105. $c = 1$; $\ln(2)$

107. $8\pi[(\ln 2)^2/3 - (\ln 4)/9 + 2/27] \approx 2.01545$

109. $\int_0^1 2 \sin(u^2)\, du$; 0.6278 **111.** Proof

Review Exercises for Chapter 8 *(page 579)*

1. $\frac{1}{3}(x^2 - 36)^{3/2} + C$ **3.** $\frac{1}{2}\ln|x^2 - 49| + C$

5. $\ln(2) + \frac{1}{2} \approx 1.1931$ **7.** $100 \arcsin(x/10) + C$

9. $\frac{1}{9}e^{3x}(3x - 1) + C$ **11.** $\frac{1}{13}e^{2x}(2 \sin 3x - 3 \cos 3x) + C$

13. $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C$

15. $\frac{1}{16}\left[(8x^2 - 1)\arcsin 2x + 2x\sqrt{1 - 4x^2}\right] + C$

17. $\sin(\pi x - 1)[\cos^2(\pi x - 1) + 2]/(3\pi) + C$

19. $\frac{2}{3}[\tan^3(x/2) + 3\tan(x/2)] + C$ **21.** $\tan\theta + \sec\theta + C$

23. $3\pi/16 + \frac{1}{2} \approx 1.0890$ **25.** $3\sqrt{4 - x^2}/x + C$

27. $\frac{1}{3}(x^2 + 4)^{1/2}(x^2 - 8) + C$ **29.** $256 - 62\sqrt{17} \approx 0.3675$

31. (a), (b), and (c) $\frac{1}{3}\sqrt{4 + x^2}(x^2 - 8) + C$

33. $6 \ln|x + 3| - 5 \ln|x - 4| + C$

35. $\frac{1}{4}[6 \ln|x - 1| - \ln(x^2 + 1) + 6 \arctan x] + C$

37. $x - \frac{64}{11}\ln|x + 8| + \frac{9}{11}\ln|x - 3| + C$

39. $\frac{1}{25}[4/(4 + 5x) + \ln|4 + 5x|] + C$ **41.** $1 - \sqrt{2}/2$

43. $\frac{1}{2}\ln|x^2 + 4x + 8| - \arctan[(x + 2)/2] + C$

45. $\ln|\tan \pi x|/\pi + C$ **47.** Proof

49. $\frac{1}{8}(\sin 2\theta - 2\theta \cos 2\theta) + C$

51. $\frac{4}{3}[x^{3/4} - 3x^{1/4} + 3\arctan(x^{1/4})] + C$

53. $2\sqrt{1 - \cos x} + C$ **55.** $\sin x \ln(\sin x) - \sin x + C$

57. $\frac{5}{2}\ln|(x - 5)/(x + 5)| + C$

59. $y = x \ln|x^2 + x| - 2x + \ln|x + 1| + C$ **61.** $\frac{1}{5}$

63. $\frac{1}{2}(\ln 4)^2 \approx 0.961$ **65.** π **67.** $\frac{128}{15}$

69. $(\bar{x}, \bar{y}) = (0, 4/(3\pi))$ **71.** 3.82 **73.** 0 **75.** ∞

77. 1 **79.** $1000e^{0.09} \approx 1094.17$ **81.** Converges; $\frac{32}{3}$

83. Diverges **85.** Converges; 1 **87.** Converges; $\pi/4$

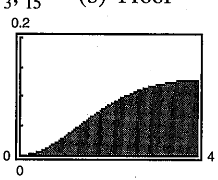
89. (a) \$6,321,205.59 (b) \$10,000,000

91. (a) 0.4581 (b) 0.0135

P.S. Problem Solving *(page 581)*

1. (a) $\frac{4}{3}, \frac{16}{15}$ (b) Proof **3.** ln 3 **5.** Proof

7. (a)



(b) $\ln 3 - \frac{4}{5}$ (c) $\ln 3 - \frac{4}{5}$

Area ≈ 0.2986

9. $\ln 3 - \frac{1}{2} \approx 0.5986$

11. (a) ∞ (b) 0 (c) $-\frac{2}{3}$

The form $0 \cdot \infty$ is indeterminant.

