13.

n	0	1	2	3	4	5	6
x_n	0	0.05	0.1	0.15	0.2	0.25	0.3
y_n	4	3.8	3.6125	3.4369	3.2726	3.1190	2.9756

n	7	8	9	10
x_n	0.35	0.4	0.45	0.5
y_n	2.8418	2.7172	2.6038	2.4986

15. $y = -\frac{5}{3}x^3 + x^2 + C$

17. $y = -3 - 1/(x + C)$ **19.** $y = Ce^x/(2 + x)^2$

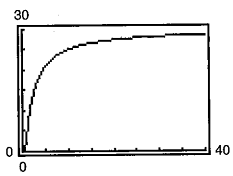
21. $\frac{dy}{dt} = \frac{k}{t^3}$; $y = -\frac{k}{2t^2} + C$ **23.** $y \approx \frac{3}{4}e^{0.379t}$

25. $y = \frac{9}{20}e^{(1/2)\ln(10/3)t}$ **27.** About 7.79 in.

29. About 37.5 yr

31. (a) $S \approx 30e^{-1.7918/t}$ (b) 20,965 units

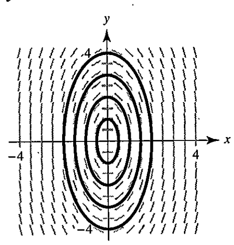
(c)



33. $y^2 = 5x^2 + C$ **35.** $y = Ce^{8x^2}$

37. $y^4 = 6x^2 - 8$ **39.** $y^4 = 2x^4 + 1$

41.



Graphs will vary.
$4x^2 + y^2 = C$

43. (a) 0.55 (b) 5250 (c) 150 (d) 6.41 yr

(e) $\frac{dP}{dt} = 0.55P\left(1 - \frac{P}{5250}\right)$

45. $y = \frac{80}{1 + 9e^{-t}}$

47. (a) $P(t) = \frac{20{,}400}{1 + 16e^{-0.553t}}$ (b) 17,118 trout (c) 4.94 yr

49. $y = -10 + Ce^x$ **51.** $y = e^{x/4}\left(\frac{1}{4}x + C\right)$

53. $y = (x + C)/(x - 2)$ **55.** $y = \frac{1}{10}e^{5x} + \frac{29}{10}e^{-5x}$

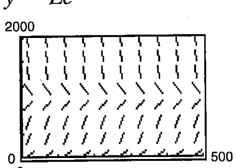
P.S. Problem Solving *(page 433)*

1. (a) $y = 1/(1 - 0.01t)^{100}$; $T = 100$

(b) $y = 1/\left[\left(\frac{1}{y_0}\right)^{\varepsilon} - k\varepsilon t\right]^{1/\varepsilon}$; Explanations will vary.

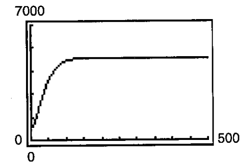
3. (a) $y = Le^{-Ce^{-kt}}$

(b)



(c) As $t \to \infty$, $y \to L$, the carrying capacity.

(d) $y_0 = 500 = 5000e^{-C} \Rightarrow e^C = 10 \Rightarrow C = \ln 10$



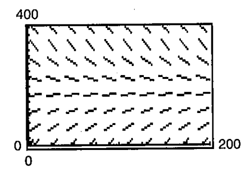
The graph is concave upward on $(0, 41.7)$ and downward on $(41.7, \infty)$.

5. 1481.45 sec ≈ 24 min, 41 sec

7. 2575.95 sec ≈ 42 min, 56 sec

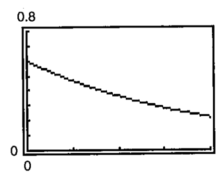
9. (a) $s = 184.21 - Ce^{-0.019t}$

(b)

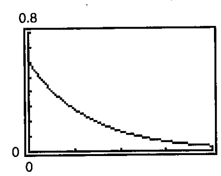


(c) As $t \to \infty$, $Ce^{-0.019t} \to 0$, and $s \to 184.21$.

