BC Calculus Chapter 8 Review Problems

First Step through Integral Checklist order: 1) Expand/Power Rule 2) U-Substitution 3) ArcTrig/Complete the square 4) Partial Fraction 5)Long Division 6) IBP/Tab

8.1

1. $\int \frac{3x+2}{x^2+9} dx$

8.2

1. $\int x^3 cos3xdx$

8.3

3. $\int \frac{sin^5 t}{\sqrt{cost}} dt$

4. $\int \sqrt{tanx}\, sec^4 xdx$

8.4

5. $\int \frac{1}{\sqrt{x^2-9}} dx$

6. $\int \frac{-5}{(x^2+5)^{3/2}} dx$

7. $\int \frac{1}{\sqrt{25-x^2}}\,dx$

8.5 8. $\int \frac{1}{4x^2-9}\,dx$

9. $\int \frac{4x^2+2x-1}{x^3+x^2}\,dx$

10. $\int \frac{x^2-x+9}{(x^2+9)^2}\,dx$

8.7 Step through Limits checklist: If real number, 1) plug in 2) if indeterminate (0/0) further evaluate

If limit approaching ∞ , compare degrees between numerator, denominator (finding H.A.) . If 0/0 or ∞/∞, then apply L'Hopital's Rule

11. $\lim_{x\to\infty} \frac{x^3}{e^{x/2}}$

12. $\lim_{x\to 0^+} (1+x)^{1/x}$

8.8

13. $\int_0^2 \frac{2}{(x-1)^{2/3}}dx$

14. $\int_1^{\infty} x^2 \ln x dx$

BC Calculus Chapter 8 Review Problems

Key

First Step through Integral Checklist order: 1) Expand/Power Rule 2) U-Substitution 3) ArcTrig/Complete the square 4) Partial Fraction 5)Long Division 6) IBP/Tab (LIPET)

8.1

1. $\int \frac{3x+2}{x^2+9}dx$

(u-sub) (arctan)

$$\int \frac{3x}{x^2+9}dx + \int \frac{2}{x^2+9}dx$$

$u = x^2+9$, $\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$

$$\int \frac{3x}{u}\cdot\frac{du}{2x} = \frac{3}{2}\int \frac{1}{u}du$$

$u = x$, $a = 3$

$\frac{1}{a}\arctan\left(\frac{u}{a}\right)$

$\frac{2}{3}\arctan\left(\frac{x}{3}\right)$

$$\boxed{\frac{3}{2}\ln|x^2+9| + \frac{2}{3}\arctan(x/3) + C}$$

8.2

1. $\int x^3\cos 3x\,dx$

Tab method

u	dv
$+x^3$	$\cos 3x$
$-3x^2$	$\frac{1}{3}\sin 3x$
$+6x$	$-\frac{1}{9}\cos 3x$
-6	$-\frac{1}{27}\sin 3x$
$+0$	$+\frac{1}{81}\cos 3x$

$$\boxed{\frac{1}{3}x^3\sin 3x + \frac{1}{3}x^2\cos 3x - \frac{6}{27}x\sin 3x - \frac{6}{81}\cos 3x + C}$$

8.3

3. $\int \frac{\sin^5 t}{\sqrt{\cos t}}dt$

$$\int \sin t\,(\sin^2 t)^2\cdot(\cos t)^{-1/2}dt$$

$$\int \sin t\,(1-\cos^2 t)^2(\cos t)^{-1/2}dt$$

$$\int \sin t\,(1-2\cos^2 t+\cos^4 t)(\cos t)^{-1/2}dt$$

$u = \cos t$, $du = -\sin t$

$$\int \sin t\left[(\cos t)^{-1/2} - 2(\cos t)^{3/2} + (\cos t)^{7/2}\right]dt$$

$$-\left(\frac{(\cos t)^{1/2}}{1/2} - 2\frac{(\cos t)^{5/2}}{5/2} + \frac{(\cos t)^{9/2}}{9/2}\right) + C$$

$$\boxed{-2(\cos t)^{1/2} + \frac{4}{5}(\cos t)^{5/2} - \frac{2}{9}(\cos t)^{9/2} + C}$$

