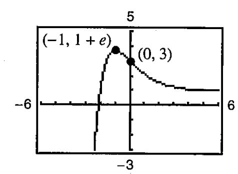
73. Relative maximum:
$(2, 1/\sqrt{2\pi})$
Points of inflection:
$\left(1, \dfrac{e^{-0.5}}{\sqrt{2\pi}}\right), \left(3, \dfrac{e^{-0.5}}{\sqrt{2\pi}}\right)$



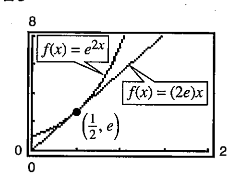
75. Relative minimum: $(0, 0)$
Relative maximum: $(2, 4e^{-2})$
Points of inflection:
$\left(2 \pm \sqrt{2}, (6 \pm 4\sqrt{2})e^{-(2 \pm \sqrt{2})}\right)$



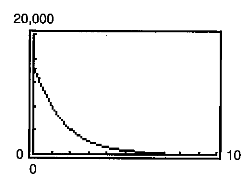
77. Relative maximum:
$(-1, 1 + e)$
Point of inflection: $(0, 3)$



79. $A = \sqrt{2}e^{-1/2}$
81. $\left(\frac{1}{2}, e\right)$

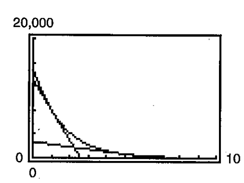


83. (a)

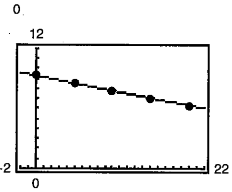


(b) When $t = 1$,
$\dfrac{dV}{dt} \approx -5028.84$.
When $t = 5$,
$\dfrac{dV}{dt} \approx -406.89$.

(c)



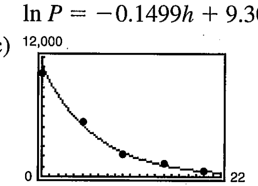
85. (a)



$\ln P = -0.1499h + 9.3018$

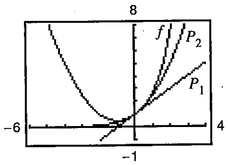
(b) $P = 10{,}957.7e^{-0.1499h}$

(c)



(d) $h = 5$: -776
$h = 18$: -111

87. $P_1 = 1 + x;\ P_2 = 1 + x + \frac{1}{2}x^2$



The values of f, P_1, and P_2 and their first derivatives agree at $x = 0$.

89. $12! = 479{,}001{,}600$
Stirling's Formula: $12! \approx 475{,}687{,}487$

91. $e^{5x} + C$ **93.** $\frac{1}{2}e^{2x-1} + C$ **95.** $\frac{1}{3}e^{x^3} + C$

97. $2e^{\sqrt{x}} + C$

99. $x - \ln(e^x + 1) + C_1$ or $-\ln(1 + e^{-x}) + C_2$

101. $-\frac{2}{3}(1 - e^x)^{3/2} + C$ **103.** $\ln|e^x - e^{-x}| + C$

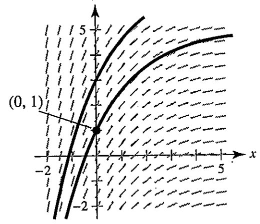
105. $-\frac{5}{2}e^{-2x} + e^{-x} + C$ **107.** $\ln|\cos e^{-x}| + C$

109. $(e^2 - 1)/(2e^2)$ **111.** $(e - 1)/(2e)$

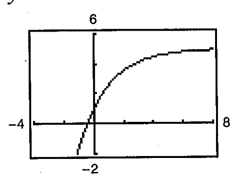
113. $(e/3)(e^2 - 1)$ **115.** $\ln\left(\frac{1 + e^6}{2}\right)$

117. $(1/\pi)[e^{\sin(\pi^2/2)} - 1]$

119. (a)

