Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding or Evaluating an Integral In Exercises 1–8, use the basic integration rules to find or evaluate the integral.

1. $\int x\sqrt{x^2 - 36}\, dx$

2. $\int xe^{x^2-1}\, dx$

3. $\int \frac{x}{x^2 - 49}\, dx$

4. $\int \frac{x}{\sqrt[3]{4 - x^2}}\, dx$

5. $\int_1^e \frac{\ln(2x)}{x}\, dx$

6. $\int_{3/2}^{2} 2x\sqrt{2x - 3}\, dx$

7. $\int \frac{100}{\sqrt{100 - x^2}}\, dx$

8. $\int \frac{2x}{x - 3}\, dx$

Using Integration by Parts In Exercises 9–16, use integration by parts to find the indefinite integral.

9. $\int xe^{3x}\, dx$

10. $\int x^3 e^x\, dx$

11. $\int e^{2x} \sin 3x\, dx$

12. $\int x\sqrt{x - 1}\, dx$

13. $\int x^2 \sin 2x\, dx$

14. $\int \ln\sqrt{x^2 - 4}\, dx$

15. $\int x \arcsin 2x\, dx$

16. $\int \arctan 2x\, dx$

Finding a Trigonometric Integral In Exercises 17–22, find the trigonometric integral.

17. $\int \cos^3(\pi x - 1)\, dx$

18. $\int \sin^2 \frac{\pi x}{2}\, dx$

19. $\int \sec^4 \frac{x}{2}\, dx$

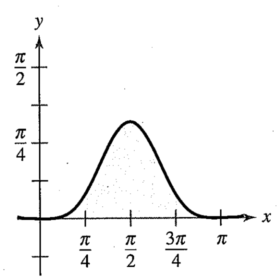
20. $\int \tan\theta \sec^4 \theta\, d\theta$

21. $\int \frac{1}{1 - \sin\theta}\, d\theta$

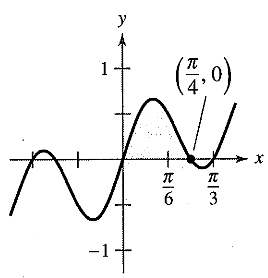
22. $\int \cos 2\theta(\sin\theta + \cos\theta)^2\, d\theta$

Area In Exercises 23 and 24, find the area of the region.

23. $y = \sin^4 x$



24. $y = \sin 3x \cos 2x$



Using Trigonometric Substitution In Exercises 25–30, use trigonometric substitution to find or evaluate the integral.

25. $\int \frac{-12}{x^2\sqrt{4 - x^2}}\, dx$

26. $\int \frac{\sqrt{x^2 - 9}}{x}\, dx, \quad x > 3$

27. $\int \frac{x^3}{\sqrt{4 + x^2}}\, dx$

28. $\int \sqrt{25 - 9x^2}\, dx$

29. $\int_0^1 \frac{6x^3}{\sqrt{16 + x^2}}\, dx$

30. $\int_3^4 x^3\sqrt{x^2 - 9}\, dx$

Using Different Methods In Exercises 31 and 32, find the indefinite integral using each method.

31. $\int \frac{x^3}{\sqrt{4 + x^2}}\, dx$

(a) Trigonometric substitution

(b) Substitution: $u^2 = 4 + x^2$

(c) Integration by parts: $dv = \frac{x}{\sqrt{4 + x^2}}\, dx$

32. $\int x\sqrt{4 + x}\, dx$

(a) Trigonometric substitution

(b) Substitution: $u^2 = 4 + x$

(c) Substitution: $u = 4 + x$

(d) Integration by parts: $dv = \sqrt{4 + x}\, dx$

Using Partial Fractions In Exercises 33–38, use partial fractions to find the indefinite integral.

33. $\int \frac{x - 39}{x^2 - x - 12}\, dx$

34. $\int \frac{5x - 2}{x^2 - x}\, dx$

35. $\int \frac{x^2 + 2x}{x^3 - x^2 + x - 1}\, dx$

36. $\int \frac{4x - 2}{3(x - 1)^2}\, dx$

37. $\int \frac{x^2}{x^2 + 5x - 24}\, dx$

38. $\int \frac{\sec^2\theta}{\tan\theta(\tan\theta - 1)}\, d\theta$

Integration by Tables In Exercises 39–46, use integration tables to find or evaluate the integral.

