Chapter 8

Section 8.1 *(page 512)*

1. b **3.** c

5. $\int u^n\,du$, $u = 5x - 3, n = 4$ **7.** $\int \frac{du}{u}$, $u = 1 - 2\sqrt{x}$ **9.** $\int \frac{du}{\sqrt{a^2 - u^2}}$, $u = t, a = 1$

11. $\int \sin u\,du$, $u = t^2$ **13.** $\int e^u\,du$, $u = \sin x$ **15.** $2(x - 5)^7 + C$

17. $-7/[6(z - 10)^6] + C$ **19.** $\frac{1}{2}v^2 - 1/[6(3v - 1)^2] + C$

21. $-\frac{1}{3}\ln|-t^3 + 9t + 1| + C$

23. $\frac{1}{2}x^2 + x + \ln|x - 1| + C$ **25.** $\ln(1 + e^x) + C$

27. $\frac{x}{15}(48x^4 + 200x^2 + 375) + C$ **29.** $\sin(2\pi x^2)/(4\pi) + C$

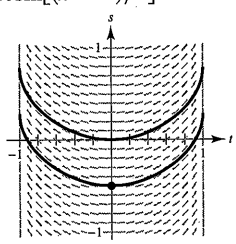
31. $-2\sqrt{\cos x} + C$ **33.** $2\ln(1 + e^x) + C$

35. $(\ln x)^2 + C$ **37.** $-\ln|\csc\alpha + \cot\alpha| + \ln|\sin\alpha| + C$

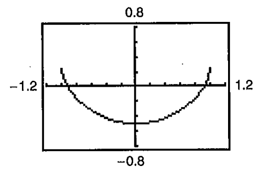
39. $-\frac{1}{4}\arcsin(4t + 1) + C$ **41.** $\frac{1}{2}\ln|\cos(2/t)| + C$

43. $6\arcsin[(x - 5)/5] + C$ **45.** $\frac{1}{4}\arctan[(2x + 1)/8] + C$

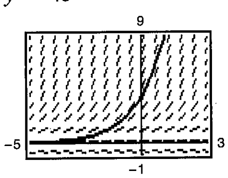
47. (a)



(b) $\frac{1}{2}\arcsin t^2 - \frac{1}{2}$



49. $y = 4e^{0.8x}$



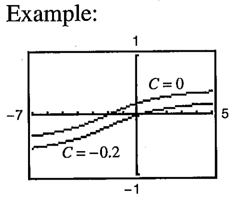
51. $y = \frac{1}{2}e^{2x} + 10e^x + 25x + C$ **53.** $r = 10\arcsin e^t + C$

55. $y = \frac{1}{2}\arctan(\tan x/2) + C$ **57.** $\frac{1}{2}$

59. $\frac{1}{2}(1 - e^{-1}) \approx 0.316$ **61.** 8 **63.** $\pi/18$

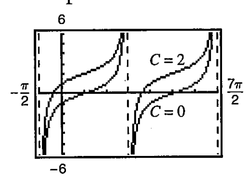
65. $18\sqrt{6}/5 \approx 8.82$ **67.** $\frac{4}{3} \approx 1.333$

69. $\frac{1}{3}\arctan\left[\frac{1}{3}(x + 2)\right] + C$
Graphs will vary.
Example:



One graph is a vertical translation of the other.

71. $\tan\theta - \sec\theta + C$
Graphs will vary.
Example:



One graph is a vertical translation of the other.

73. Power Rule: $\int u^n\,du = \frac{u^{n+1}}{n+1} + C;\ u = x^2 + 1, n = 3$

75. Log Rule: $\int \frac{du}{u} = \ln|u| + C;\ u = x^2 + 1$

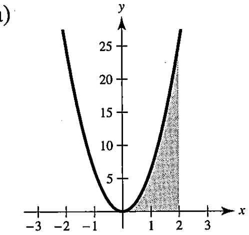
77. $a = \sqrt{2}, b = \frac{\pi}{4};\ -\frac{1}{\sqrt{2}}\ln\left|\csc\left(x + \frac{\pi}{4}\right) + \cot\left(x + \frac{\pi}{4}\right)\right| + C$

79. $a = \frac{1}{2}$

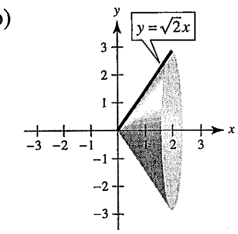
81. (a) They are equivalent because
$e^{x+C_1} = e^x \cdot e^{C_1} = Ce^x, C = e^{C_1}$.
(b) They differ by a constant.
$\sec^2 x + C_1 = (\tan^2 x + 1) + C_1 = \tan^2 x + C$

83. a

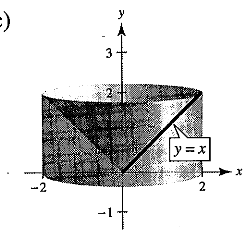
85. (a)



(b)



(c)



87. (a) $\pi(1 - e^{-1}) \approx 1.986$
(b) $b = \sqrt{\ln\left(\frac{3\pi}{3\pi - 4}\right)} \approx 0.743$

89. $\ln(\sqrt{2} + 1) \approx 0.8814$

91. $(8\pi/3)(10\sqrt{10} - 1) \approx 256.545$ **93.** $\frac{1}{3}\arctan 3 \approx 0.416$

95. About 1.0320

97. (a) $\frac{1}{3}\sin x(\cos^2 x + 2)$
(b) $\frac{1}{15}\sin x(3\cos^4 x + 4\cos^2 x + 8)$
(c) $\frac{1}{35}\sin x(5\cos^6 x + 6\cos^4 x + 8\cos^2 x + 16)$
(d) $\int \cos^{15} x\,dx = \int (1 - \sin^2 x)^7\cos x\,dx$
You would expand $(1 - \sin^2 x)^7$.