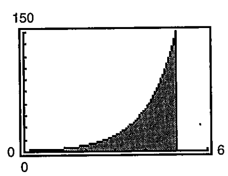


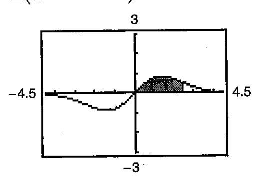
(b) $y = -4e^{-x/2} + 5$



121. $[1/(2a)]e^{ax^2} + C$ **123.** $f(x) = \frac{1}{2}(e^x + e^{-x})$

125. $e^5 - 1 \approx 147.413$ **127.** $2(1 - e^{-3/2}) \approx 1.554$





129. Midpoint Rule: 92.190; Trapezoidal Rule: 93.837;
Simpson's Rule: 92.7385

131. The probability that a given battery will last between 48 months and 60 months is approximately 47.72%.

133. $a = \ln 3$

135. $f(x) = e^x$
The domain of $f(x)$ is $(-\infty, \infty)$, and the range of $f(x)$ is $(0, \infty)$. $f(x)$ is continuous, increasing, one-to-one, and concave upward on its entire domain.
$\lim_{x\to-\infty} e^x = 0$ and $\lim_{x\to\infty} e^x = \infty$

137. (a) Log Rule (b) Substitution

139. $\int_0^x e^t\,dt \ge \int_0^x 1\,dt;\ e^x - 1 \ge x;\ e^x \ge x + 1$ for $x \ge 0$

141. (a) $t = \frac{1}{2k}\ln\frac{B}{A}$
(b) $x''(t) = k^2(Ae^{kt} + Be^{-kt})$, k^2 is the constant of proportionality.

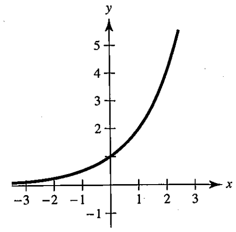
143. Proof

Section 5.5 *(page 362)*

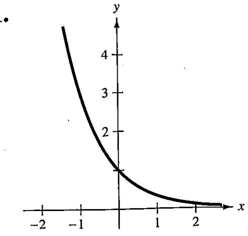
1. -3 **3.** 0 **5.** (a) $\log_2 8 = 3$ (b) $\log_3(1/3) = -1$

7. (a) $10^{-2} = 0.01$ (b) $\left(\frac{1}{2}\right)^{-3} = 8$

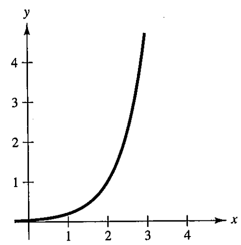
9.



11.



13.



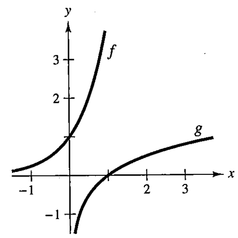
15. d **16.** c **17.** b **18.** a

19. (a) $x = 3$ (b) $x = -1$ **21.** (a) $x = \frac{1}{3}$ (b) $x = \frac{1}{16}$

23. (a) $x = -1, 2$ (b) $x = \frac{1}{3}$ **25.** 1.965 **27.** -6.288

29. 12.253 **31.** 33.000 **33.** ± 11.845

35.



37. $(\ln 4)4^x$ **39.** $(-4\ln 5)5^{-4x}$ **41.** $9^x(x\ln 9 + 1)$

43. $t2^t(t\ln 2 + 2)$ **45.** $-2^{-\theta}[(\ln 2)\cos \pi\theta + \pi \sin \pi\theta]$.

47. $5/[(\ln 4)(5x + 1)]$ **49.** $2/[(\ln 5)(t - 4)]$

51. $x/[(\ln 5)(x^2 - 1)]$ **53.** $(x - 2)/[(\ln 2)x(x - 1)]$

55. $(3x - 2)/[(2x \ln 3)(x - 1)]$ **57.** $5(1 - \ln t)/(t^2 \ln 2)$

59. $y = -2x\ln 2 - 2\ln 2 + 2$

61. $y = [1/(27\ln 3)]x + 3 - 1/\ln 3$ **63.** $2(1 - \ln x)x^{(2/x)-2}$

65. $(x - 2)^{x+1}[(x + 1)/(x - 2) + \ln(x - 2)]$

67. $y = x$ **69.** $y = \frac{\cos e}{e}x - \cos e + 1$

71. $3^x/\ln 3 + C$ **73.** $\frac{1}{3}x^3 - \frac{2^{-x}}{\ln 2} + C$

75. $[-1/(2\ln 5)](5^{-x^2}) + C$ **77.** $\ln(3^{2x} + 1)/(2\ln 3) + C$

79. $7/(2\ln 2)$ **81.** $4/\ln 5 - 2/\ln 3$ **83.** $26/\ln 3$

85. (a) $x > 0$ (b) 10^x (c) $3 \le f(x) \le 4$
(d) $0 < x < 1$ (e) 10 (f) 100^n

87. (a) \$40.64 (b) $C'(1) \approx 0.051P$, $C'(8) \approx 0.072P$
(c) $\ln 1.05$