11. (a) $C = 0.6e^{-0.25t}$



(b) $C = 0.6e^{-0.75t}$



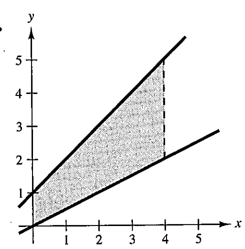
Chapter 7

Section 7.1 *(page 442)*

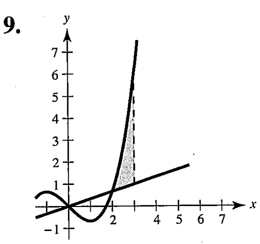
1. $-\int_0^6 (x^2 - 6x)\,dx$ **3.** $\int_0^3 (-2x^2 + 6x)\,dx$

5. $-6\int_0^1 (x^3 - x)\,dx$

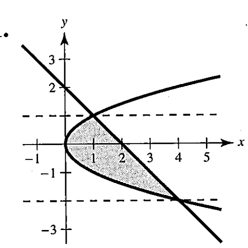
7.



9.



11.

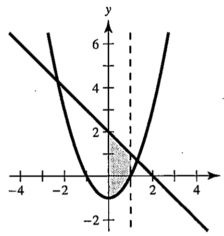


13. d

15. (a) $\frac{125}{6}$ (b) $\frac{125}{6}$

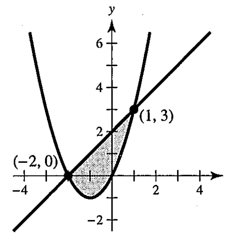
(c) Integrating with respect to y; Answers will vary.

17.



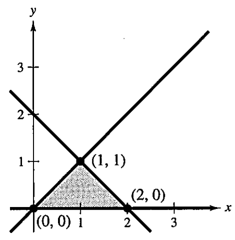
$\frac{32}{3}$

19.



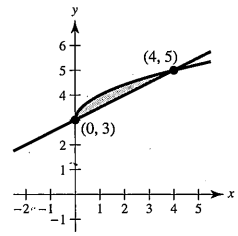
$\frac{9}{2}$

21.



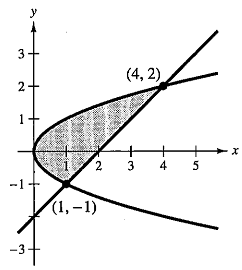
1

23.



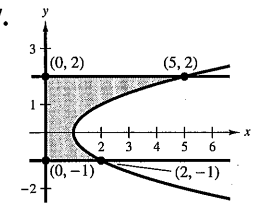
$\frac{4}{3}$

25.



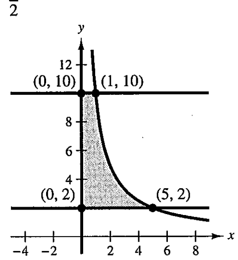
$\frac{9}{2}$

27.



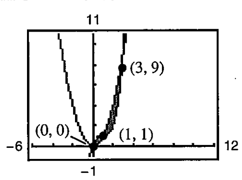
6

29.



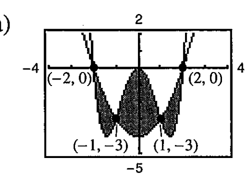
$10 \ln 5 \approx 16.094$

31. (a)



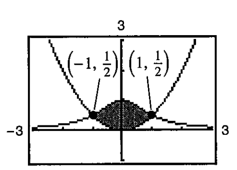
(b) $\frac{37}{12}$

33. (a)



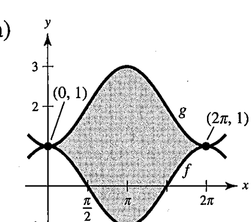
(b) 8

35. (a)



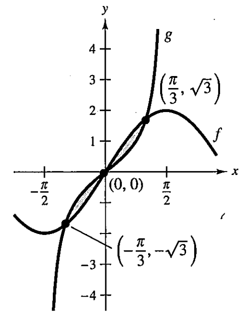
(b) $\pi/2 - 1/3 \approx 1.237$

37. (a)



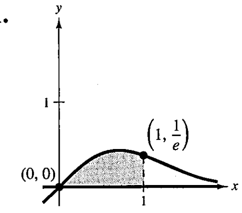
$4\pi \approx 12.566$

39.