99. Proof

Section 8.2 *(page 521)*

1. $u = x, dv = e^{2x}\,dx$ **3.** $u = (\ln x)^2, dv = dx$

5. $u = x, dv = \sec^2 x\,dx$ **7.** $\frac{1}{16}x^4(4\ln x - 1) + C$

9. $\frac{1}{9}\sin 3x - \frac{1}{3}x\cos 3x + C$ **11.** $-\frac{1}{16e^{4x}}(4x + 1) + C$

13. $e^x(x^3 - 3x^2 + 6x - 6) + C$

15. $\frac{1}{4}[2(t^2 - 1)\ln|t + 1| - t^2 + 2t] + C$ **17.** $\frac{1}{3}(\ln x)^3 + C$

19. $e^{2x}/[4(2x + 1)] + C$ **21.** $\frac{2}{15}(x - 5)^{3/2}(3x + 10) + C$

23. $x\sin x + \cos x + C$

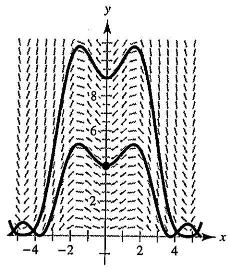
25. $(6x - x^3)\cos x + (3x^2 - 6)\sin x + C$

27. $x\arctan x - \frac{1}{2}\ln(1 + x^2) + C$

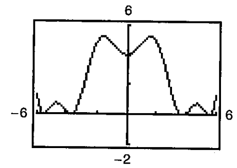
29. $-\frac{3}{34}e^{-3x}\sin 5x - \frac{5}{34}e^{-3x}\cos 5x + C$ **31.** $x\ln x - x + C$

33. $y = \frac{2}{5}t^2\sqrt{3 + 5t} - \frac{8t}{75}(3 + 5t)^{3/2} + \frac{16}{1875}(3 + 5t)^{5/2} + C$
$= \frac{2}{625}\sqrt{3 + 5t}(25t^2 - 20t + 24) + C$

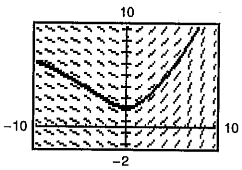
35. (a)



(b) $2\sqrt{y} - \cos x - x \sin x = 3$



37.



39. $2e^{3/2} + 4 \approx 12.963$

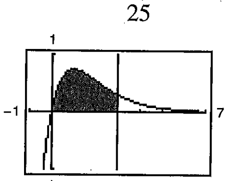
41. $\dfrac{\pi}{8} - \dfrac{1}{4} \approx 0.143$

43. $(\pi - 3\sqrt{3} + 6)/6 \approx 0.658$
45. $\frac{1}{2}[e(\sin 1 - \cos 1) + 1] \approx 0.909$
47. $8 \operatorname{arcsec} 4 + \sqrt{3}/2 - \sqrt{15}/2 - 2\pi/3 \approx 7.380$
49. $(e^{2x}/4)(2x^2 - 2x + 1) + C$
51. $(3x^2 - 6) \sin x - (x^3 - 6x) \cos x + C$
53. $x \tan x + \ln|\cos x| + C$
55. $2(\sin\sqrt{x} - \sqrt{x} \cos \sqrt{x}) + C$
57. $\frac{1}{2}(x^4e^{x^2} - 2x^2e^{x^2} + 2e^{x^2}) + C$
59. (a) Product Rule
(b) Answers will vary. Sample answer: You want dv to be the most complicated portion of the integrand.
61. (a) No, substitution (b) Yes, $u = \ln x$, $dv = x\,dx$
(c) Yes, $u = x^2$, $dv = e^{-3x}\,dx$ (d) No, substitution
(e) Yes, $u = x$ and $dv = \dfrac{1}{\sqrt{x+1}}\,dx$ (f) No, substitution
63. $\frac{1}{3}\sqrt{4 + x^2}(x^2 - 8) + C$
65. $n = 0$: $x(\ln x - 1) + C$
$n = 1$: $\frac{1}{4}x^2(2 \ln x - 1) + C$
$n = 2$: $\frac{1}{9}x^3(3 \ln x - 1) + C$
$n = 3$: $\frac{1}{16}x^4(4 \ln x - 1) + C$
$n = 4$: $\frac{1}{25}x^5(5 \ln x - 1) + C$

$$\int x^n \ln x\,dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\ln x - 1] + C$$

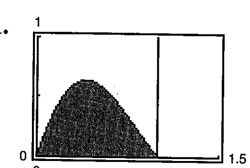
67–71. Proofs **73.** $-x^2 \cos x + 2x \sin x + 2 \cos x + C$
75. $\frac{1}{36}x^6(6 \ln x - 1) + C$
77. $\dfrac{e^{-3x}(-3 \sin 4x - 4 \cos 4x)}{25} + C$

79.



$2 - \dfrac{8}{e^3} \approx 1.602$

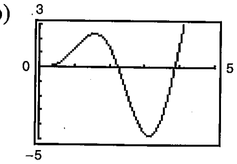
81.