89.

n	1	2	4	12
A	\$1410.60	\$1414.78	\$1416.91	\$1418.34

n	365	Continuous
A	\$1419.04	\$1419.07

91.

n	1	2	4	12
A	\$4321.94	\$4399.79	\$4440.21	\$4467.74

n	365	Continuous
A	\$4481.23	\$4481.69

93.

t	1	10	20	30
P	\$95,122.94	\$60,653.07	\$36,787.94	\$22,313.02

t	40	50
P	\$13,533.53	\$8208.50

95.

t	1	10	20	30
P	\$95,132.82	\$60,716.10	\$36,864.45	\$22,382.66

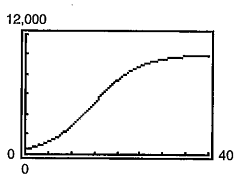
t	40	50
P	\$13,589.88	\$8251.24

97. c

99. (a) 6.7 million ft^3/acre

(b) $t = 20$: $\dfrac{dV}{dt} = 0.073$; $t = 60$: $\dfrac{dV}{dt} = 0.040$

101. (a)



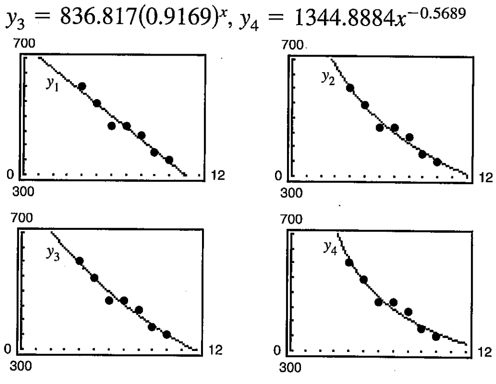
(b) 10,000 fish

(c) 1 month: About 114 fish/mo
10 months: About 403 fish/mo

(d) About 15 mo

103. (a) $y_1 = -40x + 743$, $y_2 = 968 - 265.5 \ln x$,
$y_3 = 836.817(0.9169)^x$, $y_4 = 1344.8884x^{-0.5689}$

(b)



(c) The number of pancreas transplants is decreasing by about 40 transplants each year.

(d) $y_1'(8) = -40.04$, $y_2'(8) = -33.18$, $y_3'(8) = -36.27$, $y_4'(8) = -29.30$; y_1 is decreasing at the greatest rate.

105. $y = 1200(0.6^t)$ **107.** e **109.** e^2

111. False. e is an irrational number. **113.** True **115.** True

117. (a) $(2^3)^2 = 2^6 = 64$
$2^{(3^2)} = 2^9 = 512$

(b) No. $f(x) = (x^x)^x = x^{(x^2)}$ and $g(x) = x^{(x^x)}$

(c) $f'(x) = x^{x^2}(x + 2x \ln x)$
$g'(x) = x^{x^x + x - 1}[x(\ln x)^2 + x \ln x + 1]$

119. Proof

121. (a) $\dfrac{dy}{dx} = \dfrac{y^2 - yx \ln y}{x^2 - xy \ln x}$

(b) (i) 1 when $c \neq 0$, $c \neq e$ (ii) -3.1774
(iii) -0.3147

(c) (e, e)

123. Putnam Problem B3, 1951

Section 5.6 *(page 372)*

1. $\left(-\sqrt{2}/2, 3\pi/4\right), (1/2, \pi/3), \left(\sqrt{3}/2, \pi/6\right)$ **3.** $\pi/6$

5. $\pi/3$ **7.** $\pi/6$ **9.** $-\pi/4$ **11.** 2.50

13. $\arccos(1/1.269) \approx 0.66$ **15.** x **17.** $\sqrt{1 - x^2}/x$

19. $1/x$ **21.** (a) $3/5$ (b) $5/3$

23. (a) $-\sqrt{3}$ (b) $-\frac{13}{5}$ **25.** $\sqrt{1 - 4x^2}$

27. $\sqrt{x^2 - 1}/|x|$ **29.** $\sqrt{x^2 - 9}/3$ **31.** $\sqrt{x^2 + 2}/x$

33. $x = \frac{1}{3}\left[\sin\left(\frac{1}{2}\right) + \pi\right] \approx 1.207$ **35.** $x = \frac{1}{3}$

