AP Calculus Logs and Exponentials Test Review WS #1 Name________________ Date_____

1. Find the equation of the line tangent to $y = \frac{e^{2x}}{x^2}$ at $x = -1$

2. Let f be the function defined by $f(x) = \frac{x^2}{e^x}$

a. State the domain of $f(x)$.

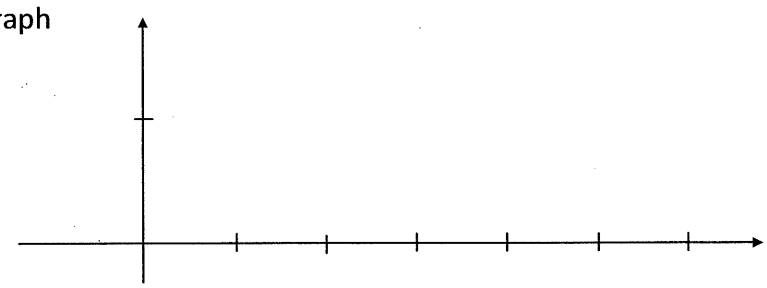
b. Find each relative maximum and relative minimum. Justify Answer

c. Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$

d. State the range of f.

e. Find each point of inflection on the graph of f. Write your answer(s) as ordered pairs and justify each answer.

f. Sketch graph



3. 1994 #4: A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t \ln t - t$.

a) Write an expression for the acceleration of the particle.
b) For what values of t is the particle moving to the right?
c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

4. Find dy/dx for $y \ln x + y^2 = 0$

5. Find dy/dx if $y = \sqrt[3]{(2 + 5x^3)^x}$

AP Calculus Logs and Exponentials Test Review WS #1 Name KEY Date

1. Find the equation of the line tangent to $y = \dfrac{e^{2x}}{x^2}$ at $x = -1$

$$y' = \frac{(e^{2x})(2)x^2 - e^{2x}(2x)}{x^4}$$

$$y' = \frac{2xe^{2x}(x-1)}{x^4} = \frac{2e^{2x}(x-1)}{x^3}$$

$$y(-1) = \frac{e^{-2}}{(-1)^2} = \frac{1}{e^2} \qquad \text{point } \left(-1, \frac{1}{e^2}\right)$$

$$y'(-1) = \frac{2e^{-2}(-2)}{(-1)^3} = \frac{-4}{-e^2} = \frac{4}{e^2} \qquad m = \frac{4}{e^2}$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - \frac{1}{e^2} = \frac{4}{e^2}(x+1)}$$

2. Let f be the function defined by $f(x) = \dfrac{x^2}{e^x}$

a. State the domain of $f(x)$.

$(-\infty, \infty)$

b. Find each relative maximum and relative minimum. Justify Answer

$f(x) = x^2e^{-x}$

$$f'(x) = 2xe^{-x} + x^2e^{-x}(-1) = e^{-x}(2x - x^2) = \frac{x(2-x)}{e^x}$$

f': $-$ (↘) on $(-\infty, 0)$, $+$ (↗) on $(0, 2)$, $-$ (↘) on $(2, \infty)$

Rel. min at $(0, 0)$ b/c $f'(x)$ changes from + to −

Rel. max at $\left(2, \frac{4}{e^2}\right)$ b/c $f'(x)$ changes from + to −

$$\boxed{f'(x) = \frac{2x - x^2}{e^x}}$$

Comparative growth rate $L<R<P<E$

c. Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$

$$\lim_{x\to-\infty} \frac{x^2}{e^x} \to \frac{(-\infty)^2}{e^{-\infty}} = (-\infty)^2 e^{\infty} = \infty$$

$$\lim_{x\to\infty} \frac{x^2}{e^x} = 0$$

d. State the range of f.

$[0, \infty)$

e)

$$f'(x) = \frac{2x - x^2}{e^x}$$

$$f''(x) = \frac{(2-2x)e^x - (2x - x^2)e^x}{e^{2x}}$$

$$= \frac{e^x(2 - 2x - 2x + x^2)}{e^{2x}} = \boxed{\frac{x^2 - 4x + 2}{e^x}}$$

$$x^2 - 4x + 2 = 0$$

$$\frac{4 \pm \sqrt{16 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

f'': $+$ (∪) on $(-\infty, 2-\sqrt{2})$, $-$ (∩) on $(2-\sqrt{2}, 2+\sqrt{2})$, $+$ (∪) on $(2+\sqrt{2}, \infty)$