$\dfrac{\pi}{1 + \pi^2}\left(\dfrac{1}{e} + 1\right) \approx 0.395$

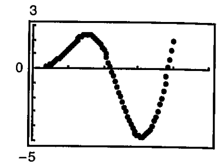
83. (a) 1 (b) $\pi(e - 2) \approx 2.257$ (c) $\frac{1}{2}\pi(e^2 + 1) \approx 13.177$
(d) $\left(\dfrac{e^2 + 1}{4}, \dfrac{e - 2}{2}\right) \approx (2.097, 0.359)$
85. In Example 6, we showed that the centroid of an equivalent region was $(1, \pi/8)$. By symmetry, the centroid of this region is $(\pi/8, 1)$.
87. $[7/(10\pi)](1 - e^{-4\pi}) \approx 0.223$ **89.** \$931,265

91. Proof **93.** $b_n = [8h/(n\pi)^2] \sin(n\pi/2)$
95. (a) $y = \frac{1}{4}(3 \sin 2x - 6x \cos 2x)$
(b)



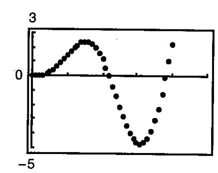
(c) You obtain the following points.

n	x_n	y_n
0	0	0
1	0.05	0
2	0.10	7.4875×10^{-4}
3	0.15	0.0037
4	0.20	0.0104
⋮	⋮	⋮
80	4.00	1.3181



(d) You obtain the following points.

n	x_n	y_n
0	5	0
1	0.1	0
2	0.2	0.0060
3	0.3	0.0293
4	0.4	0.0801
⋮	⋮	⋮
40	4.0	1.0210

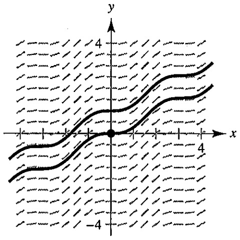


97. The graph of $y = x \sin x$ is below the graph of $y = x$ on $[0, \pi/2]$.
99. For any integrable function, $\int f(x)\,dx = C + \int f(x)\,dx$, but this cannot be used to imply that $C = 0$.

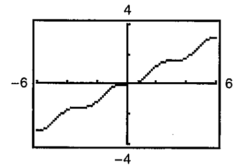
Section 8.3 *(page 530)*

1. $-\frac{1}{6}\cos^6 x + C$ **3.** $\frac{1}{16}\sin^8 2x + C$
5. $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$
7. $-\frac{1}{3}(\cos 2\theta)^{3/2} + \frac{1}{7}(\cos 2\theta)^{7/2} + C$
9. $\frac{1}{12}(6x + \sin 6x) + C$
11. $\frac{1}{8}(2x^2 - 2x \sin 2x - \cos 2x) + C$ **13.** $\frac{16}{35}$
15. $63\pi/512$ **17.** $5\pi/32$ **19.** $\frac{1}{4}\ln|\sec 4x + \tan 4x| + C$
21. $(\sec \pi x \tan \pi x + \ln|\sec \pi x + \tan \pi x|)/(2\pi) + C$
23. $\frac{1}{2}\tan^4(x/2) - \tan^2(x/2) - 2\ln|\cos(x/2)| + C$
25. $\dfrac{1}{2}\left[\dfrac{\sec^5 2t}{5} - \dfrac{\sec^3 2t}{3}\right] + C$ **27.** $\dfrac{1}{24}\sec^6 4x + C$
29. $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$
31. $\ln|\sec x + \tan x| - \sin x + C$
33. $(12\pi\theta - 8 \sin 2\pi\theta + \sin 4\pi\theta)/(32\pi) + C$
35. $y = \frac{1}{9}\sec^3 3x - \frac{1}{3}\sec 3x + C$

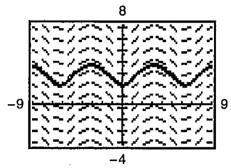
37. (a)



(b) $y = \frac{1}{2}x - \frac{1}{4}\sin 2x$



39.



41. $\frac{1}{16}(2\sin 4x + \sin 8x) + C$

43. $\frac{1}{12}(3\cos 2x - \cos 6x) + C$ **45.** $\frac{1}{8}(2\sin 2\theta - \sin 4\theta) + C$

47. $\frac{1}{4}\left(\ln|\csc^2 2x| - \cot^2 2x\right) + C$

49. $-\frac{1}{3}\cot 3x - \frac{1}{9}\cot^3 3x + C$

51. $\ln|\csc t - \cot t| + \cos t + C$

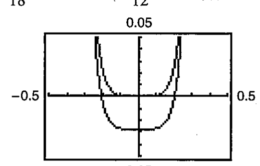
53. $\ln|\csc x - \cot x| + \cos x + C$ **55.** $t - 2\tan t + C$

57. π **59.** $3(1 - \ln 2)$ **61.** $\ln 2$ **63.** 4

65. (a) Save one sine factor and convert the remaining factors to cosines. Then expand and integrate.
(b) Save one cosine factor and convert the remaining factors to sines. Then expand and integrate.
(c) Make repeated use of the power reducing formulas to convert the integrand to odd powers of the cosine. Then proceed as in part (b).

67. (a) $\frac{1}{2}\sin^2 x + C$ (b) $-\frac{1}{2}\cos^2 x + C$
(c) $\frac{1}{2}\sin^2 x + C$ (d) $-\frac{1}{4}\cos 2x + C$
The answers are all the same; they are just written in different forms. Using trigonometric identities, you can rewrite each answer in the same form.