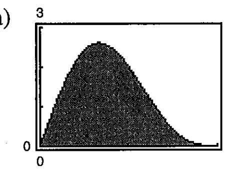
$2(1 - \ln 2) \approx 0.614$

41.



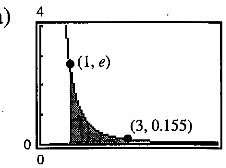
$(1/2)(1 - 1/e) \approx 0.316$

43. (a)



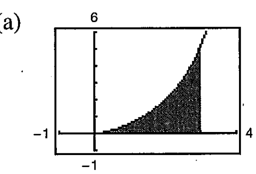
(b) 4

45. (a)



(b) About 1.323

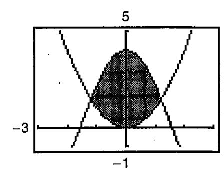
47. (a)



(b) The function is difficult to integrate.

(c) About 4.7721

49. (a)

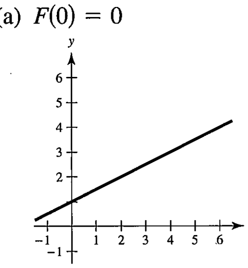


(b) The intersections are difficult to find.

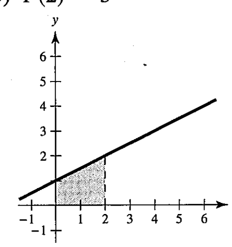
(c) About 6.3043

51. $F(x) = \frac{1}{4}x^2 + x$

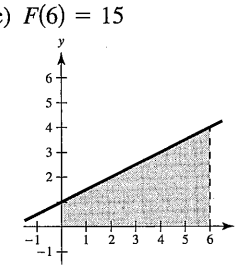
(a) $F(0) = 0$



(b) $F(2) = 3$

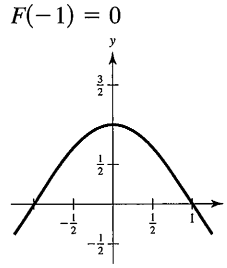


(c) $F(6) = 15$

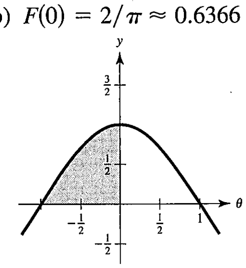


53. $F(\alpha) = (2/\pi)[\sin(\pi\alpha/2) + 1]$

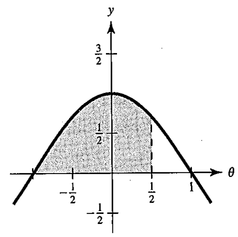
(a) $F(-1) = 0$



(b) $F(0) = 2/\pi \approx 0.6366$



(c) $F(1/2) = (\sqrt{2} + 2)/\pi \approx 1.0868$



55. 14 **57.** 16

59. Answers will vary. Sample answers:
(a) About 966 ft^2 (b) About 1004 ft^2

61. $\int_{-2}^{1} [x^3 - (3x - 2)]\, dx = \dfrac{27}{4}$

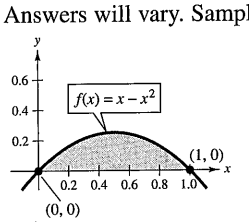
63. $\int_{0}^{1} \left[\dfrac{1}{x^2+1} - \left(-\dfrac{1}{2}x + 1\right)\right] dx \approx 0.0354$

65. Answers will vary.
Example: $x^4 - 2x^2 + 1 \le 1 - x^2$ on $[-1, 1]$
$\int_{-1}^{1} [(1 - x^2) - (x^4 - 2x^2 + 1)]\, dx = \dfrac{4}{15}$

67. (a) The integral $\int_0^5 [v_1(t) - v_2(t)]\, dt = 10$ means that the first car traveled 10 more meters than the second car between 0 and 5 seconds.
The integral $\int_0^{10} [v_1(t) - v_2(t)]\, dt = 30$ means that the first car traveled 30 more meters than the second car between 0 and 10 seconds.
The integral $\int_{20}^{30} [v_1(t) - v_2(t)]\, dt = -5$ means that the second car traveled 5 more meters than the first car between 20 and 30 seconds.
(b) No. You do not know when both cars started or the initial distance between the cars.
(c) The car with velocity v_1 is ahead by 30 meters.
(d) Car 1 is ahead by 8 meters.