39. $\int \frac{x}{(4 + 5x)^2}\, dx$

40. $\int \frac{x}{\sqrt{4 + 5x}}\, dx$

41. $\int_0^{\sqrt{\pi}/2} \frac{x}{1 + \sin x^2}\, dx$

42. $\int_0^1 \frac{x}{1 + e^{x^2}}\, dx$

43. $\int \frac{x}{x^2 + 4x + 8}\, dx$

44. $\int \frac{3}{2x\sqrt{9x^2 - 1}}\, dx, \quad x > \frac{1}{3}$

45. $\int \frac{1}{\sin \pi x \cos \pi x}\, dx$

46. $\int \frac{1}{1 + \tan \pi x}\, dx$

47. **Verifying a Formula** Verify the reduction formula

$$\int (\ln x)^n\, dx = x(\ln x)^n - n\int (\ln x)^{n-1}\, dx.$$

48. **Verifying a Formula** Verify the reduction formula

$$\int \tan^n x\, dx = \frac{1}{n - 1}\tan^{n-1} x - \int \tan^{n-2} x\, dx.$$

Finding an Indefinite Integral **In Exercises 49–56, find the indefinite integral using any method.**

49. $\int \theta \sin \theta \cos \theta \, d\theta$

50. $\int \frac{\csc\sqrt{2x}}{\sqrt{x}}\, dx$

51. $\int \frac{x^{1/4}}{1 + x^{1/2}}\, dx$

52. $\int \sqrt{1 + \sqrt{x}}\, dx$

53. $\int \sqrt{1 + \cos x}\, dx$

54. $\int \frac{3x^3 + 4x}{(x^2 + 1)^2}\, dx$

55. $\int \cos x \ln(\sin x)\, dx$

56. $\int (\sin \theta + \cos \theta)^2\, d\theta$

Differential Equation **In Exercises 57–60, solve the differential equation using any method.**

57. $\frac{dy}{dx} = \frac{25}{x^2 - 25}$

58. $\frac{dy}{dx} = \frac{\sqrt{4 - x^2}}{2x}$

59. $y' = \ln(x^2 + x)$

60. $y' = \sqrt{1 - \cos \theta}$

Evaluating a Definite Integral **In Exercises 61–66, evaluate the definite integral using any method. Use a graphing utility to verify your result.**

61. $\int_2^{\sqrt{5}} x(x^2 - 4)^{3/2}\, dx$

62. $\int_0^1 \frac{x}{(x - 2)(x - 4)}\, dx$

63. $\int_1^4 \frac{\ln x}{x}\, dx$

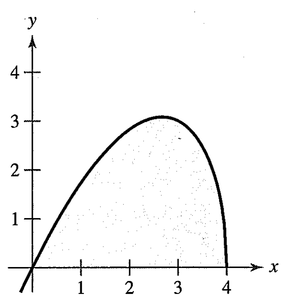
64. $\int_0^2 xe^{3x}\, dx$

65. $\int_0^{\pi} x \sin x\, dx$

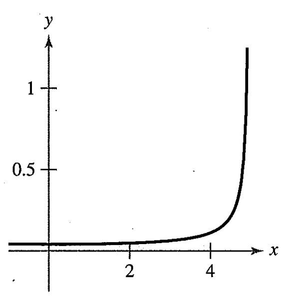
66. $\int_0^5 \frac{x}{\sqrt{4 + x}}\, dx$

Area **In Exercises 67 and 68, find the area of the region.**

67. $y = x\sqrt{4 - x}$



68. $y = \frac{1}{25 - x^2}$



Centroid **In Exercises 69 and 70, find the centroid of the region bounded by the graphs of the equations.**

69. $y = \sqrt{1 - x^2}, \quad y = 0$

70. $(x - 1)^2 + y^2 = 1, \quad (x - 4)^2 + y^2 = 4$

Arc Length **In Exercises 71 and 72, approximate to two decimal places the arc length of the curve over the given interval.**

Function	Interval
71. $y = \sin x$	$[0, \pi]$
72. $y = \sin^2 x$	$[0, \pi]$

Evaluating a Limit **In Exercises 73–80, use L'Hôpital's Rule to evaluate the limit.**

73. $\lim_{x \to 1} \frac{(\ln x)^2}{x - 1}$

74. $\lim_{x \to 0} \frac{\sin \pi x}{\sin 5\pi x}$

75. $\lim_{x \to \infty} \frac{e^{2x}}{x^2}$

76. $\lim_{x \to \infty} xe^{-x^2}$

77. $\lim_{x \to \infty} (\ln x)^{2/x}$

78. $\lim_{x \to 1^+} (x - 1)^{\ln x}$

79. $\lim_{n \to \infty} 1000\left(1 + \frac{0.09}{n}\right)^n$

80. $\lim_{x \to 1^+} \left(\frac{2}{\ln x} - \frac{2}{x - 1}\right)$

Evaluating an Improper Integral **In Exercises 81–88, determine whether the improper integral diverges or converges. Evaluate the integral if it converges.**