69. (a) $\frac{1}{18}\tan^6 3x + \frac{1}{12}\tan^4 3x + C_1, \frac{1}{18}\sec^6 3x - \frac{1}{12}\sec^4 3x + C_2$
(b)



(c) Proof

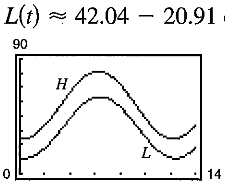
71. $\frac{1}{3}$ **73.** 1 **75.** $2\pi(1 - \pi/4) \approx 1.348$

77. (a) $\pi^2/2$ (b) $(\bar{x}, \bar{y}) = (\pi/2, \pi/8)$ **79–81.** Proofs

83. $-\frac{1}{15}\cos x(3\sin^4 x + 4\sin^2 x + 8) + C$

85. $\dfrac{5}{6\pi}\tan\dfrac{2\pi x}{5}\left(\sec^2\dfrac{2\pi x}{5} + 2\right) + C$

87. (a) $H(t) \approx 57.72 - 23.36\cos(\pi t/6) - 2.75\sin(\pi t/6)$
(b) $L(t) \approx 42.04 - 20.91\cos(\pi t/6) - 4.33\sin(\pi t/6)$
(c)



The maximum difference is at $t \approx 4.9$, or late spring.

89. Proof

Section 8.4 *(page 539)*

1. $x = 3\tan\theta$ **3.** $x = 5\sin\theta$ **5.** $x/\left(16\sqrt{16 - x^2}\right) + C$

7. $4\ln\left|\left(4 - \sqrt{16 - x^2}\right)/x\right| + \sqrt{16 - x^2} + C$

9. $\ln\left|x + \sqrt{x^2 - 25}\right| + C$

11. $\frac{1}{15}(x^2 - 25)^{3/2}(3x^2 + 50) + C$

13. $\frac{1}{3}(1 + x^2)^{3/2} + C$ **15.** $\frac{1}{2}[\arctan x + x/(1 + x^2)] + C$

17. $\frac{1}{2}x\sqrt{9 + 16x^2} + \frac{9}{8}\ln\left|4x + \sqrt{9 + 16x^2}\right| + C$

19. $\frac{25}{4}\arcsin(2x/5) + \frac{1}{2}x\sqrt{25 - 4x^2} + C$

21. $\arcsin(x/4) + C$ **23.** $4\arcsin(x/2) + x\sqrt{4 - x^2} + C$

25. $-\dfrac{(1 - x^2)^{3/2}}{3x^3} + C$ **27.** $-\dfrac{1}{3}\ln\left|\dfrac{\sqrt{4x^2 + 9} + 3}{2x}\right| + C$

29. $3/\sqrt{x^2 + 3} + C$

31. $\frac{1}{2}\left(\arcsin e^x + e^x\sqrt{1 - e^{2x}}\right) + C$

33. $\frac{1}{4}\left[x/(x^2 + 2) + \left(1/\sqrt{2}\right)\arctan\left(x/\sqrt{2}\right)\right] + C$

35. $x\,\text{arcsec}\,2x - \frac{1}{2}\ln\left|2x + \sqrt{4x^2 - 1}\right| + C$

37. $\arcsin[(x - 2)/2] + C$

39. $\sqrt{x^2 + 6x + 12} - 3\ln\left|\sqrt{x^2 + 6x + 12} + (x + 3)\right| + C$

41. (a) and (b) $\sqrt{3} - \pi/3 \approx 0.685$

43. (a) and (b) $9\left(2 - \sqrt{2}\right) \approx 5.272$

45. (a) and (b) $-(9/2)\ln\left(2\sqrt{7}/3 - 4\sqrt{3}/3 - \sqrt{21}/3 + 8/3\right) + 9\sqrt{3} - 2\sqrt{7} \approx 12.644$

47. (a) Let $u = a\sin\theta$, $\sqrt{a^2 - u^2} = a\cos\theta$, where $-\pi/2 \le \theta \le \pi/2$.
(b) Let $u = a\tan\theta$, $\sqrt{a^2 + u^2} = a\sec\theta$, where $-\pi/2 < \theta < \pi/2$.
(c) Let $u = a\sec\theta$, $\sqrt{u^2 - a^2} = \tan\theta$ if $u > a$ and $\sqrt{u^2 - a^2} = -\tan\theta$ if $u < -a$, where $0 \le \theta < \pi/2$ or $\pi/2 < \theta \le \pi$.

49. (a) $\frac{1}{2}\ln(x^2 + 9) + C$; The answers are equivalent.
(b) $x - 3\arctan(x/3) + C$; The answers are equivalent.

51. True

53. False. $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{(1 + x^2)^{3/2}} = \int_0^{\pi/3} \cos\theta\, d\theta$

55. πab

57. (a) $5\sqrt{2}$ (b) $25(1 - \pi/4)$ (c) $r^2(1 - \pi/4)$

59. $6\pi^2$ **61.** $\ln\left[\dfrac{5\left(\sqrt{2} + 1\right)}{\sqrt{26} + 1}\right] + \sqrt{26} - \sqrt{2} \approx 4.367$

63. Length of one arch of sine curve: $y = \sin x, y' = \cos x$

$$L_1 = \int_0^{\pi} \sqrt{1 + \cos^2 x}\, dx$$

Length of one arch of cosine curve: $y = \cos x, y' = -\sin x$

$$L_2 = \int_{-\pi/2}^{\pi/2} \sqrt{1 + \sin^2 x}\, dx$$
$$= \int_{-\pi/2}^{\pi/2} \sqrt{1 + \cos^2(x - \pi/2)}\, dx,\ u = x - \pi/2, du = dx$$
$$= \int_{-\pi}^{0} \sqrt{1 + \cos^2 u}\, du = \int_0^{\pi} \sqrt{1 + \cos^2 u}\, du = L_1$$

65. (0, 0.422)

67. $(\pi/32)\left[102\sqrt{2} - \ln\left(3 + 2\sqrt{2}\right)\right] \approx 13.989$

69. (a) 187.2π lb (b) $62.4\pi d$ lb **71.** Proof

73. $12 + 9\pi/2 - 25\arcsin(3/5) \approx 10.050$

75. Putnum Problem A5, 2005

Section 8.5 *(page 549)*

1. $\dfrac{A}{x} + \dfrac{B}{x - 8}$ **3.** $\dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 10}$

