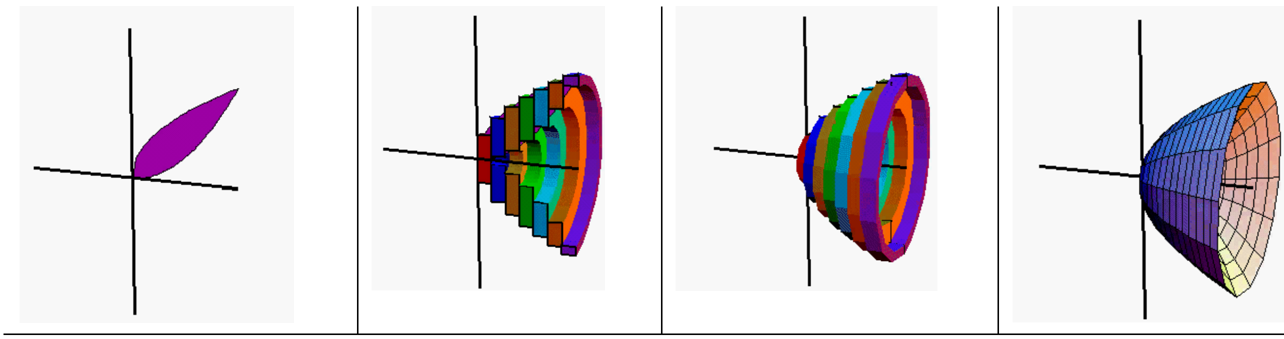
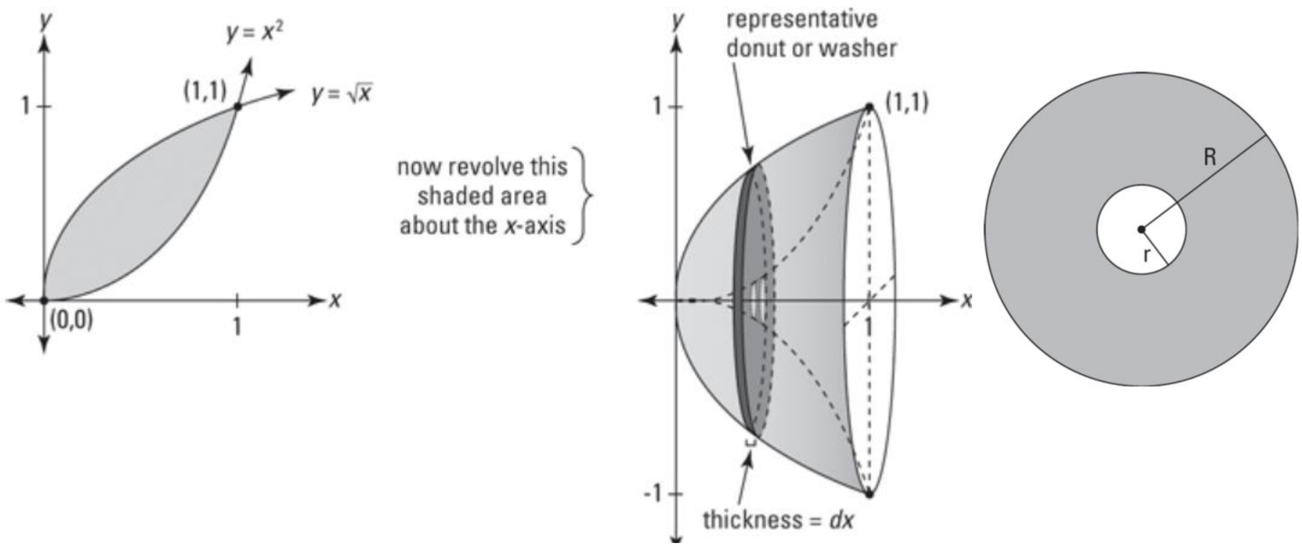
Calculus Ch. 7.2b: Volume by Washer Method

With **Disc Method**, we rotated one function around the x-axis. We used the Integral Notation to add areas of circular discs to find the volume of 3-dimensional curved objects.

Now, what if we wanted to find the volume created between 2 functions?

Take a look at the region between $y = x^2$ and $y = \sqrt{x}$. Picture taking that region and rotate that shape 360° around the x-axis. What shape do you see? What's different between this object and the object created by Disc Method? ________________________________





Each slice has the shape of a washer (circular rings) so its area equals the area of the

entire circle minus the area of the hole. Area of circular washer (ring)= ____________________

$$\boldsymbol{Volume\ (Washer\ Method)}: V = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$\boldsymbol{Volume\ (Washer\ Method)}$: $V = \pi \int_a^b [R(x)^2 - r(x)^2] dx$

<u>Washer Method Steps:</u>

1)Confirm gap exists between x-axis and the shaded region (gap indicates hole → suitable for washer method)

2) Draw dotted line across the x-axis to indicate location of Axis of Revolution **(AOR)**

3) Draw the length of **Radius R(x)**: Place pen/pencil **first** on the dotted line (AOR) and extend to further boundary of shaded region [$\boldsymbol{R(x) = Top - Bottom}$]