81. $\int_0^{16} \frac{1}{\sqrt[4]{x}}\, dx$

82. $\int_0^2 \frac{7}{x - 2}\, dx$

83. $\int_1^{\infty} x^2 \ln x\, dx$

84. $\int_0^{\infty} \frac{e^{-1/x}}{x^2}\, dx$

85. $\int_1^{\infty} \frac{\ln x}{x^2}\, dx$

86. $\int_1^{\infty} \frac{1}{\sqrt[4]{x}}\, dx$

87. $\int_2^{\infty} \frac{1}{x\sqrt{x^2 - 4}}\, dx$

88. $\int_0^{\infty} \frac{2}{\sqrt{x}(x + 4)}\, dx$

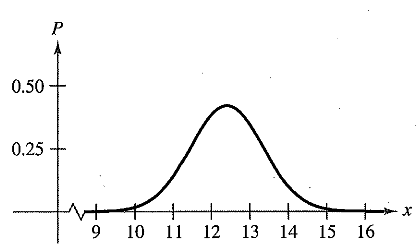
89. **Present Value** The board of directors of a corporation is calculating the price to pay for a business that is forecast to yield a continuous flow of profit of \$500,000 per year. The money will earn a nominal rate of 5% per year compounded continuously. What is the present value of the business
(a) for 20 years?
(b) forever (in perpetuity)?
(*Note:* The present value for t_0 years is $\int_0^{t_0} 500{,}000e^{-0.05t}\, dt$.)

90. **Volume** Find the volume of the solid generated by revolving the region bounded by the graphs of $y = xe^{-x}$, $y = 0$, and $x = 0$ about the x-axis.

91. **Probability** The average lengths (from beak to tail) of different species of warblers in the eastern United States are approximately normally distributed with a mean of 12.9 centimeters and a standard deviation of 0.95 centimeter (see figure). The probability that a randomly selected warbler has a length between a and b centimeters is

$$P(a \le x \le b) = \frac{1}{0.95\sqrt{2\pi}} \int_a^b e^{-(x-12.9)^2/1.805}\, dx.$$

Use a graphing utility to approximate the probability that a randomly selected warbler has a length of (a) 13 centimeters or greater and (b) 15 centimeters or greater. (*Source: Peterson's Field Guide: Eastern Birds*)



105. $c = 1$; $\ln(2)$

107. $8\pi[(\ln 2)^2/3 - (\ln 4)/9 + 2/27] \approx 2.01545$

109. $\int_0^1 2\sin(u^2)\,du$; 0.6278 **111.** Proof

Review Exercises for Chapter 8 *(page 579)*

1. $\frac{1}{3}(x^2 - 36)^{3/2} + C$ **3.** $\frac{1}{2}\ln|x^2 - 49| + C$

5. $\ln(2) + \frac{1}{2} \approx 1.1931$ **7.** $100\arcsin(x/10) + C$

9. $\frac{1}{9}e^{3x}(3x - 1) + C$ **11.** $\frac{1}{13}e^{2x}(2\sin 3x - 3\cos 3x) + C$

13. $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$

15. $\frac{1}{16}\left[(8x^2 - 1)\arcsin 2x + 2x\sqrt{1 - 4x^2}\right] + C$

17. $\sin(\pi x - 1)[\cos^2(\pi x - 1) + 2]/(3\pi) + C$

19. $\frac{2}{3}[\tan^3(x/2) + 3\tan(x/2)] + C$ **21.** $\tan\theta + \sec\theta + C$

23. $3\pi/16 + \frac{1}{2} \approx 1.0890$ **25.** $3\sqrt{4 - x^2}/x + C$

27. $\frac{1}{3}(x^2 + 4)^{1/2}(x^2 - 8) + C$ **29.** $256 - 62\sqrt{17} \approx 0.3675$