5. $\frac{1}{6}\ln|(x - 3)/(x + 3)| + C$ **7.** $\ln|(x - 1)/(x + 4)| + C$

9. $5\ln|x-2| - \ln|x+2| - 3\ln|x| + C$
11. $x^2 + \frac{3}{2}\ln|x-4| - \frac{1}{2}\ln|x+2| + C$
13. $1/x + \ln|x^4 + x^3| + C$
15. $2\ln|x-2| - \ln|x| - 3/(x-2) + C$
17. $\ln|(x^2+1)/x| + C$
19. $\frac{1}{6}\left[\ln|(x-2)/(x+2)| + \sqrt{2}\arctan\left(x/\sqrt{2}\right)\right] + C$
21. $\ln|x+1| + \sqrt{2}\arctan\left[(x-1)/\sqrt{2}\right] + C$
23. $\ln 3$ **25.** $\frac{1}{2}\ln(8/5) - \pi/4 + \arctan 2 \approx 0.557$
27. $\ln|1 + \sec x| + C$ **29.** $\ln\left|\frac{\tan x + 2}{\tan x + 3}\right| + C$
31. $\frac{1}{5}\ln\left|\frac{e^x - 1}{e^x + 4}\right| + C$ **33.** $2\sqrt{x} + 2\ln\left|\frac{\sqrt{x} - 2}{\sqrt{x} + 2}\right| + C$
35–37. Proofs **39.** First divide x^3 by $(x - 5)$.
41. (a) Substitution: $u = x^2 + 2x - 8$ (b) Partial fractions
(c) Trigonometric substitution (tan) or inverse tangent rule
43. $12\ln\left(\frac{9}{8}\right) \approx 1.4134$ **45.** 4.90 or \$490,000
47. $V = 2\pi\left(\arctan 3 - \frac{3}{10}\right) \approx 5.963$; $(\bar{x}, \bar{y}) \approx (1.521, 0.412)$
49. $x = n[e^{(n+1)kt} - 1]/[n + e^{(n+1)kt}]$ **51.** $\pi/8$

Section 8.6 *(page 555)*

1. $-\frac{1}{2}x(10 - x) + 25\ln|5 + x| + C$ **3.** $-\sqrt{1 - x^2}/x + C$
5. $\frac{1}{24}(3x + \sin 3x\cos 3x + 2\cos^3 3x\sin 3x) + C$
7. $-2\left(\cot\sqrt{x} + \csc\sqrt{x}\right) + C$ **9.** $x - \frac{1}{2}\ln(1 + e^{2x}) + C$
11. $\frac{1}{16}x^8(8\ln x - 1) + C$
13. (a) and (b) $\frac{1}{27}e^{3x}(9x^2 - 6x + 2) + C$
15. (a) and (b) $\ln|(x+1)/x| - 1/x + C$
17. $\frac{1}{2}\left[(x^2+1)\operatorname{arccsc}(x^2+1) + \ln\left(x^2 + 1 + \sqrt{x^4 + 2x^2}\right)\right] + C$
19. $\sqrt{x^2 - 4}/(4x) + C$
21. $\frac{4}{25}[\ln|2 - 5x| + 2/(2 - 5x)] + C$
23. $e^x\arccos(e^x) - \sqrt{1 - e^{2x}} + C$
25. $\frac{1}{2}(x^2 + \cot x^2 + \csc x^2) + C$
27. $\left(\sqrt{2}/2\right)\arctan\left[(1 + \sin\theta)/\sqrt{2}\right] + C$
29. $-\sqrt{2 + 9x^2}/(2x) + C$
31. $\frac{1}{4}\left(2\ln|x| - 3\ln|3 + 2\ln|x||\right) + C$
33. $(3x - 10)/[2(x^2 - 6x + 10)] + \frac{3}{2}\arctan(x - 3) + C$
35. $\frac{1}{2}\ln\left|x^2 - 3 + \sqrt{x^4 - 6x^2 + 5}\right| + C$
37. $2/(1 + e^x) - 1/[2(1 + e^x)^2] + \ln(1 + e^x) + C$
39. $\frac{1}{2}(e - 1) \approx 0.8591$ **41.** $\frac{32}{5}\ln 2 - \frac{31}{25} \approx 3.1961$
43. $\pi/2$ **45.** $\pi^3/8 - 3\pi + 6 \approx 0.4510$ **47–51.** Proofs
53. $\frac{1}{\sqrt{5}}\ln\left|\frac{2\tan(\theta/2) - 3 - \sqrt{5}}{2\tan(\theta/2) - 3 + \sqrt{5}}\right| + C$ **55.** $\ln 2$
57. $\frac{1}{2}\ln(3 - 2\cos\theta) + C$ **59.** $-2\cos\sqrt{\theta} + C$ **61.** $4\sqrt{3}$
63. (a) $\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$
$\int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$
$\int x^3\ln x\,dx = \frac{1}{4}x^4\ln x - \frac{1}{16}x^4 + C$
65. (a) Arctangent Formula, Formula 23,
$\int \frac{1}{u^2 + 1}\,du,\ u = e^x$
(b) Log Rule: $\int \frac{1}{u}\,du,\ u = e^x + 1$
(c) Substitution: $u = x^2$, $du = 2x\,dx$
Then Formula 81.
(d) Integration by parts (e) Cannot be integrated
(f) Formula 16 with $u = e^{2x}$
67. False. Substitutions may first have to be made to rewrite the integral in a form that appears in the table.
69. 1919.145 ft-lb **71.** $32\pi^2$ **73.** About 401.4

Section 8.7 *(page 564)*

1.