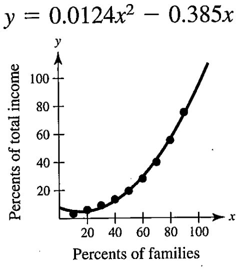
69. $b = 9(1 - 1/\sqrt[3]{4}) \approx 3.330$ **71.** $a = 4 - 2\sqrt{2} \approx 1.172$

73. Answers will vary. Sample answer: $\frac{1}{6}$

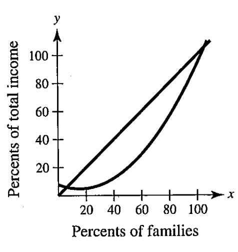


75. R_1; \$11.375 billion

77. (a) $y = 0.0124x^2 - 0.385x + 7.85$
(b)



(c)



(d) About 2006.7

79. (a) About 6.031 m^2 (b) About 12.062 m^3 (c) 60,310 lb

81. $\sqrt{3}/2 + 7\pi/24 + 1 \approx 2.7823$ **83.** True

85. False. Let $f(x) = x$ and $g(x) = 2x - x^2$. f and g intersect at (1, 1), the midpoint of [0, 2], but
$$\int_a^b [f(x) - g(x)]\, dx = \int_0^2 [x - (2x - x^2)]\, dx = \tfrac{2}{3} \neq 0.$$

87. Putnam Problem A1, 1993

Section 7.2 *(page 453)*

1. $\pi\int_0^1 (-x + 1)^2\, dx = \dfrac{\pi}{3}$ **3.** $\pi\int_1^4 (\sqrt{x})^2\, dx = \dfrac{15\pi}{2}$

5. $\pi\int_0^1 [(x^2)^2 - (x^5)^2]\, dx = \dfrac{6\pi}{55}$ **7.** $\pi\int_0^4 (\sqrt{y})^2\, dy = 8\pi$

9. $\pi\int_0^1 (y^{3/2})^2\, dy = \dfrac{\pi}{4}$

11. (a) $9\pi/2$ (b) $(36\pi\sqrt{3})/5$ (c) $(24\pi\sqrt{3})/5$
(d) $(84\pi\sqrt{3})/5$

13. (a) $32\pi/3$ (b) $64\pi/3$ **15.** 18π

17. $\pi(48 \ln 2 - \frac{27}{4}) \approx 83.318$ **19.** $124\pi/3$

21. $832\pi/15$ **23.** $\pi \ln 5$ **25.** $2\pi/3$

27. $(\pi/2)(1 - 1/e^2) \approx 1.358$ **29.** $277\pi/3$ **31.** 8π

33. $\pi^2/2 \approx 4.935$ **35.** $(\pi/2)(e^2 - 1) \approx 10.036$

37. 1.969 **39.** 15.4115 **41.** $\pi/3$ **43.** $2\pi/15$

45. $\pi/2$ **47.** $\pi/6$

49. A sine curve on $[0, \pi/2]$ revolved about the x-axis

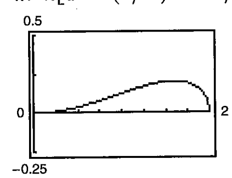
51. The parabola $y = 4x - x^2$ is a horizontal translation of the parabola $y = 4 - x^2$. Therefore, their volumes are equal.

53. (a) This statement is true. Explanations will vary.
(b) This statement is false. Explanations will vary.

