Ch. 5 Review
WS #3 (p.15-16)

*quotient Rule

1) c) $f(x) = \dfrac{\overbrace{x^3}^{f}}{\underbrace{e^x}_{g}}$

$$f'(x) = \frac{\overbrace{3x^2}^{f'} \cdot \overbrace{e^x}^{g} - \overbrace{x^3}^{f} \cdot \overbrace{e^x(1)}^{g'}}{\underbrace{(e^x)^2}_{g^2}} \rightarrow \frac{\overbrace{3x^2e^x}-\overbrace{x^3e^x}}{(e^x)^2}$$

$$f'(x) = \frac{x^2\cancel{e^x}(3-x\quad)}{(e^x)^2} \rightarrow \boxed{\frac{x^2(3-x)}{e^x}}$$

$x^2(3-x)=0$

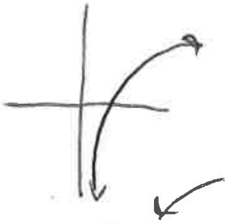
$x^2=0$ | $3-x=0$

$\boxed{x=0}$ | $\boxed{x=3}$

$e^x=0$

$\ln e^x = \cancel{\ln 0}$

$x \ln e = \text{none}$



$f'(x)$: $\nearrow$ $+$ (test -1) | 0 | $\nearrow$ (test 1) | 3 | $\searrow$ $-$ (test 4)

$\frac{3^3}{e^3} = \frac{27}{e^3}$

Rel. max at $(3, \frac{27}{e^3})$
b/c $f'(x)$ changes from + to −

$$f'(x) = \frac{\overbrace{3x^2-x^3}^{f}}{\underbrace{e^x}_{g}}$$

$$f''(x) = \frac{\overbrace{(6x-3x^2)}^{f'}\overbrace{(e^x)}^{g} - \overbrace{(3x^2-x^3)}^{f}\overbrace{(e^x)(1)}^{g'}}{\underbrace{(e^x)^2}_{g^2}}$$

$$f''(x) = \frac{6xe^x - 3x^2e^x - 3x^2e^x + x^3e^x}{(e^x)^2} \rightarrow \frac{6xe^x - 6x^2e^x + xe^{3x}}{(e^x)^2}$$

quadratic formula
$x = 3 \pm \sqrt{3}$

$$f''(x) \quad \frac{x\cancel{e^x}(x^2-6x+6)}{\cancel{e^x}e^x}$$

$x=0$

$e^x \neq 0$

$f''(x)$: $\cap$ $-$ | 0 | $\cup$ $+$ | $3-\sqrt{3}$ | $\cap$ $-$ | $3+\sqrt{3}$ | $\cup$ $+$

POI at $x=0$, $3-\sqrt{3}$, $3+\sqrt{3}$
b/c $f''(x)$ change signs

3) position function

$x(t) = \frac{4}{3}e^{3t} - 8t$

$\frac{d}{dx}e^{u} = e^{u} \cdot u'$

a) $v(t) = \frac{4}{3}e^{3t}(3) - 8 \rightarrow \boxed{v(t) = 4e^{3t} - 8}$

b) $a(t) = 4e^{3t}(3) - 0 \rightarrow \boxed{a(t) = 12e^{3t}}$

c) when is particle at rest? (*set $v(t)=0$)

$0 = 4e^{3t} - 8$

$8 = 4e^{3t}$

$\frac{4e^{3t}}{4} = \frac{8}{4}$

$e^{3t} = 2$

$\ln e^{3t} = \ln 2$

$3t\cancel{\ln e} = \ln 2$

$3t = \ln 2$

$\boxed{t = \frac{\ln 2}{3} \text{ or } \frac{1}{3}\ln 2}$

d) Find velocity when $t=3$.

$V(3) = 4e^{3(3)} - 8 = \boxed{4e^{9} - 8}$

e) Find acceleration when $t=3$ $\quad a(3) = 12e^{3(3)} = \boxed{12e^{9}}$

f) At $t=3$, is velocity of particle increasing or decreasing?