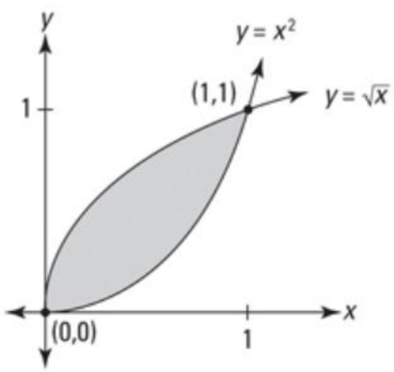
4) Draw the length of **radius r(x)**: Place pen/pencil **first** on the dotted line (AOR) and extend to closer boundary of shaded region [$\boldsymbol{r(x) = top - bottom}$]

5) Identify the left and right bounds (a and b). If needed, set the equations equal to find bounds.

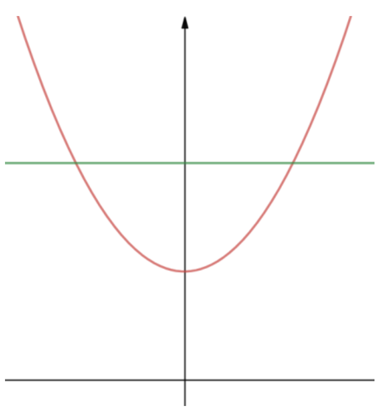
6) Enter expressions for R(x) and r(x) into Washer Method Volume formula

7) Enter Integral into calculator to find Volume. (TI-84: Math 9→ FnInt or TI-36X Pro: 2nd → e)

Example 1: Find the volume of the solid bounded by $y = x^2$ and $y = \sqrt{x}$ revolved about the x-axis.



Example 2: Find the volume of the solid bounded by $y = x^2 + 1$ and $y = 2$ revolved about the x-axis.



Washer Method Steps: $V = \pi \int_a^b [R(x)^2 - r(x)^2]dx$

1)Confirm gap exists between x-axis and the shaded region (gap indicates hole → suitable for washer method)

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$[\boldsymbol{r(x) = top - bottom}]$

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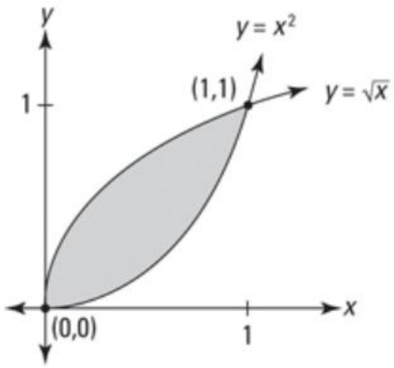
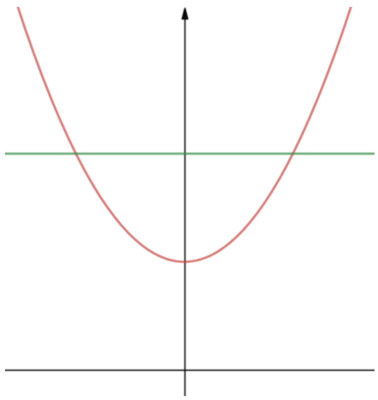
7) Enter Integral into calculator to find Volume. (TI-84: Math 9→ FnInt or TI-36X Pro: 2^{nd} → e)

Example 3: Find the volume of the solid bounded by $y = x^2$ and $y = \sqrt{x}$ revolved about the line y = 1



Example 4: Find the volume of the solid bounded by $y = x^2 + 1$ and $y = 2$ revolved about line y = 4



<u>Washer Method Steps:</u> $V = \pi \int_a^b [R(x)^2 - r(x)^2]dx$

1)Confirm gap exists between x-axis and the shaded region (gap indicates hole → suitable for washer method)

2) Draw dotted line across the x-axis to indicate location of Axis of Revolution **(AOR)**

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4) Draw the length of **radius r(x)**: Place pen/pencil **first** on the dotted line (AOR) and extend to closer boundary of shaded region $[\boldsymbol{r(x) = top - bottom}]$

5) Identify the left and right bounds (a and b). If needed, set the equations equal to find bounds.

6) Enter expressions for R(x) and r(x) into Washer Method Volume formula

7) Enter Integral into calculator to find Volume. (TI-84: Math 9→ FnInt or TI-36X Pro: 2^{nd} → e)

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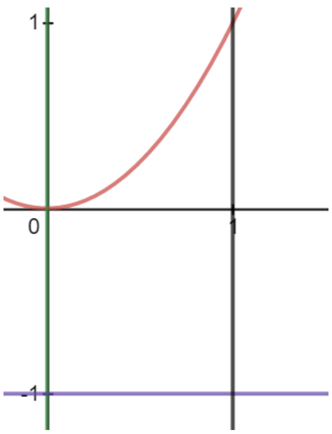
5. **Find the volume of the solid bounded by x = 1, y =- 1, y-axis, and the graph $y = x^2$ rotated about the line y = -3**



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6. **Find the volume of the solid bounded by equations y = x^2 – x and y = 6 rotated about the line y = 8**