55. $2\sqrt{2}$ **57.** $V = \frac{4}{3}\pi(R^2 - r^2)^{3/2}$ **59.** Proof

61. $\pi r^2 h[1 - (h/H) + h^2/(3H^2)]$

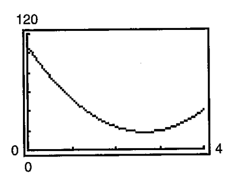
63.



$\pi/30$

65. (a) 60π (b) 50π

67. (a) $V = \pi(4b^2 - \frac{64}{3}b + \frac{512}{15})$
(b)



(c) $b = \frac{8}{3} \approx 2.67$

$b \approx 2.67$

69. (a) ii; right circular cylinder of radius r and height h
(b) iv; ellipsoid whose underlying ellipse has the equation $(x/b)^2 + (y/a)^2 = 1$
(c) iii, sphere of radius r
(d) i; right circular cone of radius r and height h
(e) v; torus of cross-sectional radius r and other radius R

71. (a) $\frac{81}{10}$ (b) $\frac{9}{2}$ **73.** $\frac{16}{3}r^3$

75. (a) $\frac{2}{3}r^3$ (b) $\frac{2}{3}r^3 \tan\theta$; As $\theta \to 90°$, $V \to \infty$.

Section 7.3 (page 462)

1. $2\pi\int_0^2 x^2\,dx = \frac{16\pi}{3}$ **3.** $2\pi\int_0^4 x\sqrt{x}\,dx = \frac{128\pi}{5}$

5. $2\pi\int_0^4 \frac{1}{4}x^3\,dx = 32\pi$ **7.** $2\pi\int_0^2 x(4x - 2x^2)\,dx = \frac{16\pi}{3}$

9. $2\pi\int_0^2 x(x^2 - 4x + 4)\,dx = \frac{8\pi}{3}$

11. $2\pi\int_2^4 x\sqrt{x-2}\,dx = \frac{128\pi}{15}\sqrt{2}$

13. $2\pi\int_0^1 x\left(\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\right)dx = \sqrt{2\pi}\left(1 - \frac{1}{\sqrt{e}}\right) \approx 0.986$

15. $2\pi\int_0^2 y(2 - y)\,dy = \frac{8\pi}{3}$

17. $2\pi\left[\int_0^{1/2} y\,dy + \int_{1/2}^1 y\left(\frac{1}{y} - 1\right)dy\right] = \frac{\pi}{2}$

19. $2\pi\int_0^8 y^{4/3}\,dy = \frac{768\pi}{7}$

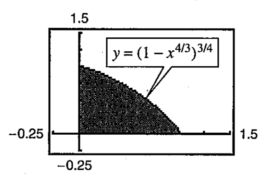
21. $2\pi\int_0^2 y(4 - 2y)\,dy = 16\pi/3$ **23.** 8π **25.** 16π

27. Shell method; it is much easier to put x in terms of y rather than vice versa.

29. (a) $128\pi/7$ (b) $64\pi/5$ (c) $96\pi/5$

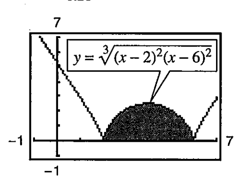
31. (a) $\pi a^3/15$ (b) $\pi a^3/15$ (c) $4\pi a^3/15$

33. (a)



(b) 1.506

35. (a)



(b) 187.25

37. (a) The rectangles would be vertical.
(b) The rectangles would be horizontal.

39. Both integrals yield the volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x-1}$, $y = 0$, and $x = 5$ about the x-axis.

41. a, c, b

43. (a) Region bounded by $y = x^2, y = 0, x = 0, x = 2$
(b) Revolved about the y-axis

45. (a) Region bounded by $x = \sqrt{6-y}, y = 0, x = 0$
(b) Revolved about $y = -2$

47. Diameter $= 2\sqrt{4 - 2\sqrt{3}} \approx 1.464$ **49.** $4\pi^2$

51. (a) Proof (b) (i) $V = 2\pi$ (ii) $V = 6\pi^2$ **53.** Proof

55. (a) $R_1(n) = n/(n+1)$ (b) $\lim_{n\to\infty} R_1(n) = 1$
(c) $V = \pi ab^{n+2}[n/(n+2)]$; $R_2(n) = n/(n+2)$
(d) $\lim_{n\to\infty} R_2(n) = 1$
(e) As $n \to \infty$, the graph approaches the line $x = b$.

