

1. Let f be the function defined by $f(x) = \frac{x^3}{e^x}$

a. State the domain of $f(x)$.

b. Find the range of f . (find this after sketching your curve)

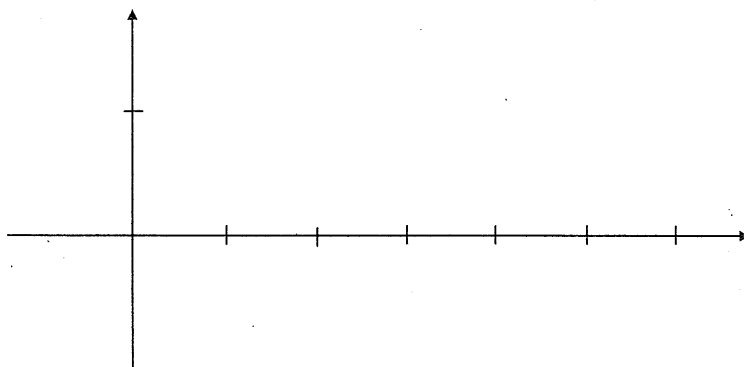
c. Find each relative maximum and relative minimum. (ordered pairs) Justify Answer

Find each point of inflection on graph of f . (Provide only x value) Justify answer.

d. Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$

e. Find x -intercept(s)

f. Sketch graph.



2. If $y = (e^{-x})(\ln x)$, then dy/dx when $x = 1$ is

3. The position of a particle moving on the x-axis is given by $x(t) = \frac{4}{3}e^{3t} - 8t$

a) Write an expression for the velocity at any given time t

b) Write an expression for the acceleration at any given time t

c) For what values of t is the particle at rest?

d) Find the velocity when $t = 3$

e) Find the acceleration when $t = 3$

f) At $t = 3$, is the velocity of the particle increasing or decreasing?

g) At $t = 3$, is the speed of the particle increasing or decreasing?

4. Find dy/dx if $\ln(yx) = y^2 - x^3 - e$

1. Let f be the function defined by $f(x) = \frac{x^3}{e^x}$

a. State the domain of $f(x)$.

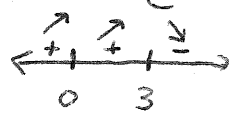
$(-\infty, \infty)$

b. Find the range of f . (find this after sketching your curve)

$(-\infty, \frac{27}{e^3})$

c. Find each relative maximum and relative minimum. (ordered pairs) Justify Answer

$f'(x) = \frac{3x^2 e^x - x^3 e^x}{e^{2x}} = \frac{3x^2 - x^3}{e^x} = \frac{x^2(3-x)}{e^x} \quad x=0,3$

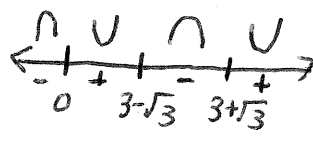

 Rel. max at $(3, \frac{27}{e^3})$
 b/c $f'(x)$ changes from + to -

Comparative growth rate

$\lim_{x \rightarrow \infty} \frac{x^3}{e^x} = 0$
 $\lim_{x \rightarrow -\infty} \frac{x^3}{e^x} = \frac{(-\infty)^3}{e^{-\infty}} = (-\infty)^3 e^{\infty} = -\infty$

Find each point of inflection on graph of f . (Provide only x value) Justify answer.

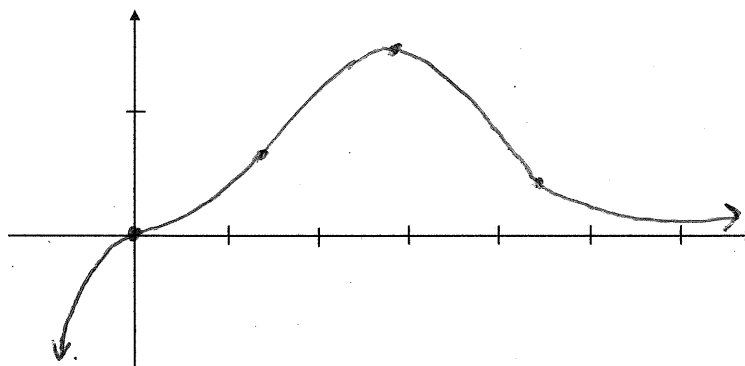
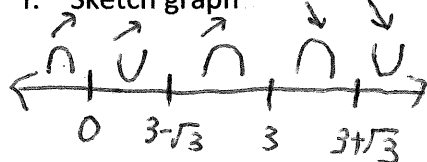
$f''(x) = \frac{(6x - 3x^2)e^x - (3x^2 - x^3)e^x}{e^{2x}} = \frac{6x - 3x^2 - 3x^2 + x^3}{e^x}$
 $f''(x) = \frac{x^3 - 6x^2 + 6x}{e^x} = \frac{x(x^2 - 6x + 6)}{e^x} \quad x=0, 3+\sqrt{3}, 3-\sqrt{3}$


 POI at $x=0, 3+\sqrt{3}$
 b/c $f''(x)$ change signs.

d. Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$

e. Find x -intercept(s) $(0,0)$

f. Sketch graph



2. If $y = (e^{-x})(\ln x)$, then dy/dx when $x = 1$ is

$y' = (e^{-x})(-1)\ln x + e^{-x}(\frac{1}{x})$

$y' = \frac{-\ln x}{e^x} + \frac{1}{xe^x}$

$y'(1) = \frac{-\ln 1}{e^1} + \frac{1}{e^1} = \frac{1}{e}$

3. The position of a particle moving on the x-axis is given by $x(t) = \frac{4}{3}e^{3t} - 8t$

a) Write an expression for the velocity at any given time t

$$v(t) = \frac{4}{3}e^{3t} \cdot (3) - 8$$

$$v(t) = 4e^{3t} - 8$$

b) Write an expression for the acceleration at any given time t

$$a(t) = 4e^{3t} \cdot 3 - 0$$

$$a(t) = 12e^{3t}$$

c) For what values of t is the particle at rest?

$$0 = 4e^{3t} - 8 \quad \left| \quad e^{3t} = 2 \quad \left| \quad 3t = \ln 2 \right. \right.$$

$$4e^{3t} = 8 \quad \left| \quad \ln e^{3t} = \ln 2 \quad \left| \quad t = \frac{\ln 2}{3} \right. \right.$$

d) Find the velocity when $t = 3$

$$v(3) = 4e^{3(3)} - 8 = 4e^9 - 8$$

e) Find the acceleration when $t = 3$

$$a(3) = 12e^9$$

f) At $t = 3$, is the velocity of the particle increasing or decreasing?

velocity is increasing at $t = 3$
b/c $a(t) > 0$

g) At $t = 3$, is the speed of the particle increasing or decreasing?

At $t = 3$, speed is increasing b/c
 $v(t)$ and $a(t)$ have same signs.

4. Find dy/dx if $\ln(yx) = y^2 - x^3 - e$

$$\ln y + \ln x = y^2 - x^3 - e$$

$$\frac{1}{y} \frac{dy}{dx} + \frac{1}{x} = 2y \left(\frac{dy}{dx} \right) - 3x^2 - 0$$

$$\frac{dy}{dx} \left[\frac{1}{y} - 2y \right] = -3x^2 - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{-3x^2 - \frac{1}{x}}{\frac{1}{y} - 2y} \cdot \frac{(xy)}{(xy)} = \boxed{\frac{-3x^3 - y}{x - 2xy^2}}$$