4. $\int \sqrt{\tan x}\,\sec^4 x\,dx$

$$\int (\tan x)^{1/2}\sec^2 x\cdot\sec^2 x\,dx$$

$$\int (\tan x)^{1/2}(1+\tan^2 x)\cdot\sec^2 x\,dx$$

$$\int (\tan x)^{1/2} + (\tan x)^{5/2}\cdot\sec^2 x\,dx$$

$u = \tan x$, $\frac{du}{dx} = \sec^2 x$, $dx = \frac{du}{\sec^2 x}$

$$\int u^{1/2} + u^{5/2}\cdot\sec^2 x\cdot\frac{du}{\sec^2 x}$$

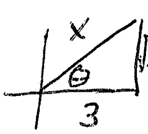
$$\frac{u^{3/2}}{3/2} + \frac{u^{7/2}}{7/2} + C$$

$$\boxed{\frac{2}{3}(\tan x)^{3/2} + \frac{2}{7}(\tan x)^{7/2} + C}$$

8.4

5. $\int \frac{1}{\sqrt{x^2-9}}dx$

$*\sqrt{u^2-a^2}$ use $\sec\theta = \frac{u}{a}$



$\sec\theta = \frac{x}{3}$ $\tan\theta = \frac{\sqrt{x^2-9}}{3}$

$\sec\theta\tan\theta\,d\theta = \frac{1}{3}dx$

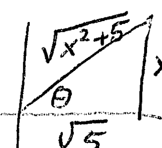
$3\sec\theta\tan\theta\,d\theta = dx$

$$\int \frac{1}{3\tan\theta}\cdot 3\sec\theta\tan\theta\,d\theta$$

$$\int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| = \boxed{\ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + C}$$

6. $\int \frac{-5}{(x^2+5)^{3/2}}dx$

$\sqrt{a^2+u^2}$, $\tan\theta = \frac{u}{a}$



$\tan\theta = \frac{x}{\sqrt{5}}$

$\sqrt{5}\sec^2\theta\,d\theta = dx$

$\sec\theta = \frac{\sqrt{x^2+5}}{\sqrt{5}}$ $\sqrt{5}\sec\theta = \sqrt{x^2+5}$

$$\int \frac{-5}{(\sqrt{5}\sec\theta)^3}\cdot\sqrt{5}\sec^2\theta\,d\theta$$

$$-\int \frac{1}{\sec\theta}d\theta = -\int \cos\theta\,d\theta$$

$$-\sin\theta + C = \boxed{\frac{-x}{\sqrt{x^2+5}} + C}$$

7. $\int \frac{1}{\sqrt{25-x^2}}dx$ $\quad * \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$

$a=5 \quad u=x$

$\boxed{\arcsin\left(\frac{x}{5}\right)+C}$

8.5 8. $\int \frac{1}{4x^2-9}dx = \int \frac{1}{(2x+3)(2x-3)}dx$

$A=-1/6 \quad B=1/6$

$\int \frac{1}{(2x+3)(2x-3)}dx = \frac{A}{2x+3} + \frac{B}{2x-3} = \int \frac{-1/6}{2x+3} + \frac{1/6}{2x-3}$

$x=-3/2 \quad x=3/2$

$u=2x+3$

$\frac{du}{dx}=2$

$dx=\frac{du}{2}$

$-\frac{1}{12}\ln|2x+3| + \frac{1}{12}\ln|2x-3| + C$

$\frac{1}{12}[\ln|2x-3| - \ln|2x+3|] = \boxed{\frac{1}{12}\ln\left|\frac{2x-3}{2x+3}\right|+C}$