13. About 0.8670 **15.** $\frac{1/12}{x} + \frac{1/42}{x - 3} + \frac{1/10}{x - 1} + \frac{111/140}{x + 4}$

17–19. Proofs **21.** About 0.0158

Chapter 9

Section 9.1 *(page 592)*

1. 3, 9, 27, 81, 243 **3.** 1, 0, −1, 0, 1 **5.** $2, -1, \frac{2}{3}, -\frac{1}{2}, \frac{2}{5}$

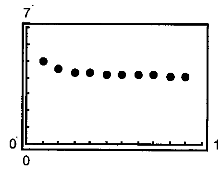
7. 3, 4, 6, 10, 18 **9.** c **10.** a **11.** d **12.** b

13. 14, 17; add 3 to preceding term.

15. 80, 160; multiply preceding term by 2. **17.** $n + 1$

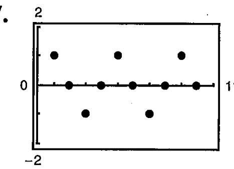
19. $1/[(2n + 1)(2n)]$ **21.** 5 **23.** 2

25.



Converges to 4

27.



Diverges

29. Converges to 0 **31.** Diverges **33.** Converges to 5

35. Converges to 0 **37.** Diverges **39.** Converges to 0

41. Converges to 1 **43.** Converges to 0

45. Answers will vary. Sample answer: $6n - 4$

47. Answers will vary. Sample answer: $n^2 - 3$

49. Answers will vary. Sample answer: $(n + 1)/(n + 2)$

51. Answers will vary. Sample answer: $(n + 1)/n$

53. Monotonic, bounded **55.** Not monotonic, bounded

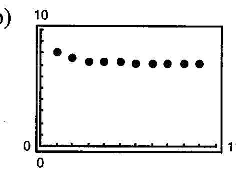
57. Monotonic, bounded **59.** Not monotonic, bounded

61. (a) $\left|7 + \frac{1}{n}\right| \ge 7 \implies$ bounded

$a_n > a_{n+1} \implies$ monotonic

So, $\{a_n\}$ converges.

(b)



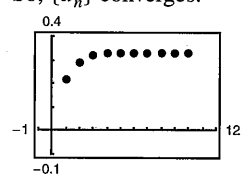
Limit = 7

63. (a) $\left|\frac{1}{3}\left(1 - \frac{1}{3^n}\right)\right| < \frac{1}{3} \implies$ bounded

$a_n < a_{n+1} \implies$ monotonic

So, $\{a_n\}$ converges.

(b)



Limit $= \frac{1}{3}$

65. $\{a_n\}$ has a limit because it is bounded and monotonic; because $2 \le a_n \le 4$, $2 \le L \le 4$.

67. (a) No. $\lim_{n\to\infty} A_n$ does not exist.

(b)

n	1	2	3	4
A_n	\$10,045.83	\$10,091.88	\$10,138.13	\$10,184.60

n	5	6	7
A_n	\$10,231.28	\$10,278.17	\$10,325.28

n	8	9	10
A_n	\$10,372.60	\$10,420.14	\$10,467.90

69. No. A sequence is said to converge when its terms approach a real number.

71. (a) $10 - \frac{1}{n}$

(b) Impossible. The sequence converges by Theorem 9.5.

(c) $a_n = \frac{3n}{4n + 1}$

(d) Impossible. An unbounded sequence diverges.

73. (a) $\$4{,}500{,}000{,}000(0.8)^n$

(b)

Year	1	2
Budget	\$3,600,000,000	\$2,880,000,000

Year	3	4
Budget	\$2,304,000,000	\$1,843,200,000

(c) Converges to 0

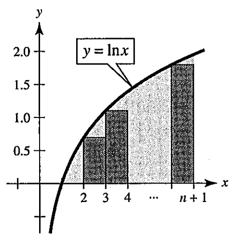
75. 1, 1.4142, 1.4422, 1.4142, 1.3797, 1.3480; Converges to 1

77. Proof **79.** True **81.** True

83. (a) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144
(b) 1, 2, 1.5, 1.6667, 1.6, 1.6250, 1.6154, 1.6190, 1.6176, 1.6182 (c) Proof
(d) $\rho = (1 + \sqrt{5})/2 \approx 1.6180$

85. (a) 1.4142, 1.8478, 1.9616, 1.9904, 1.9976
(b) $a_n = \sqrt{2 + a_{n-1}}$ (c) $\lim_{n\to\infty} a_n = 2$

87. (a) Proof
(b)