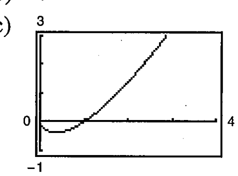
x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	1.3177	1.3332	1.3333	1.3333	1.3332	1.3177

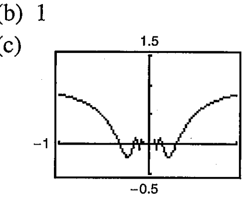
$\frac{4}{3}$

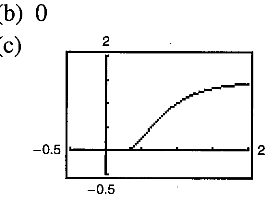
3.

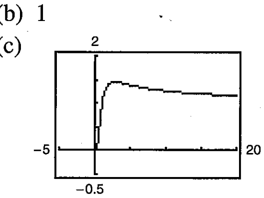
x	1	10	10^2	10^3	10^4	10^5
$f(x)$	0.9900	90,483.7	3.7×10^9	4.5×10^{10}	0	0

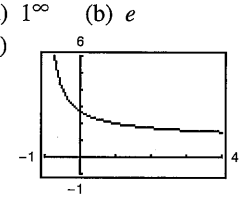
0

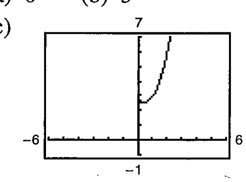
5. $\frac{3}{8}$ **7.** $\frac{1}{8}$ **9.** $\frac{5}{3}$ **11.** 4 **13.** 0 **15.** ∞ **17.** $\frac{11}{4}$
19. $\frac{3}{5}$ **21.** 1 **23.** $\frac{5}{4}$ **25.** ∞ **27.** 0 **29.** 1
31. 0 **33.** 0 **35.** ∞ **37.** $\frac{5}{9}$ **39.** 1 **41.** ∞
43. (a) Not indeterminate
(b) ∞
(c)


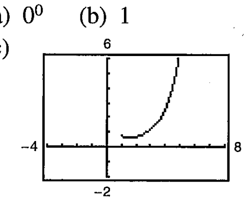
45. (a) $0 \cdot \infty$
(b) 1
(c)


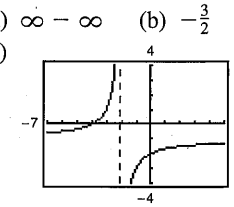
47. (a) Not indeterminate
(b) 0
(c)


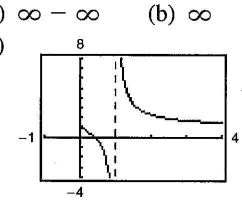
49. (a) ∞^0
(b) 1
(c)


51. (a) 1^∞ (b) e
(c)


53. (a) 0^0 (b) 3
(c)


55. (a) 0^0 (b) 1
(c)


57. (a) $\infty - \infty$ (b) $-\frac{3}{2}$
(c)


59. (a) $\infty - \infty$ (b) ∞
(c)


61. $\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 1^\infty, 0^0, \infty - \infty$

63. Answers will vary. Examples:
(a) $f(x) = x^2 - 25, g(x) = x - 5$
(b) $f(x) = (x - 5)^2, g(x) = x^2 - 25$
(c) $f(x) = x^2 - 25, g(x) = (x - 5)^3$

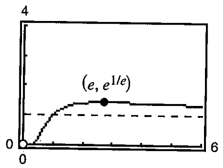
65. (a) Yes: $\frac{0}{0}$ (b) No: $\frac{0}{-1}$ (c) Yes: $\frac{\infty}{\infty}$ (d) Yes: $\frac{0}{0}$
(e) No: $\frac{-1}{0}$ (f) Yes: $\frac{0}{0}$

67.

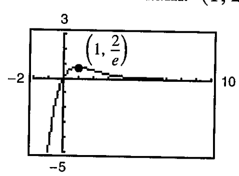
x	10	10^2	10^4	10^6	10^8	10^{10}
$\frac{(\ln x)^4}{x}$	2.811	4.498	0.720	0.036	0.001	0.000

69. 0 **71.** 0 **73.** 0

75. Horizontal asymptote: $y = 1$
Relative maximum: $(e, e^{1/e})$



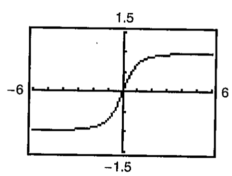
77. Horizontal asymptote: $y = 0$
Relative maximum: $(1, 2/e)$



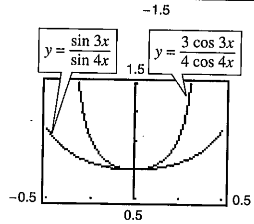
79. Limit is not of the form $0/0$ or ∞/∞.

81. Limit is not of the form $0/0$ or ∞/∞.

83. (a) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$
Applying L'Hôpital's Rule twice results in the original limit, so L'Hôpital's Rule fails.
(b) 1
(c)



85.



As $x \to 0$, the graphs get closer together (they both approach 0.75). By L'Hôpital's Rule,
$\lim_{x\to 0} \frac{\sin 3x}{\sin 4x} = \lim_{x\to 0} \frac{3 \cos 3x}{4 \cos 4x} = \frac{3}{4}.$

87. $v = 32t + v_0$ **89.** Proof **91.** $c = \frac{2}{3}$ **93.** $c = \pi/4$

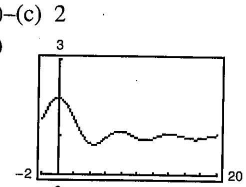
95. False. L'Hôpital's Rule does not apply because $\lim_{x\to 0}(x^2 + x + 1) \neq 0$.

