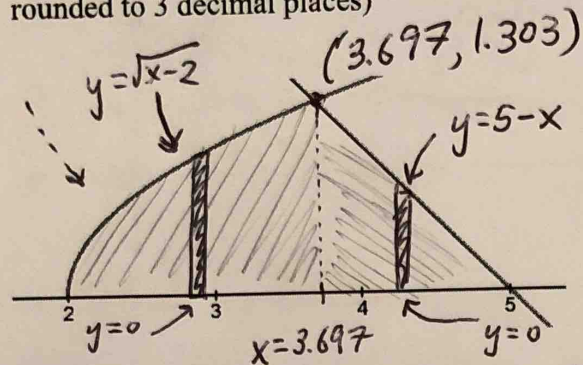


A.P. Calculus AB Chapter 7.-7.2 Area & Volume Unit Review WS #1

Key

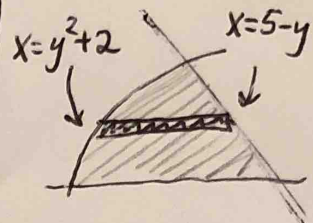
1) Given the region below enclosed by $f(x) = \sqrt{x-2}$, $g(x) = 5-x$, and the x-axis.

a) Find the area of the below region. (Write the integral notation(s) as well as the numeric approximation rounded to 3 decimal places)



Method 2: Right-Left

$$\begin{array}{l|l} y = \sqrt{x-2} & y = 5-x \\ y^2 = x-2 & x = 5-y \\ y^2+2 = x & \end{array}$$

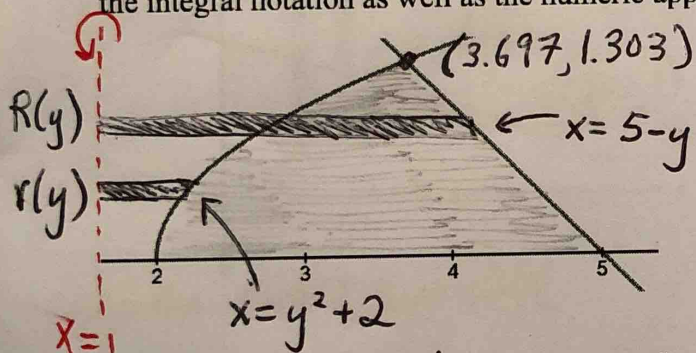


$$\text{Area} = \int_0^{1.303} (5-y - (y^2+2)) dy = \boxed{2.323}$$

Method 1: Top-Bottom, split into 2 regions:

$$\text{Area} = \int_2^{3.697} (\sqrt{x-2} - 0) dx + \int_{3.697}^5 (5-x - 0) dx = \boxed{2.323}$$

b) Find the Volume of solid generated when the enclosed region is revolved about the line $x=1$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



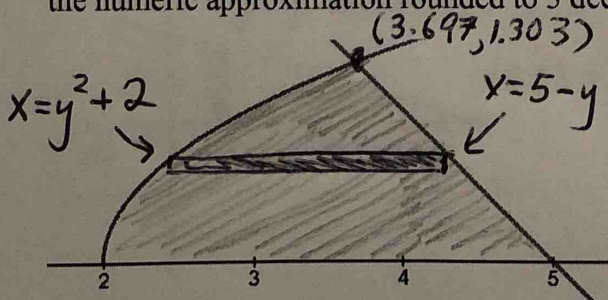
$$\begin{aligned} R(y) &= 5-y - (1) = 4-y \\ r(y) &= y^2+2 - (1) = y^2+1 \end{aligned}$$

$$V = \pi \int_0^{1.303} [(4-y)^2 - (y^2+1)^2] dy$$

$$V = \boxed{11.265\pi \text{ units}^3}$$

* Washer Method, Right-Left

c) The enclosed region is the base of a solid. The cross section of the solid taken perpendicular to the y-axis is an equilateral triangle. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