(c) Proof (d) Proof
(e) $\dfrac{\sqrt[20]{20!}}{20} \approx 0.4152$; $\dfrac{\sqrt[50]{50!}}{50} \approx 0.3897$; $\dfrac{\sqrt[100]{100!}}{100} \approx 0.3799$

89–91. Proofs **93.** Putnam Problem A1, 1990

Section 9.2 *(page 601)*

1. 1, 1.25, 1.361, 1.424, 1.464

3. 3, −1.5, 5.25, −4.875, 10.3125

5. 3, 4.5, 5.25, 5.625, 5.8125 **7.** Geometric series: $r = \frac{7}{6} > 1$

9. $\lim_{n\to\infty} a_n = 1 \neq 0$ **11.** $\lim_{n\to\infty} a_n = 1 \neq 0$

13. $\lim_{n\to\infty} a_n = \frac{1}{2} \neq 0$ **15.** Geometric series: $r = \frac{5}{6} < 1$

17. Geometric series: $r = 0.9 < 1$

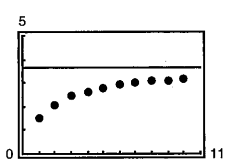
19. Telescoping series: $a_n = 1/n - 1/(n + 1)$; Converges to 1.

21. (a) $\frac{11}{3}$

(b)

n	5	10	20	50	100
S_n	2.7976	3.1643	3.3936	3.5513	3.6078

(c)



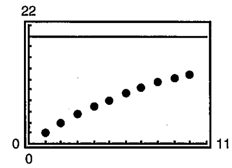
(d) The terms of the series decrease in magnitude relatively slowly, and the sequence of partial sums approaches the sum of the series relatively slowly.

23. (a) 20

(b)

n	5	10	20	50	100
S_n	8.1902	13.0264	17.5685	19.8969	19.9995

(c)



(d) The terms of the series decrease in magnitude relatively slowly, and the sequence of partial sums approaches the sum of the series relatively slowly.

25. 15 **27.** 3 **29.** 32 **31.** $\frac{1}{2}$ **33.** $\dfrac{\sin(1)}{1 - \sin(1)}$

35. (a) $\sum_{n=0}^{\infty} \frac{4}{10}(0.1)^n$ (b) $\frac{4}{9}$ **37.** (a) $\sum_{n=0}^{\infty} \frac{81}{100}(0.01)^n$ (b) $\frac{9}{11}$

39. (a) $\sum_{n=0}^{\infty} \frac{3}{40}(0.01)^n$ (b) $\frac{5}{66}$ **41.** Diverges **43.** Diverges

45. Converges **47.** Diverges **49.** Diverges

51. Diverges **53.** Diverges **55.** See definitions on page 595.

57. The series given by

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots + ar^n + \cdots, a \neq 0$$

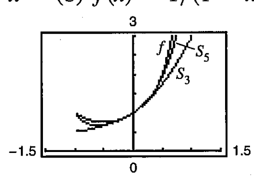
is a geometric series with ratio r. When $0 < |r| < 1$, the series converges to the sum $\sum_{n=0}^{\infty} ar^n = \dfrac{a}{1 - r}$.

59. The series in (a) and (b) are the same. The series in (c) is different unless $a_1 = a_2 = \cdots = a$ is constant.

61. $|x| < \frac{1}{3}$; $\dfrac{3x}{1 - 3x}$ **63.** $0 < x < 2$; $(x - 1)/(2 - x)$

65. $-1 < x < 1$; $1/(1 + x)$

67. (a) x (b) $f(x) = 1/(1 - x)$, $|x| < 1$
(c)



Answers will vary.

69. The required terms for the two series are $n = 100$ and $n = 5$, respectively. The second series converges at a higher rate.

71. $160{,}000(1 - 0.95^n)$ units

73. $\sum_{i=0}^{\infty} 200(0.75)^i$; Sum = \$800 million **75.** 152.42 feet

77. $\frac{1}{8}$; $\sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{1}{2}\right)^n = \dfrac{1/2}{1 - 1/2} = 1$

79. (a) $-1 + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n = -1 + \dfrac{a}{1 - r} = -1 + \dfrac{1}{1 - 1/2} = 1$
(b) No (c) 2

81. (a) 126 in.^{2} (b) 128 in.^{2}

83. The \$2,000,000 sweepstakes has a present value of \$1,146,992.12. After accruing interest over the 20-year period, it attains its full value.

