Chapter 8

Section 8.1 *(page 512)*

1. b **3.** c

5. $\int u^n\,du$; $u = 5x - 3, n = 4$ **7.** $\int \frac{du}{u}$; $u = 1 - 2\sqrt{x}$ **9.** $\int \frac{du}{\sqrt{a^2 - u^2}}$; $u = t, a = 1$

11. $\int \sin u\,du$; $u = t^2$ **13.** $\int e^u\,du$; $u = \sin x$ **15.** $2(x - 5)^7 + C$

17. $-7/[6(z - 10)^6] + C$ **19.** $\frac{1}{2}v^2 - 1/[6(3v - 1)^2] + C$

21. $-\frac{1}{3}\ln|-t^3 + 9t + 1| + C$

23. $\frac{1}{2}x^2 + x + \ln|x - 1| + C$ **25.** $\ln(1 + e^x) + C$

27. $\frac{x}{15}(48x^4 + 200x^2 + 375) + C$ **29.** $\sin(2\pi x^2)/(4\pi) + C$

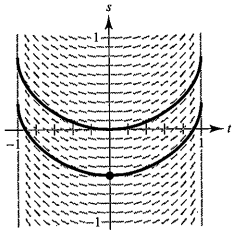
31. $-2\sqrt{\cos x} + C$ **33.** $2\ln(1 + e^x) + C$

35. $(\ln x)^2 + C$ **37.** $-\ln|\csc\alpha + \cot\alpha| + \ln|\sin\alpha| + C$

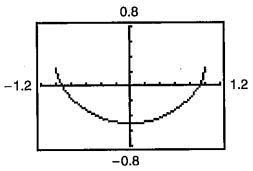
39. $-\frac{1}{4}\arcsin(4t + 1) + C$ **41.** $\frac{1}{2}\ln|\cos(2/t)| + C$

43. $6\arcsin[(x - 5)/5] + C$ **45.** $\frac{1}{4}\arctan[(2x + 1)/8] + C$

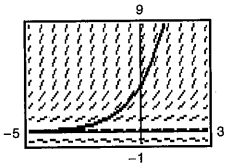
47. (a)



(b) $\frac{1}{2}\arcsin t^2 - \frac{1}{2}$



49. $y = 4e^{0.8x}$



51. $y = \frac{1}{2}e^{2x} + 10e^x + 25x + C$ **53.** $r = 10\arcsin e^t + C$

55. $y = \frac{1}{2}\arctan(\tan x/2) + C$ **57.** $\frac{1}{2}$

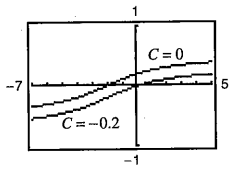
59. $\frac{1}{2}(1 - e^{-1}) \approx 0.316$ **61.** 8 **63.** $\pi/18$

65. $18\sqrt{6}/5 \approx 8.82$ **67.** $\frac{4}{3} \approx 1.333$

69. $\frac{1}{3}\arctan\left[\frac{1}{3}(x + 2)\right] + C$

Graphs will vary.

Example:

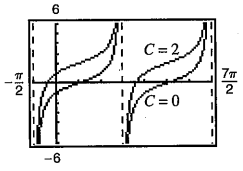


One graph is a vertical translation of the other.

71. $\tan\theta - \sec\theta + C$

Graphs will vary.

Example:



One graph is a vertical translation of the other.

73. Power Rule: $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$; $u = x^2 + 1, n = 3$

75. Log Rule: $\int \frac{du}{u} = \ln|u| + C$; $u = x^2 + 1$

77. $a = \sqrt{2}, b = \frac{\pi}{4}$; $-\frac{1}{\sqrt{2}}\ln\left|\csc\left(x + \frac{\pi}{4}\right) + \cot\left(x + \frac{\pi}{4}\right)\right| + C$

79. $a = \frac{1}{2}$

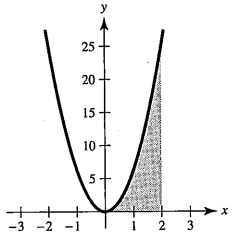
81. (a) They are equivalent because $e^{x+C_1} = e^x \cdot e^{C_1} = Ce^x$, $C = e^{C_1}$.

(b) They differ by a constant.