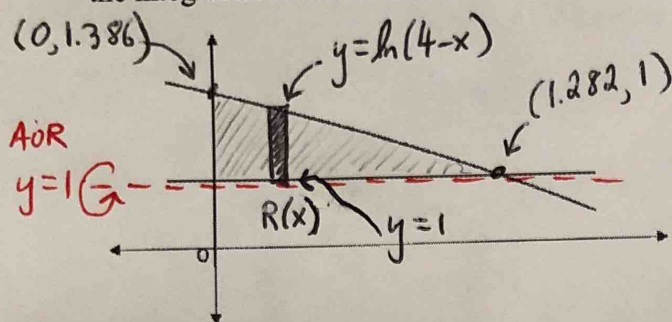
$$\begin{aligned} \text{base} &= 5-y - (y^2+2) = 5-y-y^2-2 \\ &= 3-y^2-y \end{aligned}$$

$$\text{Area} = \frac{\sqrt{3}}{4} (\text{base})^2 \rightarrow \frac{\sqrt{3}}{4} (3-y^2-y)^2$$

$$V = \int_0^{1.303} \frac{\sqrt{3}}{4} (3-y^2-y)^2 dy = \boxed{2.225 \text{ units}^3}$$

2) Given the region below enclosed by $f(x) = \ln(4-x)$, the line $y = 1$, and the y-axis.

a) Find the Volume of solid generated when the enclosed region is revolved about the line $y = 1$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



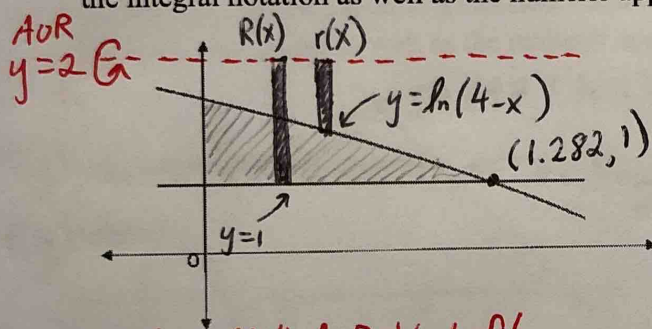
* Disc Method, Top-Bottom

$$R(x) = \ln(4-x) - (1)$$

$$V = \pi \int_0^{1.282} [\ln(4-x) - 1]^2 dx$$

$$V = 0.069\pi \text{ units}^3$$

b) Find the Volume of solid generated when the enclosed region is revolved about the line $y = 2$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



* washer Method, Right-Left

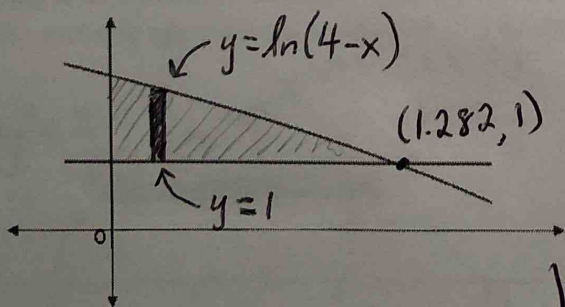
$$R(x) = 2 - (1) = 1$$

$$r(x) = 2 - (\ln(4-x))$$

$$V = \pi \int_0^{1.282} [1]^2 - [2 - \ln(4-x)]^2 dx$$

$$V = 0.457\pi \text{ units}^3$$

c) The enclosed region is the base of a solid. The cross section of the solid taken perpendicular to the x-axis is a rectangle whose height is twice the base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



$$\text{base} = \ln(4-x) - (1)$$

$$\text{height} = 2[\ln(4-x) - 1]$$

$$\text{Area} = (\text{base})(\text{height})$$

$$\text{Area} = 2[\ln(4-x) - 1]^2$$

$$V = \int_0^{1.282} 2[\ln(4-x) - 1]^2 dx$$

$$V = 0.139 \text{ units}^3$$