85. (a) \$5,368,709.11 (b) \$10,737,418.23 (c) \$21,474,836.47

87. (a) \$14,773.59 (b) \$14,779.65

89. (a) \$91,373.09 (b) \$91,503.32

91. False. $\lim_{n\to\infty} \frac{1}{n} = 0$, but $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

93. False. $\sum_{n=1}^{\infty} ar^n = \left(\dfrac{a}{1 - r}\right) - a$; The formula requires that the geometric series begins with $n = 0$.

95. True **97.** Answers will vary. Example: $\sum_{n=0}^{\infty} 1$, $\sum_{n=0}^{\infty} (-1)$

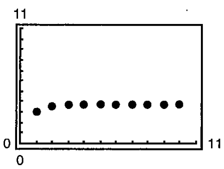
99–101. Proofs **103.** 2

Section 9.3 *(page 609)*

1. Diverges **3.** Converges **5.** Converges
7. Converges **9.** Diverges **11.** Diverges
13. Converges **15.** Converges **17.** Converges
19. Diverges **21.** Converges **23.** Diverges
25. $f(x)$ is not positive for $x \geq 1$.
27. $f(x)$ is not always decreasing. **29.** Converges
31. Diverges **33.** Diverges **35.** Converges
37. Converges

39. (a)

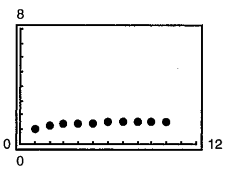
n	5	10	20	50	100
S_n	3.7488	3.75	3.75	3.75	3.75



The partial sums approach the sum 3.75 very quickly.

(b)

n	5	10	20	50	100
S_n	1.4636	1.5498	1.5962	1.6251	1.635

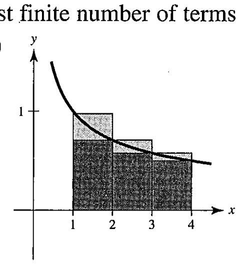


The partial sums approach the sum $\pi^2/6 \approx 1.6449$ more slowly than the series in part (a).

41. See Theorem 9.10 on page 605. Answers will vary. For example, convergence or divergence can be determined for the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

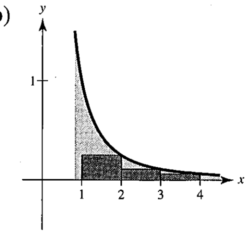
43. No. Because $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, $\sum_{n=10,000}^{\infty} \frac{1}{n}$ also diverges. The convergence or divergence of a series is not determined by the first finite number of terms of the series.

45. (a)



The area under the rectangles is greater than the area under the curve. Because $\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_1^{\infty} = \infty$ diverges, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges.

(b)



The area under the rectangles is less than the area under the curve. Because $\int_1^{\infty} \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^{\infty} = 1$ converges, $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges $\left(\text{and so does } \sum_{n=1}^{\infty} \frac{1}{n^2}\right)$.

47. $p > 1$ **49.** $p > 1$ **51.** $p > 3$ **53.** Proof
55. $S_5 = 1.4636$, $R_5 = 0.20$ **57.** $S_{10} \approx 0.9818$, $R_{10} \approx 0.0997$ **59.** $S_4 \approx 0.4049$, $R_4 \approx 5.6 \times 10^{-8}$
61. $N \geq 7$ **63.** $N \geq 16$

65. (a) $\sum_{n=2}^{\infty} \frac{1}{n^{1.1}}$ converges by the p-Series Test because $1.1 > 1$. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges by the Integral Test because $\int_2^{\infty} \frac{1}{x \ln x}\,dx$ diverges.

(b) $\sum_{n=2}^{\infty} \frac{1}{n^{1.1}} = 0.4665 + 0.2987 + 0.2176 + 0.1703 + 0.1393 + \cdots$

$\sum_{n=2}^{\infty} \frac{1}{n \ln n} = 0.7213 + 0.3034 + 0.1803 + 0.1243 + 0.0930 + \cdots$

(c) $n \geq 3.431 \times 10^{15}$

67. (a) Let $f(x) = 1/x$. f is positive, continuous, and decreasing on $[1, \infty)$.

$S_n - 1 \leq \int_1^n \frac{1}{x}\,dx = \ln n$

$S_n \geq \int_1^{n+1} \frac{1}{x}\,dx = \ln(n+1)$

So, $\ln(n+1) \leq S_n \leq 1 + \ln n$.