37. (a) and (b) Proofs **39.** $2/\sqrt{2x - x^2}$

41. $-3/\sqrt{4 - x^2}$ **43.** $e^x/(1 + e^{2x})$

45. $\left(3x - \sqrt{1 - 9x^2} \arcsin 3x\right)/\left(x^2\sqrt{1 - 9x^2}\right)$

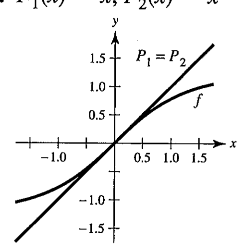
47. $-t/\sqrt{1 - t^2}$ **49.** $2 \arccos x$ **51.** $1/(1 - x^4)$

53. $\arcsin x$ **55.** $x^2/\sqrt{16 - x^2}$ **57.** $2/(1 + x^2)^2$

59. $y = \frac{1}{3}\left(4\sqrt{3}x - 2\sqrt{3} + \pi\right)$ **61.** $y = \frac{1}{4}x + (\pi - 2)/4$

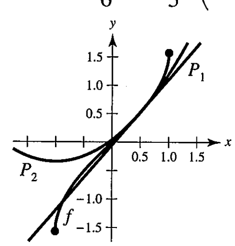
63. $y = (2\pi - 4)x + 4$

65. $P_1(x) = x$; $P_2(x) = x$



67. $P_1(x) = \dfrac{\pi}{6} + \dfrac{2\sqrt{3}}{3}\left(x - \dfrac{1}{2}\right)$

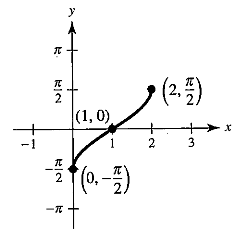
$P_2(x) = \dfrac{\pi}{6} + \dfrac{2\sqrt{3}}{3}\left(x - \dfrac{1}{2}\right) + \dfrac{2\sqrt{3}}{9}\left(x - \dfrac{1}{2}\right)^2$



69. Relative maximum: $(1.272, -0.606)$
Relative minimum: $(-1.272, 3.747)$

71. Relative maximum: $(2, 2.214)$

73.

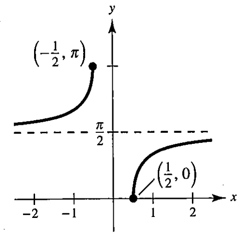


Maximum: $\left(2, \frac{\pi}{2}\right)$

Minimum: $\left(0, -\frac{\pi}{2}\right)$

Point of inflection: $(1, 0)$

75.



Maximum: $\left(-\frac{1}{2}, \pi\right)$

Minimum: $\left(\frac{1}{2}, 0\right)$

Asymptote: $y = \frac{\pi}{2}$

77. $y = -2\pi x/(\pi + 8) + 1 - \pi^2/(2\pi + 16)$

79. $y = -x + \sqrt{2}$

81. If the domains were not restricted, then the trigonometric functions would have no inverses, because they would not be one-to-one.

83. (a) $\arcsin(\arcsin 0.5) \approx 0.551$
$\arcsin(\arcsin 1)$ does not exist.
(b) $\sin(-1) \le x \le \sin(1)$

85. False. The range of arccos is $[0, \pi]$.

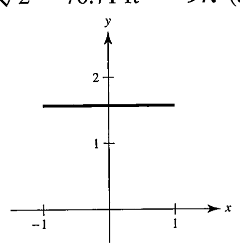
87. True **89.** True

91. (a) $\theta = \operatorname{arccot}(x/5)$
(b) $x = 10$: 16 rad/h; $x = 3$: 58.824 rad/h

93. (a) $h(t) = -16t^2 + 256$; $t = 4$ sec
(b) $t = 1$: -0.0520 rad/sec; $t = 2$: -0.1116 rad/sec

95. $50\sqrt{2} \approx 70.71$ ft **97.** (a) and (b) Proofs

99. (a)



(b) The graph is a horizontal line at $\frac{\pi}{2}$.
(c) Proof

101. $c = 2$ **103.** Proof

Section 5.7 *(page 380)*

1. $\arcsin \frac{x}{3} + C$ **3.** $\operatorname{arcsec}|2x| + C$

5. $\arcsin(x + 1) + C$ **7.** $\frac{1}{2}\arcsin t^2 + C$

9. $\frac{1}{10}\arctan \frac{t^2}{5} + C$ **11.** $\frac{1}{4}\arctan(e^{2x}/2) + C$