9. $\int \frac{4x^2+2x-1}{x^3+x^2}dx = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$x^2(x+1)$

$4x^2+2x-1 = Ax(x+1) + B(x+1) + Cx^2$

$4x^2+2x-1 = Ax^2+Ax+Bx+B+Cx^2$

$\boxed{x=0}$, $-1=B$

$x=-1$, $4-3=C$, $C=1$

$x=1$, $5=2A+(-2)+1$, $A=3$

$\int \frac{3}{x} + \frac{-1}{x^2} + \frac{1}{x+1}dx = \boxed{3\ln|x| + \frac{1}{x} + \ln|x+1| + C}$

10. $\int \frac{x^2-x+9}{(x^2+9)^2}dx = \frac{Ax+B}{x^2+9} + \frac{Cx+D}{(x^2+9)^2}$

$x^2-x+9 = (Ax+B)(x^2+9) + Cx+D$

$= Ax^3+Bx^2+9Ax+Cx+9B+D$

$x^2-x+9 = Ax^3+Bx^2+x(9A+C)+9B+D$

$A=0$

$B=1$

$9A+C=-1 \quad 9B+D=9$

$9(0)+C=-1 \quad 9+D=9$

$C=-1 \quad D=0$

$\int \frac{1}{x^2+9} + \frac{-x}{(x^2+9)^2}$

(arctan) (u-sub)

$\boxed{\frac{1}{3}\arctan\left(\frac{x}{3}\right) + \frac{1}{2(x^2+9)} + C}$

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11. $\lim_{x\to\infty} \frac{x^3}{e^{x/2}} = \frac{\infty}{\infty}$ L'Hopital's

$\lim_{x\to\infty} \frac{3x^2}{\frac{1}{2}e^{x/2}} \rightarrow \frac{6x}{\frac{1}{4}e^{x/2}} \rightarrow \frac{6}{\frac{1}{8}e^{x/2}} = \boxed{0}$

12. $\lim_{x\to 0^+}(1+x)^{1/x}$

$y = \lim_{x\to 0^+}(1+x)^{1/x}$

$\ln y = \frac{1}{x}\ln(1+x)$

$\lim_{x\to 0^+} \frac{\ln(1+x)}{x} \quad \frac{\infty}{\infty} \rightarrow \frac{\frac{1}{1+x}}{1} \quad \lim_{x\to 0^+}\frac{1}{1+x}=1$

$e^{\ln y} = e^1 \quad \boxed{y=e}$

8.8

13. $\int_0^2 \frac{2}{(x-1)^{2/3}}dx$ $\quad \int_0^1 \frac{2}{(x-1)^{2/3}} + \int_1^2 \frac{2}{(x-1)^{2/3}}dx$

$\lim_{b\to 1^-}\int_0^b \frac{2}{(x-1)^{2/3}}dx + \lim_{b\to 1^+} \frac{2}{(x-1)^{2/3}}dx$

$\int 2u^{-2/3} = \frac{2u^{1/3}}{1/3}$

$\int 2u^{-2/3} = \frac{2u^{1/3}}{1/3}$

$6(x-1)\Big]_b^2$

$\lim_{b\to 1^-} 6(x-1)^{1/3}\Big]_0^b = 0-(-6) + 6-0$

$6+6 = \boxed{12}$

14. $\int_1^\infty x^2 \ln x\, dx$ $\quad uv - \int v\,du$

$u=\ln x \quad dv=x^2$

$\frac{du}{dx}=\frac{1}{x} \quad v=\frac{x^3}{3}$

$du=\frac{1}{x}dx$

$\frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)dx$

$\frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx$

$\lim_{b\to\infty} \frac{x^3}{3}\ln x - \frac{x^3}{9}\Big]_1^b$

$\frac{b^3}{3}\ln b - \frac{b^3}{9} - \left(0-\frac{1}{9}\right) \rightarrow \infty$ $\boxed{\text{diverges}}$