(b) $\ln(n+1) - \ln n \leq S_n - \ln n \leq 1$
Also, $\ln(n+1) - \ln n > 0$ for $n \geq 1$. So, $0 \leq S_n - \ln n \leq 1$, and the sequence $\{a_n\}$ is bounded.

(c) $a_n - a_{n+1} = [S_n - \ln n] - [S_{n+1} - \ln(n+1)]$

$= \int_n^{n+1} \frac{1}{x}\,dx - \frac{1}{n+1} \geq 0$

So, $a_n \geq a_{n+1}$.

(d) Because the sequence is bounded and monotonic, it converges to a limit, γ.

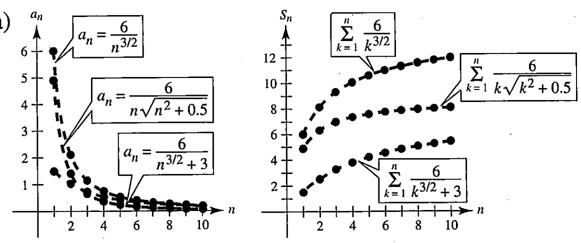
(e) 0.5822

69. (a) Diverges (b) Diverges
(c) $\sum_{n=2}^{\infty} x^{\ln n}$ converges for $x < 1/e$.

71. Diverges **73.** Converges **75.** Converges
77. Diverges **79.** Diverges **81.** Converges

Section 9.4 *(page 616)*

1. (a)



(b) $\sum_{n=1}^{\infty} \frac{6}{n^{3/2}}$; Converges

(c) The magnitudes of the terms are less than the magnitudes of the terms of the p-series. Therefore, the series converges.

(d) The smaller the magnitudes of the terms, the smaller the magnitudes of the terms of the sequence of partial sums.

3. Diverges **5.** Diverges **7.** Diverges **9.** Converges
11. Converges **13.** Diverges **15.** Diverges
17. Converges **19.** Converges **21.** Diverges
23. Diverges; p-Series Test
25. Converges; Direct Comparison Test with $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$
27. Diverges; nth-Term Test **29.** Converges; Integral Test
31. $\lim_{n\to\infty} \frac{a_n}{1/n} = \lim_{n\to\infty} na_n$; $\lim_{n\to\infty} na_n \neq 0$, but is finite.
The series diverges by the Limit Comparison Test.
33. Diverges **35.** Converges
37. $\lim_{n\to\infty} n\left(\frac{n^3}{5n^4+3}\right) = \frac{1}{5} \neq 0$; So, $\sum_{n=1}^{\infty} \frac{n^3}{5n^4+3}$ diverges.
39. Diverges **41.** Converges
43. Convergence or divergence is dependent on the form of the general term for the series and not necessarily on the magnitudes of the terms.
45. See Theorem 9.13 on page 614. Answers will vary. For example, $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n-1}}$ diverges because $\lim_{n\to\infty} \frac{1/\sqrt{n-1}}{1/\sqrt{n}} = 1$ and $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ diverges (p-series).
47. (a) Proof
(b)

n	5	10	20	50	100
S_n	1.1839	1.2087	1.2212	1.2287	1.2312

(c) 0.1226 (d) 0.0277
49. False. Let $a_n = 1/n^3$ and $b_n = 1/n^2$. **51.** True
53. True **55.** Proof **57.** $\sum_{n=1}^{\infty} \frac{1}{n^2}, \sum_{n=1}^{\infty} \frac{1}{n^3}$ **59–65.** Proofs
67. Putnam Problem B4, 1988

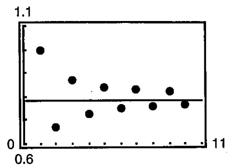
Section 9.5 *(page 625)*

1. (a)

n	1	2	3	4	5
S_n	1.0000	0.6667	0.8667	0.7238	0.8349

n	6	7	8	9	10
S_n	0.7440	0.8209	0.7543	0.8131	0.7605

(b)



(c) The points alternate sides of the horizontal line $y = \pi/4$ that represents the sum of the series. The distances between the successive points and the line decrease.

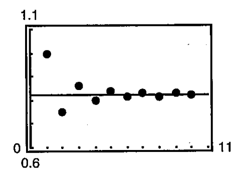
(d) The distance in part (c) is always less than the magnitude of the next term of the series.