13. $\arcsin\left(\frac{\tan x}{5}\right) + C$ **15.** $2\arcsin\sqrt{x} + C$

17. $\frac{1}{2}\ln(x^2 + 1) - 3\arctan x + C$

19. $8\arcsin[(x - 3)/3] - \sqrt{6x - x^2} + C$ **21.** $\pi/6$

23. $\pi/6$ **25.** $\frac{1}{5}\arctan\frac{3}{5} \approx 0.108$

27. $\arctan 5 - \pi/4 \approx 0.588$ **29.** $\pi/4$ **31.** $\frac{1}{32}\pi^2 \approx 0.308$

33. $\pi/2$ **35.** $\ln|x^2 + 6x + 13| - 3\arctan[(x + 3)/2] + C$

37. $\arcsin[(x + 2)/2] + C$ **39.** $4 - 2\sqrt{3} + \frac{1}{6}\pi \approx 1.059$

41. $\frac{1}{2}\arctan(x^2 + 1) + C$

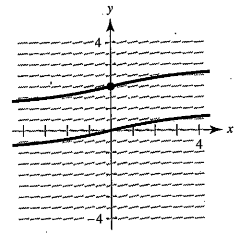
43. $2\sqrt{e^t - 3} - 2\sqrt{3}\arctan(\sqrt{e^t - 3}/\sqrt{3}) + C$ **45.** $\pi/6$

47. a and b **49.** a, b, and c

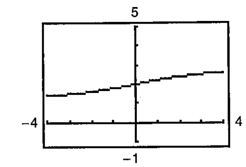
51. No. This integral does not correspond to any of the basic integration rules.

53. $y = \arcsin(x/2) + \pi$

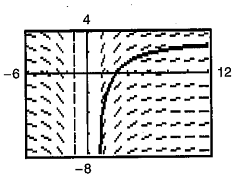
55. (a)



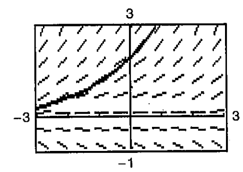
(b) $y = \frac{2}{3}\arctan\frac{x}{3} + 2$



57.

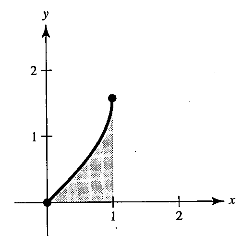


59.



61. $\pi/3$ **63.** $\pi/8$ **65.** $3\pi/2$

67. (a)



(b) 0.5708
(c) $(\pi - 2)/2$

69. (a) $F(x)$ represents the average value of $f(x)$ over the interval $[x, x + 2]$. Maximum at $x = -1$
(b) Maximum at $x = -1$

71. False. $\int \frac{dx}{3x\sqrt{9x^2 - 16}} = \frac{1}{12}\operatorname{arcsec}\frac{|3x|}{4} + C$

73. True **75–77.** Proofs

79. (a) $\int_0^1 \frac{1}{1 + x^2}\,dx$ (b) About 0.7847
(c) Because $\int_0^1 \frac{1}{1 + x^2}\,dx = \frac{\pi}{4}$, you can use the Trapezoidal Rule to approximate $\frac{\pi}{4}$. Multiplying the result by 4 gives an estimation of π.

Section 5.8 *(page 390)*

1. (a) 10.018 (b) -0.964 **3.** (a) $\frac{4}{3}$ (b) $\frac{13}{12}$

5. (a) 1.317 (b) 0.962 **7–13.** Proofs

15. $\cosh x = \sqrt{13}/2$; $\tanh x = 3\sqrt{13}/13$; $\operatorname{csch} x = 2/3$; $\operatorname{sech} x = 2\sqrt{13}/13$; $\coth x = \sqrt{13}/3$

17. ∞ **19.** 0 **21.** 1 **23.** $3\cosh 3x$

25. $-10x[\operatorname{sech}(5x^2)\tanh(5x^2)]$ **27.** $\coth x$ **29.** $\sinh^2 x$

31. $\operatorname{sech} t$ **33.** $y = -2x + 2$ **35.** $y = 1 - 2x$

37. Relative maxima: $(\pm\pi, \cosh \pi)$; Relative minimum: $(0, -1)$

39. Relative maximum: $(1.20, 0.66)$;
Relative minimum: $(-1.20, -0.66)$