97. True **99.** $\frac{3}{4}$ **101.** $\frac{4}{3}$ **103.** $a = 1, b = \pm 2$

105. Proof **107.** (a) $0 \cdot \infty$ (b) 0 **109.** Proof

111. (a)–(c) 2

113. (a)



(b) $\lim_{x\to\infty} h(x) = 1$
(c) No

115. Putnam Problem A1, 1956

Section 8.8 *(page 575)*

1. Improper; $0 \le \frac{3}{5} \le 1$

3. Not improper; continuous on $[0, 1]$

5. Not improper; continuous on $[0, 2]$

7. Improper; infinite limits of integration

9. Infinite discontinuity at $x = 0$; 4

11. Infinite discontinuity at $x = 1$; diverges

13. Infinite discontinuity at $x = 0$; diverges

15. Infinite limit of integration; converges to 1 **17.** $\frac{1}{2}$

19. Diverges **21.** Diverges **23.** 2 **25.** $1/[2(\ln 4)^2]$

27. π **29.** $\pi/4$ **31.** Diverges **33.** Diverges

35. 0 **37.** $-\frac{1}{4}$ **39.** Diverges **41.** $\pi/3$ **43.** $\ln 3$

45. $\pi/6$ **47.** $2\pi\sqrt{6}/3$ **49.** $p > 1$ **51.** Proof

53. Diverges **55.** Converges **57.** Converges

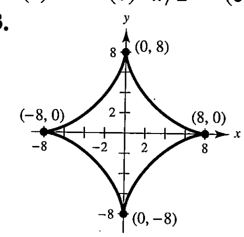
59. Diverges **61.** Converges

63. An integral with infinite integration limits, an integral with an infinite discontinuity at or between the integration limits

65. The improper integral diverges. **67.** e **69.** π

71. (a) 1 (b) $\pi/2$ (c) 2π

73.



Perimeter = 48

75. $8\pi^2$ **77.** (a) $W = 20{,}000$ mile-tons (b) 4000 mi

79. (a) Proof (b) $P = 43.53\%$ (c) $E(x) = 7$

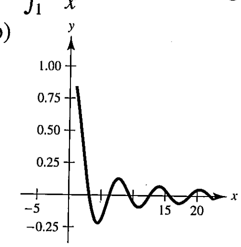
81. (a) \$757,992.41 (b) \$837,995.15 (c) \$1,066,666.67

83. $P = \left[2\pi NI\left(\sqrt{r^2 + c^2} - c\right)\right] / \left(kr\sqrt{r^2 + c^2}\right)$

85. False. Let $f(x) = 1/(x + 1)$. **87.** True

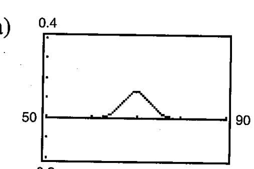
89. (a) and (b) Proofs
(c) The definition of the improper integral $\int_{-\infty}^{\infty}$ is not $\lim_{a\to\infty} \int_{-a}^{a}$ but rather that if you rewrite the integral that diverges, you can find that the integral converges.

91. (a) $\int_1^\infty \frac{1}{x^n}\, dx$ will converge if $n > 1$ and diverge if $n \le 1$.
(b)



(c) Converges

93. (a)



(b) About 0.2525
(c) 0.2525; same by symmetry

95. $1/s, s > 0$ **97.** $2/s^3, s > 0$ **99.** $s/(s^2 + a^2), s > 0$

101. $s/(s^2 - a^2), s > |a|$

103. (a) $\Gamma(1) = 1, \Gamma(2) = 1, \Gamma(3) = 2$ (b) Proof
(c) $\Gamma(n) = (n - 1)!$

105. $c = 1$; $\ln(2)$

107. $8\pi[(\ln 2)^2/3 - (\ln 4)/9 + 2/27] \approx 2.01545$

109. $\int_0^1 2\sin(u^2)\,du$; 0.6278 **111.** Proof

Review Exercises for Chapter 8 *(page 579)*

1. $\frac{1}{3}(x^2 - 36)^{3/2} + C$ **3.** $\frac{1}{2}\ln|x^2 - 49| + C$

5. $\ln(2) + \frac{1}{2} \approx 1.1931$ **7.** $100\arcsin(x/10) + C$

9. $\frac{1}{9}e^{3x}(3x - 1) + C$ **11.** $\frac{1}{13}e^{2x}(2\sin 3x - 3\cos 3x) + C$

13. $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$

15. $\frac{1}{16}\left[(8x^2 - 1)\arcsin 2x + 2x\sqrt{1 - 4x^2}\right] + C$

17. $\sin(\pi x - 1)[\cos^2(\pi x - 1) + 2]/(3\pi) + C$

19. $\frac{2}{3}[\tan^3(x/2) + 3\tan(x/2)] + C$ **21.** $\tan\theta + \sec\theta + C$

23. $3\pi/16 + \frac{1}{2} \approx 1.0890$ **25.** $3\sqrt{4 - x^2}/x + C$

27. $\frac{1}{3}(x^2 + 4)^{1/2}(x^2 - 8) + C$ **29.** $256 - 62\sqrt{17} \approx 0.3675$