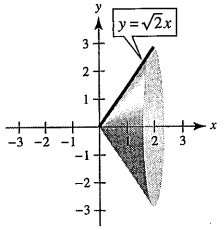
$\sec^2 x + C_1 = (\tan^2 x + 1) + C_1 = \tan^2 x + C$

83. a

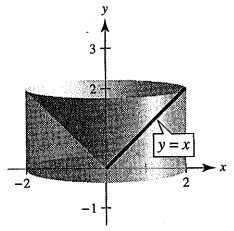
85. (a)



(b)



(c)



87. (a) $\pi(1 - e^{-1}) \approx 1.986$

(b) $b = \sqrt{\ln\left(\frac{3\pi}{3\pi - 4}\right)} \approx 0.743$

89. $\ln\left(\sqrt{2} + 1\right) \approx 0.8814$

91. $(8\pi/3)\left(10\sqrt{10} - 1\right) \approx 256.545$ **93.** $\frac{1}{3}\arctan 3 \approx 0.416$

95. About 1.0320

97. (a) $\frac{1}{3}\sin x(\cos^2 x + 2)$

(b) $\frac{1}{15}\sin x(3\cos^4 x + 4\cos^2 x + 8)$

(c) $\frac{1}{35}\sin x(5\cos^6 x + 6\cos^4 x + 8\cos^2 x + 16)$

(d) $\int \cos^{15} x\,dx = \int (1 - \sin^2 x)^7 \cos x\,dx$

You would expand $(1 - \sin^2 x)^7$.

99. Proof

Section 8.2 *(page 521)*

1. $u = x, dv = e^{2x}dx$ **3.** $u = (\ln x)^2, dv = dx$

5. $u = x, dv = \sec^2 x\,dx$ **7.** $\frac{1}{16}x^4(4\ln x - 1) + C$

9. $\frac{1}{9}\sin 3x - \frac{1}{3}x\cos 3x + C$ **11.** $-\frac{1}{16e^{4x}}(4x + 1) + C$

13. $e^x(x^3 - 3x^2 + 6x - 6) + C$

15. $\frac{1}{4}[2(t^2 - 1)\ln|t + 1| - t^2 + 2t] + C$ **17.** $\frac{1}{3}(\ln x)^3 + C$

19. $e^{2x}/[4(2x + 1)] + C$ **21.** $\frac{2}{15}(x - 5)^{3/2}(3x + 10) + C$

23. $x\sin x + \cos x + C$

25. $(6x - x^3)\cos x + (3x^2 - 6)\sin x + C$

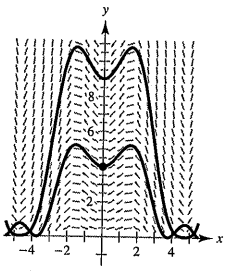
27. $x\arctan x - \frac{1}{2}\ln(1 + x^2) + C$

29. $-\frac{3}{34}e^{-3x}\sin 5x - \frac{5}{34}e^{-3x}\cos 5x + C$ **31.** $x\ln x - x + C$

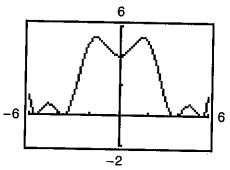
33. $y = \frac{2}{5}t^2\sqrt{3 + 5t} - \frac{8t}{75}(3 + 5t)^{3/2} + \frac{16}{1875}(3 + 5t)^{5/2} + C$

$= \frac{2}{625}\sqrt{3 + 5t}(25t^2 - 20t + 24) + C$

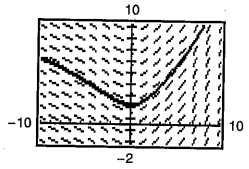
35. (a)



(b) $2\sqrt{y} - \cos x - x \sin x = 3$



37.



39. $2e^{3/2} + 4 \approx 12.963$

41. $\frac{\pi}{8} - \frac{1}{4} \approx 0.143$

43. $(\pi - 3\sqrt{3} + 6)/6 \approx 0.658$

45. $\frac{1}{2}[e(\sin 1 - \cos 1) + 1] \approx 0.909$

47. $8 \operatorname{arcsec} 4 + \sqrt{3}/2 - \sqrt{15}/2 - 2\pi/3 \approx 7.380$

49. $(e^{2x}/4)(2x^2 - 2x + 1) + C$

51. $(3x^2 - 6)\sin x - (x^3 - 6x)\cos x + C$

53. $x \tan x + \ln|\cos x| + C$

55. $2(\sin\sqrt{x} - \sqrt{x}\cos\sqrt{x}) + C$

57. $\frac{1}{2}(x^4e^{x^2} - 2x^2e^{x^2} + 2e^{x^2}) + C$

59. (a) Product Rule

(b) Answers will vary. Sample answer: You want dv to be the most complicated portion of the integrand.

