AP Calculus Logs and Exponentials Test Review WS #2 Name________________Date_____

1. The position of a particle moving on the x-axis is given by $x(t) = \frac{2t}{e^t}$

a) Show that $v(t) = \frac{2-2t}{e^t}$

b) Show that $a(t) = \frac{2t-4}{e^t}$

c) For what values of t is the particle at rest?

d) For the value found in part c, is the particle located to the right or the left of the origin? Justify your answer

e) For the same value from part c, is the velocity increasing or decreasing?

f) For the same value in part c, is the acceleration increasing or decreasing?

g) What is the velocity when the acceleration is zero?

h) At t = 3, is the velocity of the particle increasing or decreasing?

i) At t = 3,is the speed of the particle increasing or decreasing?

j) For what value(s) of t is the particle located at the origin?

2. Find dy/dx for $\ln(xy^2) = y^2 - 3x$

3. Given $f(x) = \ln(4 - x^2)$

a) Find Domain of f(x)	b) Find the Range of f(x)
c) Find intercepts for f(x)	d) Determine the asymptotes for f(x)
e) Find $f'(x)$	f) Find $f''(x)$
g) Find intervals of increase/decrease and relative extrema. Justify your answer	h) Determine intervals of concave up/down and POI

Sketch Graph:

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1. The position of a particle moving on the x-axis is given by $x(t) = \frac{2t}{e^t}$

a) Show that $v(t) = \frac{2-2t}{e^t}$

$v(t) = \frac{2(e^t) - 2t(e^t)}{e^{2t}} = \frac{e^t(2-2t)}{e^{2t}} = \boxed{\frac{2-2t}{e^t}}$

b) Show that $a(t) = \frac{2t-4}{e^t}$

$a(t) = \frac{(-2)(e^t) - (2-2t)(e^t)}{e^{2t}} = \frac{e^t(-2-2+2t)}{e^{2t}} = \boxed{\frac{2t-4}{e^t}}$

c) For what values of t is the particle at rest?

$2-2t=0$ $\boxed{t=1}$

d) For the value found in part c, is the particle located to the right or the left of the origin? Justify your answer

$x(1) = \frac{2}{e}$ particle is located to the right of origin since $\frac{2}{e} > 0$

e) For the same value from part c, is the velocity increasing or decreasing?

$a(1) = \frac{2-4}{e} = \frac{-2}{e} < 0$

velocity is decreasing b/c $a(t) < 0$

f) For what value of t is acceleration $=0$?

$0 = 2t-4$

$2t = 4$

$\boxed{t=2}$

g) What is the velocity when the acceleration is zero?

$v(2) = \frac{2-4}{e^2} = \boxed{\frac{-2}{e^2}}$

h) At t = 3, is the velocity of the particle increasing or decreasing?

$a(3) = \frac{6-4}{e^3} = \frac{2}{e^3} > 0$ Since $a(3) > 0$ velocity is increasing.

i) At t = 3,is the speed of the particle increasing or decreasing?

$v(3) = \frac{2-6}{e^3} = \frac{-4}{e^3} < 0$

Since $v(3)$ and $a(3)$ have opposite signs, speed is decreasing.

j) For what value(s) of t is the particle located at the origin?

$0 = \frac{2t}{e^t}$ $\boxed{t=0}$

2. Find dy/dx for $\ln(xy^2) = y^2 - 3x$

$$\ln x + 2\ln y = y^2 - 3x$$

$$\frac{1}{x} + 2\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) = 2y\left(\frac{dy}{dx}\right) - 3$$

$$\frac{dy}{dx}\left(\frac{2}{y} - 2y\right) = -3 - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{-3-\frac{1}{x}}{\frac{2}{y}-2y} \cdot \frac{xy}{xy} = \boxed{\frac{-3xy - y}{2x - 4xy^2}}$$

3. Given $f(x) = \ln(4 - x^2)$

$- \quad + \quad -$; -2, 2

a) Find Domain of f(x)

$(-2, 2)$

b) Find the Range of f(x)

$(-\infty, \ln 4)$

c) Find intercepts for f(x)

$\ln(4-x^2) = 0$

$e^0 = 4 - x^2$

$1 = 4 - x^2$

$-3 = -x^2$

$x = \pm\sqrt{3}$

y-int at $(0, \ln 4)$

d) Determine the asymptotes for f(x)

$x = -2, \ x = 2$

e) Find f'(x)

$$\frac{1}{4-x^2}(-2x) = \boxed{\frac{-2x}{4-x^2}}$$

f) Find f "(x)

$$\frac{(-2)(4-x^2) - (-2x)(-2x)}{(4-x^2)^2} = \frac{-8+2x^2-4x^2}{(4-x^2)^2}$$

$$\frac{-8-2x^2}{(4-x^2)^2} = \boxed{\frac{-2(4+x^2)}{(4-x^2)^2}}$$

g) Find intervals of increase/decrease and relative extrema. Justify your answer

$-2x = 0 \quad x = 0$

$+ \quad -$; $-2 \quad 0 \quad 2$

Rel. max at $(0, \ln 4)$
b/c $f'(x)$ changes from + to −.

h) Determine intervals of concave up/down and POI

No critical values since $-2(4+x^2) \neq 0$

$-$; $-2 \quad 2$

$f(x)$ concave down $(-2, 2)$ b/c $f''(x) < 0$

Sketch Graph:

$-2 \quad 0 \quad 2$