31. (a), (b), and (c) $\frac{1}{3}\sqrt{4 + x^2}(x^2 - 8) + C$

33. $6\ln|x + 3| - 5\ln|x - 4| + C$

35. $\frac{1}{4}[6\ln|x - 1| - \ln(x^2 + 1) + 6\arctan x] + C$

37. $x - \frac{64}{11}\ln|x + 8| + \frac{9}{11}\ln|x - 3| + C$

39. $\frac{1}{25}[4/(4 + 5x) + \ln|4 + 5x|] + C$ **41.** $1 - \sqrt{2}/2$

43. $\frac{1}{2}\ln|x^2 + 4x + 8| - \arctan[(x + 2)/2] + C$

45. $\ln|\tan\pi x|/\pi + C$ **47.** Proof

49. $\frac{1}{8}(\sin 2\theta - 2\theta\cos 2\theta) + C$

51. $\frac{4}{3}[x^{3/4} - 3x^{1/4} + 3\arctan(x^{1/4})] + C$

53. $2\sqrt{1 - \cos x} + C$ **55.** $\sin x\ln(\sin x) - \sin x + C$

57. $\frac{5}{2}\ln|(x - 5)/(x + 5)| + C$

59. $y = x\ln|x^2 + x| - 2x + \ln|x + 1| + C$ **61.** $\frac{1}{5}$

63. $\frac{1}{2}(\ln 4)^2 \approx 0.961$ **65.** π **67.** $\frac{128}{15}$

69. $(\bar{x}, \bar{y}) = (0, 4/(3\pi))$ **71.** 3.82 **73.** 0 **75.** ∞

77. 1 **79.** $1000e^{0.09} \approx 1094.17$ **81.** Converges; $\frac{32}{3}$

83. Diverges **85.** Converges; 1 **87.** Converges; $\pi/4$

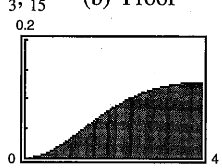
89. (a) \$6,321,205.59 (b) \$10,000,000

91. (a) 0.4581 (b) 0.0135

P.S. Problem Solving *(page 581)*

1. (a) $\frac{4}{3}, \frac{16}{15}$ (b) Proof **3.** $\ln 3$ **5.** Proof

7. (a)



(b) $\ln 3 - \frac{4}{5}$ (c) $\ln 3 - \frac{4}{5}$

Area ≈ 0.2986

9. $\ln 3 - \frac{1}{2} \approx 0.5986$

11. (a) ∞ (b) 0 (c) $-\frac{2}{3}$

The form $0 \cdot \infty$ is indeterminant.

13. About 0.8670 **15.** $\dfrac{1/12}{x} + \dfrac{1/42}{x - 3} + \dfrac{1/10}{x - 1} + \dfrac{111/140}{x + 4}$

17–19. Proofs **21.** About 0.0158

Chapter 9

Section 9.1 *(page 592)*

1. 3, 9, 27, 81, 243 **3.** 1, 0, −1, 0, 1 **5.** $2, -1, \frac{2}{3}, -\frac{1}{2}, \frac{2}{5}$

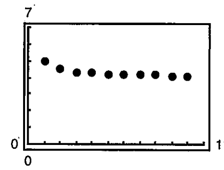
7. 3, 4, 6, 10, 18 **9.** c **10.** a **11.** d **12.** b

13. 14, 17; add 3 to preceding term.

15. 80, 160; multiply preceding term by 2. **17.** $n + 1$

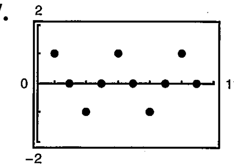
19. $1/[(2n + 1)(2n)]$ **21.** 5 **23.** 2

25.



Converges to 4

27.



Diverges

29. Converges to 0 **31.** Diverges **33.** Converges to 5

35. Converges to 0 **37.** Diverges **39.** Converges to 0

41. Converges to 1 **43.** Converges to 0

45. Answers will vary. Sample answer: $6n - 4$

47. Answers will vary. Sample answer: $n^2 - 3$

49. Answers will vary. Sample answer: $(n + 1)/(n + 2)$

51. Answers will vary. Sample answer: $(n + 1)/n$

53. Monotonic, bounded **55.** Not monotonic, bounded

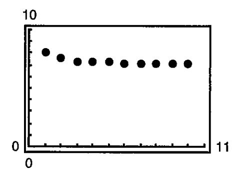
57. Monotonic, bounded **59.** Not monotonic, bounded

61. (a) $\left|7 + \frac{1}{n}\right| \geq 7 \Rightarrow$ bounded

$a_n > a_{n+1} \Rightarrow$ monotonic

So, $\{a_n\}$ converges.

(b)



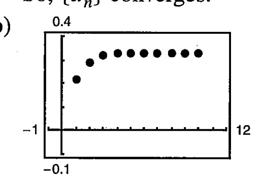
Limit = 7

63. (a) $\left|\frac{1}{3}\left(1 - \frac{1}{3^n}\right)\right| < \frac{1}{3} \Rightarrow$ bounded

$a_n < a_{n+1} \Rightarrow$ monotonic

So, $\{a_n\}$ converges.

(b)



Limit = $\frac{1}{3}$

65. $\{a_n\}$ has a limit because it is bounded and monotonic; because $2 \leq a_n \leq 4$, $2 \leq L \leq 4$.

67. (a) No. $\lim_{n\to\infty} A_n$ does not exist.

(b)

n	1	2	3	4
A_n	\$10,045.83	\$10,091.88	\$10,138.13	\$10,184.60

n	5	6	7
A_n	\$10,231.28	\$10,278.17	\$10,325.28

n	8	9	10
A_n	\$10,372.60	\$10,420.14	\$10,467.90

69. No. A sequence is said to converge when its terms approach a real number.

71. (a) $10 - \frac{1}{n}$

(b) Impossible. The sequence converges by Theorem 9.5.

(c) $a_n = \dfrac{3n}{4n + 1}$

(d) Impossible. An unbounded sequence diverges.