57. (a) and (b) About 121,475 ft^3 **59.** $c = 2$

61. (a) $64\pi/3$ (b) $2048\pi/35$ (c) $8192\pi/105$

Section 7.4 (page 473)

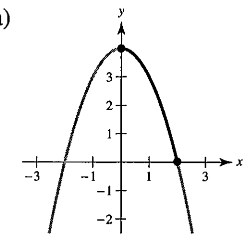
1. (a) and (b) 17 **3.** $\frac{5}{3}$ **5.** $\frac{2}{3}(2\sqrt{2} - 1) \approx 1.219$

7. $5\sqrt{5} - 2\sqrt{2} \approx 8.352$ **9.** 309.3195

11. $\ln[(\sqrt{2}+1)/(\sqrt{2}-1)] \approx 1.763$

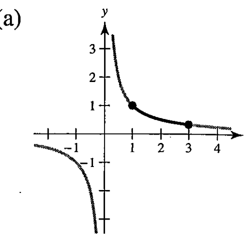
13. $\frac{1}{2}(e^2 - 1/e^2) \approx 3.627$ **15.** $\frac{76}{3}$

17. (a)



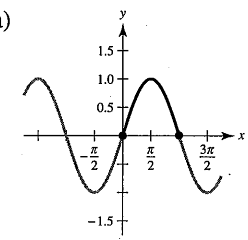
(b) $\int_0^2 \sqrt{1 + 4x^2}\,dx$
(c) About 4.647

19. (a)



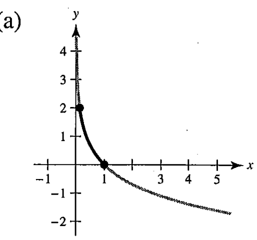
(b) $\int_1^3 \sqrt{1 + \frac{1}{x^4}}\,dx$
(c) About 2.147

21. (a)



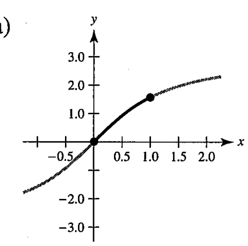
(b) $\int_0^\pi \sqrt{1 + \cos^2 x}\,dx$
(c) About 3.820

23. (a)



(b) $\int_0^2 \sqrt{1 + e^{-2y}}\,dy$
$= \int_{e^{-2}}^1 \sqrt{1 + \frac{1}{x^2}}\,dx$
(c) About 2.221

25. (a)



(b) $\int_0^1 \sqrt{1 + \left(\frac{2}{1+x^2}\right)^2}\,dx$
(c) About 1.871

27. b

29. (a) 64.125 (b) 64.525 (c) 64.666 (d) 64.672

31. $20[\sinh 1 - \sinh(-1)] \approx 47.0$ m **33.** About 1480

35. $3\arcsin\frac{2}{3} \approx 2.1892$

37. $2\pi\int_0^3 \frac{1}{3}x^3\sqrt{1 + x^4}\,dx = \frac{\pi}{9}(82\sqrt{82} - 1) \approx 258.85$

39. $2\pi\int_1^2 \left(\frac{x^3}{6} + \frac{1}{2x}\right)\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)dx = \frac{47\pi}{16} \approx 9.23$

41. $2\pi\int_{-1}^1 2\,dx = 8\pi \approx 25.13$

43. $2\pi\int_1^8 x\sqrt{1 + \frac{1}{9x^{4/3}}}\,dx = \frac{\pi}{27}(145\sqrt{145} - 10\sqrt{10}) \approx 199.48$

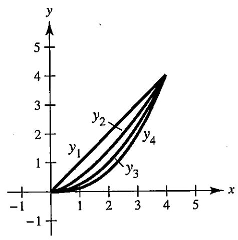
45. $2\pi\int_0^2 x\sqrt{1 + \frac{x^2}{4}}\,dx = \frac{\pi}{3}(16\sqrt{2} - 8) \approx 15.318$

47. 14.424

49. A rectifiable curve is a curve with a finite arc length.

51. The integral formula for the area of a surface of revolution is derived from the formula for the lateral surface area of the frustum of a right circular cone. The formula is $S = 2\pi rL$, where $r = \frac{1}{2}(r_1 + r_2)$, which is the average radius of the frustum, and L is the length of a line segment on the frustum. The representative element is $2\pi f(d_i)\sqrt{1 + (\Delta y_i/\Delta x_i)^2}\,\Delta x_i$.

