Ch.5 Test Review Problem: Curve Sketching

$e \approx 2.7$

$f(x) = \dfrac{x}{\ln x}$ a) find domain b) Rel. max/min c) POI d) $\lim\limits_{x\to 1^+} f(x)$ e) $\lim\limits_{x\to\infty} f(x)$ f) sketch graph

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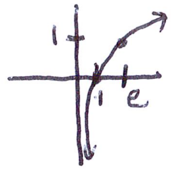
a) find domain: * consider individual restrictions of family of functions.
* set denominator $= 0$

$y = \ln x$  $\rightarrow$ D: $(0, \infty)$

$\ln x = 0$ | $x = 1$
$\log_e x = 0$ | $\boxed{x \neq 1}$
$e^0 = x$ | VA: $x = 1$

$\boxed{D: (0,1) \cup (1, \infty)}$

b) 1^{st} derivative test: $y = \dfrac{\overbrace{x}^{f}}{\underbrace{\ln x}_{g}}$ $\quad y' = \dfrac{\overbrace{(1)}^{f'}\overbrace{(\ln x)}^{g} - \overbrace{(x)}^{f}\overbrace{\left(\frac{1}{x}\right)}^{g'}}{\underbrace{(\ln x)^2}_{g^2}} = \boxed{\dfrac{\ln x - 1}{(\ln x)^2}}$

*critical points:

$\ln x - 1 = 0$
$\ln x = 1$
$\log_e x = 1$
$\underline{\underline{e^1 = x}}$

$(\ln x)^2 = 0$
$\ln x = 0$
$\log_e x = 0$
$e^0 = x$
$\underline{\underline{x = 1}}$

$f'(x)$: $-$ ($\tfrac{1}{2}$), $-$ (2), $+$ (4); marks at 0, 1, e

Rel. minimum at $\left(e, \dfrac{e}{\ln e}\right)$ → $\left(e, \underline{e}\right)$ b/c $f'(x)$ changes from $-$ to $+$

c) $y' = \dfrac{\overbrace{\ln x - 1}^{f}}{\underbrace{(\ln x)^2}_{g}}$ $\quad y'' = \dfrac{\overbrace{\left(\frac{1}{x}\right)}^{f'}\overbrace{(\ln x)^2}^{g} - \overbrace{(\ln x - 1)}^{f}\cdot\overbrace{2(\ln x)\left(\frac{1}{x}\right)}^{g'}}{\underbrace{(\ln x)^4}_{g^2}}$ chain: out: $(\ )^2$ in: $\ln x$

$= \dfrac{\frac{1}{x}(\ln x)^2 - 2\left(\frac{1}{x}\right)\ln x(\ln x - 1)}{(\ln x)^4}$

$y'' = \dfrac{\frac{1}{x}(\ln x)\left[\ln x - 2(\ln x - 1)\right]}{(\ln x)^4} = \dfrac{\frac{1}{x}\ln x\left[\ln x - 2\ln x + 2\right]}{(\ln x)^4} = \boxed{\dfrac{2 - \ln x}{x(\ln x)^3}}$

*critical pts:

$2 - \ln x = 0$
$2 = \ln x$
$\ln x = 2$
$\log_e x = 2$
$\underline{\underline{e^2 = x}}$

$x = 0$

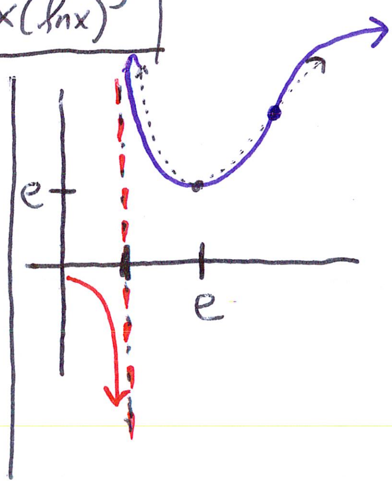
$(\ln x)^3 = 0$
$x = 1$

$f''(x)$: $\cap$ $-$ ($\tfrac{1}{2}$), $\cup$ $+$ (2), $\cap$ $-$ (e^4); marks at 0, 1, e^2

POI: $\left(e^2, \dfrac{e^2}{\ln e^2}\right)$
$\left(e^2, \tfrac{1}{2}e^2\right)$

$\lim\limits_{x\to\infty} \dfrac{x}{\ln x} = \infty$

$\lim\limits_{x\to 1^+} \dfrac{x}{\ln x} = \infty$



Find $\frac{dy}{dx}$ for $\ln(xy) + xy = 50$

* expand log expression
* implicit diff
* product rule

$$\ln x + \ln y + \overset{f}{x}\overset{g}{y} = 50$$

$$\frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) + \overset{f'}{1}\overset{g}{y} + \overset{f}{x}\overset{g'}{\left(\frac{dy}{dx}\right)} = 0$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) + x\left(\frac{dy}{dx}\right) = -\frac{1}{x} - y$$

$$\frac{dy}{dx}\left(\frac{1}{y} + x\right) = -\frac{1}{x} - y$$

$$\frac{dy}{dx} = \frac{-\frac{1}{x} - y}{\frac{1}{y} + x}$$

xy

$$\boxed{\frac{dy}{dx} = \frac{-y - xy^2}{x + x^2 y}}$$

$$\frac{dy}{dx} = \frac{-y(1+xy)}{x(1+xy)} = \boxed{\frac{-y}{x}}$$