$0.59 \to 2-\sqrt{2}$ $\qquad$ $2+\sqrt{2} \leftarrow 3.4$

e. Find each point of inflection on the graph of f. Write your answer(s) as ordered pairs and justify each answer. POI at $x = 2-\sqrt{2},\ 2+\sqrt{2}$ b/c $f''(x)$ change signs

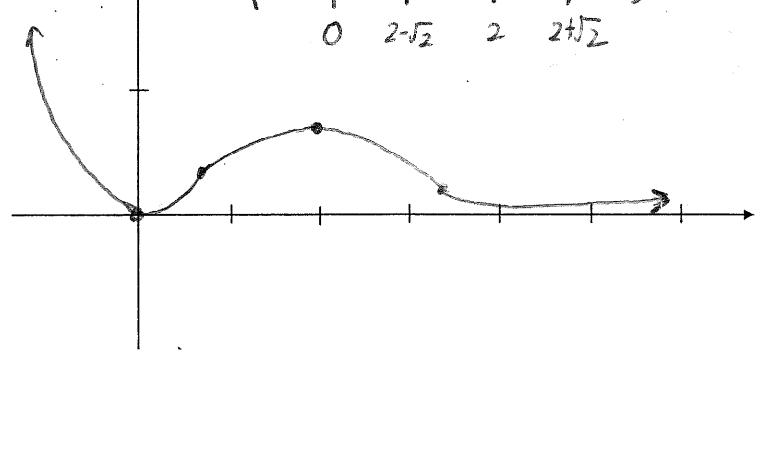
f. Sketch graph.

0 $\quad 2-\sqrt{2} \quad$ 2 $\quad 2+\sqrt{2}$

x-int: $\frac{x^2}{e^x} = 0$ $\quad x = 0$ $\quad (0, 0)$

POI: $(0.59, 0.19)$

$(3.4, 0.39)$



3. 1994 #4: A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t \ln t - t$.

a) Write an expression for the acceleration of the particle.

b) For what values of t is the particle moving to the right?

c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

a) $a(t) = (1)\ln t + t\left(\frac{1}{t}\right) - 1 = \ln t + 1 - 1 = \ln t$ $\boxed{a(t) = \ln t}$

b) set $v(t) = 0$, make sign line.

$v(t) = t\ln t - 1$

$0 = t(\ln t - 1)$

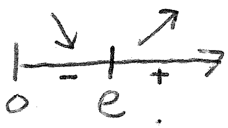
$t = 0$ | $\ln t - 1 = 0$

$\ln t = 1$

$\log_e t = 1$

$e^1 = t$

$\boxed{t = e}$



particle moving right (e, ∞) b/c $v(t) > 0$

c) * minimum velocity occurs where $v'(t)$ changes from − to +, (POI)

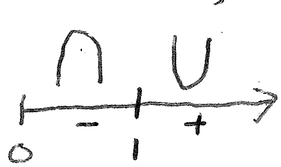
set $a(t) = 0$

$\ln t = 0$

$\log_e t = 0$

$e^0 = t$

$1 = t$



Minimum velocity at $t = 1$ since $a(t)$ changes from − to +

4. Find $\frac{dy}{dx}$. $y\ln x + y^2 = 0$

$\frac{dy}{dx}\ln x + y\left(\frac{1}{x}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}(\ln x + 2y) = \frac{-y}{x}$

$\frac{dy}{dx} = \frac{\frac{-y}{x}}{\ln x + 2y} = \boxed{\frac{-y}{x(\ln x + 2y)}}$

5. Find dy/dx if y = $\sqrt[3]{(2 + 5x^3)^x}$

$y = (2+5x^3)^{x/3}$

$\ln y = \ln (2+5x^3)^{x/3}$

$\ln y = \left(\frac{x}{3}\right)\left[\ln(2+5x^3)\right]$

$\frac{1}{y}\frac{dy}{dx} = \left(\frac{1}{3}\right)\ln(2+5x^3) + \left(\frac{x}{3}\right)\left(\frac{1}{2+5x^3}\right)(15x^2)$

$\frac{dy}{dx} = y\left[\frac{1}{3}\ln(2+5x^3) + \frac{5x^3}{2+5x^3}\right] = \boxed{(2+5x^3)^{x/3}\left[\frac{1}{3}\ln(2+5x^3) + \frac{5x^3}{2+5x^3}\right]}$