61. (a) No, substitution (b) Yes, $u = \ln x$, $dv = x\,dx$

(c) Yes, $u = x^2$, $dv = e^{-3x}\,dx$ (d) No, substitution

(e) Yes, $u = x$ and $dv = \dfrac{1}{\sqrt{x+1}}\,dx$ (f) No, substitution

63. $\frac{1}{3}\sqrt{4 + x^2}(x^2 - 8) + C$

65. $n = 0$: $x(\ln x - 1) + C$

$n = 1$: $\frac{1}{4}x^2(2 \ln x - 1) + C$

$n = 2$: $\frac{1}{9}x^3(3 \ln x - 1) + C$

$n = 3$: $\frac{1}{16}x^4(4 \ln x - 1) + C$

$n = 4$: $\frac{1}{25}x^5(5 \ln x - 1) + C$

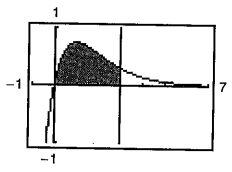
$$\int x^n \ln x\,dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\ln x - 1] + C$$

67–71. Proofs **73.** $-x^2 \cos x + 2x \sin x + 2 \cos x + C$

75. $\frac{1}{36}x^6(6 \ln x - 1) + C$

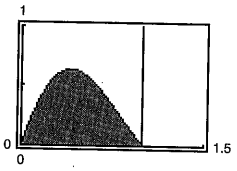
77. $\dfrac{e^{-3x}(-3 \sin 4x - 4 \cos 4x)}{25} + C$

79.



$2 - \dfrac{8}{e^3} \approx 1.602$

81.



$\dfrac{\pi}{1 + \pi^2}\left(\dfrac{1}{e} + 1\right) \approx 0.395$

83. (a) 1 (b) $\pi(e - 2) \approx 2.257$ (c) $\frac{1}{2}\pi(e^2 + 1) \approx 13.177$

(d) $\left(\dfrac{e^2 + 1}{4}, \dfrac{e - 2}{2}\right) \approx (2.097, 0.359)$

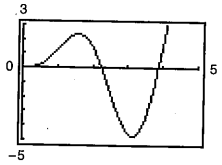
85. In Example 6, we showed that the centroid of an equivalent region was $(1, \pi/8)$. By symmetry, the centroid of this region is $(\pi/8, 1)$.

87. $[7/(10\pi)](1 - e^{-4\pi}) \approx 0.223$ **89.** \$931,265

91. Proof **93.** $b_n = [8h/(n\pi)^2]\sin(n\pi/2)$

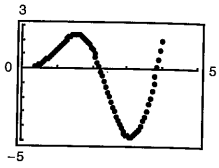
95. (a) $y = \frac{1}{4}(3 \sin 2x - 6x \cos 2x)$

(b)



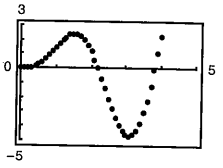
(c) You obtain the following points.

n	x_n	y_n
0	0	0
1	0.05	0
2	0.10	7.4875×10^{-4}
3	0.15	0.0037
4	0.20	0.0104
⋮	⋮	⋮
80	4.00	1.3181



(d) You obtain the following points.

n	x_n	y_n
0	5	0
1	0.1	0
2	0.2	0.0060
3	0.3	0.0293
4	0.4	0.0801
⋮	⋮	⋮
40	4.0	1.0210



97. The graph of $y = x \sin x$ is below the graph of $y = x$ on $[0, \pi/2]$.

99. For any integrable function, $\int f(x)\,dx = C + \int f(x)\,dx$, but this cannot be used to imply that $C = 0$.

Section 8.3 *(page 530)*

1. $-\frac{1}{6}\cos^6 x + C$ **3.** $\frac{1}{16}\sin^8 2x + C$

5. $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

7. $-\frac{1}{3}(\cos 2\theta)^{3/2} + \frac{1}{7}(\cos 2\theta)^{7/2} + C$

9. $\frac{1}{12}(6x + \sin 6x) + C$

11. $\frac{1}{8}(2x^2 - 2x \sin 2x - \cos 2x) + C$ **13.** $\frac{16}{35}$

15. $63\pi/512$ **17.** $5\pi/32$ **19.** $\frac{1}{4}\ln|\sec 4x + \tan 4x| + C$

21. $(\sec \pi x \tan \pi x + \ln|\sec \pi x + \tan \pi x|)/(2\pi) + C$

23. $\frac{1}{2}\tan^4(x/2) - \tan^2(x/2) - 2\ln|\cos(x/2)| + C$

25. $\dfrac{1}{2}\left[\dfrac{\sec^5 2t}{5} - \dfrac{\sec^3 2t}{3}\right] + C$ **27.** $\dfrac{1}{24}\sec^6 4x + C$

29. $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

31. $\ln|\sec x + \tan x| - \sin x + C$

33. $(12\pi\theta - 8 \sin 2\pi\theta + \sin 4\pi\theta)/(32\pi) + C$

35. $y = \frac{1}{9}\sec^3 3x - \frac{1}{3}\sec 3x + C$