$a(3) = 12e^{9} > 0$, velocity is increasing at $t=3$.

g) At $t=3$, is speed increasing or decreasing?

Since $v(3) > 0$, $a(3) > 0$, speed is increasing at $t=3$.

4) Find $\frac{dy}{dx}$ if $\ln(yx)=y^2-x^3-e$

* implicit differentiation

* expand using log properties.

$$\ln y+\ln x=y^2-x^3-e$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right)+\frac{1}{x}=2y\left(\frac{dy}{dx}\right)-3x^2-0$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right)-2y\left(\frac{dy}{dx}\right)=-3x^2-\frac{1}{x}$$

$$\frac{dy}{dx}\left(\frac{1}{y}-2y\right)=-3x^2-\frac{1}{x}$$

$$\frac{dy}{dx}=\frac{-\frac{\overset{(x)}{3x^2}}{1}-\frac{1}{x}}{\frac{1}{y}-\frac{2y}{1}(y)}\rightarrow\frac{\frac{-3x^3}{x}-\frac{1}{x}}{\frac{1}{y}-\frac{2y^2}{y}}\rightarrow\frac{\frac{-3x^3-1}{x}}{\frac{1-2y^2}{y}}$$

$$\frac{(-3x^3-1)}{x}\cdot\frac{y}{(1-2y^2)}\rightarrow\boxed{\frac{-3yx^3-y}{x-2xy^2}}$$

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5) Find $f'(x)$ $\quad f(x)=(3x-1)^{\sqrt{x}}$ $\quad (\text{variable})^{(\text{variable})}$

$$y=(3x-1)^{x^{1/2}}$$

implicit

$$\ln y=\ln(3x-1)^{x^{1/2}}$$

$$\ln y=\overbrace{x^{1/2}}\cdot\overbrace{\ln(3x-1)}$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right)=\overbrace{\frac{1}{2}x^{-1/2}}^{f'}\cdot\overbrace{\ln(3x-1)}^{g}+\overbrace{x^{1/2}}^{f}\cdot\overbrace{\frac{3}{3x-1}}^{g'}$$

$$\frac{dy}{dx}=y\left[\frac{\ln(3x-1)}{2\sqrt{x}}+\frac{3\sqrt{x}}{3x-1}\right]$$

$$\frac{dy}{dx}=(3x-1)^{\sqrt{x}}\left[\frac{\ln(3x-1)}{2\sqrt{x}}+\frac{3\sqrt{x}}{3x-1}\right]$$

*log differentiation ⤴

2b) $y = \sqrt[x]{(5-4x)^7}$

$$y = (5-4x)^{7/x}$$

$$\ln y = \ln (5-4x)^{7x^{-1}}$$

$$\ln y = 7x^{-1} \cdot \ln(5-4x)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \overbrace{-7x^{-2}}^{f'} \cdot \overbrace{\ln(5-4x)}^{g} \overset{+}{+} \overbrace{7x^{-1}}^{f} \cdot \overbrace{\frac{-4}{5-4x}}^{g'}$$

$$\frac{dy}{dx} = y\left[-\frac{7\ln(5-4x)}{x^2} + \frac{-28}{x(5-4x)}\right]$$

$$\boxed{\frac{dy}{dx} = \sqrt[x]{(5-4x)^7}\left[\frac{-7\ln(5-4x)}{x^2} - \frac{28}{x(5-4x)}\right]}$$

6) $f(x) = 2x^3 - 3x^2 + 1$ find $(f^{-1})'(-4)$

$f(-1) = -4$	$(f^{-1})(-4) = \underline{-1}$
$f'(-1) = \underline{12}$	$(f^{-1})'(-4) = \boxed{\frac{1}{12}}$

$$-4 = 2x^3 - 3x^2 + 1$$

$$0 = 2x^3 - 3x^2 + 5$$

$$\boxed{x = -1}$$

$-3 \le x \le 3$

$$f'(x) = 6x^2 - 6x$$

$$f'(-1) = 6(-1)^2 - 6(-1) = 12$$