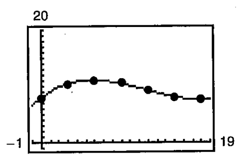
53. (a)



(b) y_1, y_2, y_3, y_4
(c) $s_1 \approx 5.657$; $s_2 \approx 5.759$; $s_3 \approx 5.916$; $s_4 \approx 6.063$

55. 20π **57.** $6\pi(3 - \sqrt{5}) \approx 14.40$

59. (a) Answers will vary. Sample answer: 5207.62 in.^{3}
(b) Answers will vary. Sample answer: 1168.64 in.^{2}
(c) $r = 0.0040y^3 - 0.142y^2 + 1.23y + 7.9$



(d) 5279.64 in.^{3}; 1179.5 in.^{2}

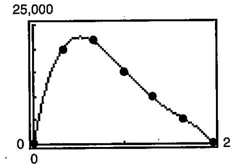
61. (a) $\pi(1 - 1/b)$ (b) $2\pi\int_1^b \sqrt{x^4 + 1}/x^3\,dx$
(c) $\lim_{b\to\infty} V = \lim_{b\to\infty} \pi(1 - 1/b) = \pi$
(d) Because $\dfrac{\sqrt{x^4+1}}{x^3} > \dfrac{\sqrt{x^4}}{x^3} = \dfrac{1}{x} > 0$ on $[1, b]$,
you have $\displaystyle\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx > \int_1^b \frac{1}{x}\,dx = \Big[\ln x\Big]_1^b = \ln b$
and $\lim_{b\to\infty} \ln b \to \infty$. So, $\displaystyle\lim_{b\to\infty} 2\pi\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx = \infty$.

63. Fleeing object: $\frac{2}{3}$ unit
Pursuer: $\displaystyle\frac{1}{2}\int_0^1 \frac{x+1}{\sqrt{x}}\,dx = \frac{4}{3} = 2\left(\frac{2}{3}\right)$

65. $384\pi/5$ **67–69.** Proofs

Section 7.5 *(page 483)*

1. 48,000 ft-lb **3.** 896 N-m **5.** 40.833 in.-lb $\approx$ 3.403 ft-lb
7. 160 in.-lb $\approx$ 13.3 ft-lb **9.** 37.125 ft-lb
11. (a) 487.805 mile-tons $\approx 5.151 \times 10^9$ ft-lb
(b) 1395.349 mile-tons $\approx 1.473 \times 10^{10}$ ft-lb
13. (a) 2.93×10^4 mile-tons $\approx 3.10 \times 10^{11}$ ft-lb
(b) 3.38×10^4 mile-tons $\approx 3.57 \times 10^{11}$ ft-lb
15. (a) 2496 ft-lb (b) 9984 ft-lb **17.** $470{,}400\pi$ N-m
19. 2995.2π ft-lb **21.** $20{,}217.6\pi$ ft-lb **23.** 2457π ft-lb
25. 600 ft-lb **27.** 450 ft-lb **29.** 168.75 ft-lb
31. If an object is moved a distance D in the direction of an applied constant force F, then the work W done by the force is defined as $W = FD$.
33. The situation in part (a) requires more work. There is no work required for part (b) because the distance is 0.
35. (a) 54 ft-lb (b) 160 ft-lb (c) 9 ft-lb (d) 18 ft-lb
37. $2000 \ln(3/2) \approx 810.93$ ft-lb **39.** 3249.4 ft-lb
41. 10,330.3 ft-lb
43. (a) $16{,}000\pi$ ft-lb (b) 24,888.889 ft-lb
(c) $F(x) = -16{,}261.36x^4 + 85{,}295.45x^3 - 157{,}738.64x^2 + 104{,}386.36x - 32.4675$