31. (a), (b), and (c) $\frac{1}{3}\sqrt{4 + x^2}(x^2 - 8) + C$

33. $6\ln|x + 3| - 5\ln|x - 4| + C$

35. $\frac{1}{4}[6\ln|x - 1| - \ln(x^2 + 1) + 6\arctan x] + C$

37. $x - \frac{64}{11}\ln|x + 8| + \frac{9}{11}\ln|x - 3| + C$

39. $\frac{1}{25}[4/(4 + 5x) + \ln|4 + 5x|] + C$ **41.** $1 - \sqrt{2}/2$

43. $\frac{1}{2}\ln|x^2 + 4x + 8| - \arctan[(x + 2)/2] + C$

45. $\ln|\tan\pi x|/\pi + C$ **47.** Proof

49. $\frac{1}{8}(\sin 2\theta - 2\theta\cos 2\theta) + C$

51. $\frac{4}{3}[x^{3/4} - 3x^{1/4} + 3\arctan(x^{1/4})] + C$

53. $2\sqrt{1 - \cos x} + C$ **55.** $\sin x\ln(\sin x) - \sin x + C$

57. $\frac{5}{2}\ln|(x - 5)/(x + 5)| + C$

59. $y = x\ln|x^2 + x| - 2x + \ln|x + 1| + C$ **61.** $\frac{1}{5}$

63. $\frac{1}{2}(\ln 4)^2 \approx 0.961$ **65.** π **67.** $\frac{128}{15}$

69. $(\bar{x}, \bar{y}) = (0, 4/(3\pi))$ **71.** 3.82 **73.** 0 **75.** ∞

77. 1 **79.** $1000e^{0.09} \approx 1094.17$ **81.** Converges; $\frac{32}{3}$

83. Diverges **85.** Converges; 1 **87.** Converges; $\pi/4$

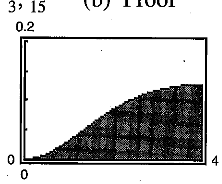
89. (a) $6,321,205.59 (b) $10,000,000

91. (a) 0.4581 (b) 0.0135

P.S. Problem Solving *(page 581)*

1. (a) $\frac{4}{3}, \frac{16}{15}$ (b) Proof **3.** $\ln 3$ **5.** Proof

7. (a)



Area $\approx$ 0.2986

(b) $\ln 3 - \frac{4}{5}$ (c) $\ln 3 - \frac{4}{5}$

9. $\ln 3 - \frac{1}{2} \approx 0.5986$

11. (a) ∞ (b) 0 (c) $-\frac{2}{3}$
The form $0 \cdot \infty$ is indeterminant.

13. About 0.8670 **15.** $\dfrac{1/12}{x} + \dfrac{1/42}{x - 3} + \dfrac{1/10}{x - 1} + \dfrac{111/140}{x + 4}$

17–19. Proofs **21.** About 0.0158

Chapter 9

Section 9.1 *(page 592)*

1. 3, 9, 27, 81, 243 **3.** 1, 0, −1, 0, 1 **5.** $2, -1, \frac{2}{3}, -\frac{1}{2}, \frac{2}{5}$

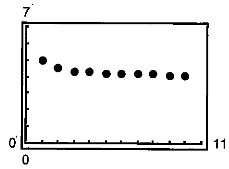
7. 3, 4, 6, 10, 18 **9.** c **10.** a **11.** d **12.** b

13. 14, 17; add 3 to preceding term.

15. 80, 160; multiply preceding term by 2. **17.** $n + 1$

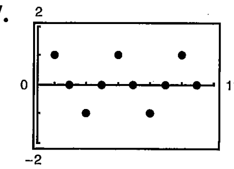
19. $1/[(2n + 1)(2n)]$ **21.** 5 **23.** 2

25.



Converges to 4

27.



Diverges

29. Converges to 0 **31.** Diverges **33.** Converges to 5

35. Converges to 0 **37.** Diverges **39.** Converges to 0

41. Converges to 1 **43.** Converges to 0

45. Answers will vary. Sample answer: $6n - 4$

47. Answers will vary. Sample answer: $n^2 - 3$

49. Answers will vary. Sample answer: $(n + 1)/(n + 2)$

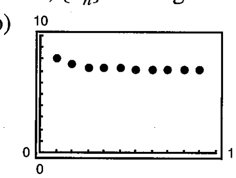
51. Answers will vary. Sample answer: $(n + 1)/n$

53. Monotonic, bounded **55.** Not monotonic, bounded

57. Monotonic, bounded **59.** Not monotonic, bounded

61. (a) $\left|7 + \frac{1}{n}\right| \geq 7 \Rightarrow$ bounded
$a_n > a_{n+1} \Rightarrow$ monotonic
So, $\{a_n\}$ converges.

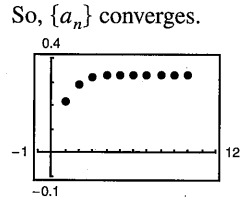
(b)



Limit = 7

63. (a) $\left|\frac{1}{3}\left(1 - \frac{1}{3^n}\right)\right| < \frac{1}{3} \Rightarrow$ bounded
$a_n < a_{n+1} \Rightarrow$ monotonic
So, $\{a_n\}$ converges.

(b)



Limit = $\frac{1}{3}$

65. $\{a_n\}$ has a limit because it is bounded and monotonic; because $2 \leq a_n \leq 4$, $2 \leq L \leq 4$.

67. (a) No. $\lim_{n\to\infty} A_n$ does not exist.

(b)

n	1	2	3	4
A_n	$10,045.83	$10,091.88	$10,138.13	$10,184.60

n	5	6	7
A_n	$10,231.28	$10,278.17	$10,325.28

n	8	9	10
A_n	$10,372.60	$10,420.14	$10,467.90

69. No. A sequence is said to converge when its terms approach a real number.

71. (a) $10 - \frac{1}{n}$
(b) Impossible. The sequence converges by Theorem 9.5.
(c) $a_n = \dfrac{3n}{4n + 1}$
(d) Impossible. An unbounded sequence diverges.