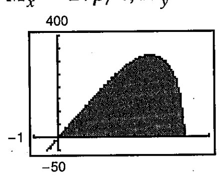


(d) 0.524 ft (e) 25,180.5 ft-lb

Section 7.6 *(page 494)*

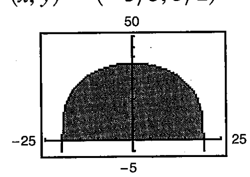
1. $\bar{x} = -\frac{4}{3}$ **3.** $\bar{x} = 4$ **5.** (a) $\bar{x} = 8$ (b) $\bar{x} = -\frac{3}{4}$
7. $x = 6$ ft **9.** $(\bar{x}, \bar{y}) = \left(\frac{10}{9}, -\frac{1}{9}\right)$ **11.** $(\bar{x}, \bar{y}) = \left(2, \frac{48}{25}\right)$
13. $M_x = \rho/3$, $M_y = 4\rho/3$, $(\bar{x}, \bar{y}) = (4/3, 1/3)$
15. $M_x = 4\rho$, $M_y = 64\rho/5$, $(\bar{x}, \bar{y}) = (12/5, 3/4)$
17. $M_x = \rho/35$, $M_y = \rho/20$, $(\bar{x}, \bar{y}) = (3/5, 12/35)$
19. $M_x = 99\rho/5$, $M_y = 27\rho/4$, $(\bar{x}, \bar{y}) = (3/2, 22/5)$
21. $M_x = 192\rho/7$, $M_y = 96\rho$, $(\bar{x}, \bar{y}) = (5, 10/7)$
23. $M_x = 0$, $M_y = 256\rho/15$, $(\bar{x}, \bar{y}) = (8/5, 0)$
25. $M_x = 27\rho/4$, $M_y = -27\rho/10$, $(\bar{x}, \bar{y}) = (-3/5, 3/2)$

27.



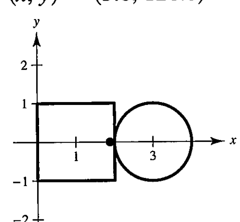
$(\bar{x}, \bar{y}) = (3.0, 126.0)$

29.



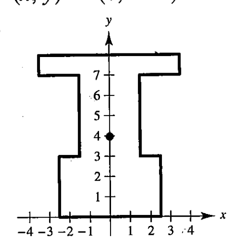
$(\bar{x}, \bar{y}) = (0, 16.2)$

31.



$(\bar{x}, \bar{y}) = \left(\dfrac{4 + 3\pi}{4 + \pi}, 0\right)$

33.



$(\bar{x}, \bar{y}) = \left(0, \dfrac{135}{34}\right)$

35. $(\bar{x}, \bar{y}) = \left(\dfrac{2 + 3\pi}{2 + \pi}, 0\right)$ **37.** $160\pi^2 \approx 1579.14$
39. $128\pi/3 \approx 134.04$
41. The center of mass $(\bar{x}, \bar{y})$ is $\bar{x} = M_y/m$ and $\bar{y} = M_x/m$, where:
1. $m = m_1 + m_2 + \cdots + m_n$ is the total mass of the system.
2. $M_y = m_1x_1 + m_2x_2 + \cdots + m_nx_n$ is the moment about the y-axis.
3. $M_x = m_1y_1 + m_2y_2 + \cdots + m_ny_n$ is the moment about the x-axis.

43. See Theorem 7.1 on page 493. **45.** $(\bar{x}, \bar{y}) = \left(\dfrac{b}{3}, \dfrac{c}{3}\right)$
47. $(\bar{x}, \bar{y}) = \left(\dfrac{(a + 2b)c}{3(a + b)}, \dfrac{a^2 + ab + b^2}{3(a + b)}\right)$
49. $(\bar{x}, \bar{y}) = (0, 4b/(3\pi))$