3. (a)

n	1	2	3	4	5
S_n	1.0000	0.7500	0.8611	0.7986	0.8386

n	6	7	8	9	10
S_n	0.8108	0.8312	0.8156	0.8280	0.8180

(b)



(c) The points alternate sides of the horizontal line $y = \pi^2/12$ that represents the sum of the series. The distances between the successive points and the line decrease.

(d) The distance in part (c) is always less than the magnitude of the next term of the series.

5. Converges **7.** Converges **9.** Diverges **11.** Diverges
13. Converges **15.** Diverges **17.** Diverges
19. Converges **21.** Converges **23.** Converges
25. Converges **27.** $1.8264 \leq S \leq 1.8403$
29. $1.7938 \leq S \leq 1.8054$ **31.** 10 **33.** 7
35. 7 terms (Note that the sum begins with $n = 0$.)
37. Converges absolutely **39.** Converges absolutely
41. Converges conditionally **43.** Diverges
45. Converges conditionally **47.** Converges absolutely
49. Converges absolutely **51.** Converges conditionally
53. Converges absolutely
55. An alternating series is a series whose terms alternate in sign.
57. $|S - S_N| = |R_N| \leq a_{N+1}$
59. (a) False. For example, let $a_n = \frac{(-1)^n}{n}$.

Then $\sum a_n = \sum \frac{(-1)^n}{n}$ converges

and $\sum(-a_n) = \sum \frac{(-1)^{n+1}}{n}$ converges.

But, $\sum |a_n| = \sum \frac{1}{n}$ diverges.

(b) True. For if $\sum |a_n|$ converged, then so would $\sum a_n$ by Theorem 9.16.

61. True **63.** $p > 0$
65. Proof; The converse is false. For example: Let $a_n = 1/n$.
67. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, hence so does $\sum_{n=1}^{\infty} \frac{1}{n^4}$.
69. (a) No. $a_{n+1} \leq a_n$ is not satisfied for all n. For example, $\frac{1}{9} < \frac{1}{8}$.
(b) Yes. 0.5

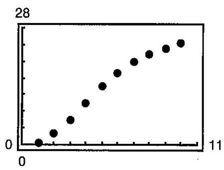
71. Converges; p-Series Test **73.** Diverges; nth-Term Test
75. Converges; Geometric Series Test
77. Converges; Integral Test
79. Converges; Alternating Series Test
81. The first term of the series is 0, not 1. You cannot regroup series terms arbitrarily.

Section 9.6 *(page 633)*

1–3. Proofs **5.** d **6.** c **7.** f **8.** b **9.** a
10. e
11. (a) Proof
(b)

n	5	10	15	20	25
S_n	13.7813	24.2363	25.8468	25.9897	25.9994

(c)



(d) 26

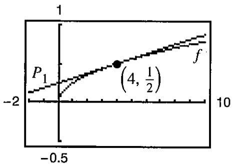
(e) The more rapidly the terms of the series approach 0, the more rapidly the sequence of partial sums approaches the sum of the series.

13. Converges **15.** Diverges **17.** Diverges
19. Converges **21.** Converges **23.** Converges
25. Diverges **27.** Converges **29.** Converges
31. Diverges **33.** Converges **35.** Converges
37. Converges **39.** Diverges **41.** Converges
43. Diverges **45.** Converges **47.** Converges
49. Converges **51.** Converges; Alternating Series Test
53. Converges; p-Series Test **55.** Diverges; nth-Term Test
57. Diverges; Geometric Series Test
59. Converges; Limit Comparison Test with $b_n = 1/2^n$
61. Converges; Direct Comparison Test with $b_n = 1/3^n$
63. Diverges; Ratio Test **65.** Converges; Ratio Test
67. Converges; Ratio Test **69.** a and c **71.** a and b
73. $\sum_{n=0}^{\infty} \frac{n+1}{7^{n+1}}$ **75.** (a) 9 (b) -0.7769
77. Diverges; $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$
79. Converges; $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$ **81.** Diverges; $\lim a_n \neq 0$
83. Converges **85.** Converges **87.** $(-3, 3)$
89. $(-2, 0]$ **91.** $x = 0$
93. See Theorem 9.17 on page 627.
95. No; the series $\sum_{n=1}^{\infty} \frac{1}{n + 10{,}000}$ diverges.
97. Absolutely; by Theorem 9.17 **99–105.** Proofs
107. (a) Diverges (b) Converges (c) Converges
(d) Converges for all integers $x \geq 2$
109. Putnam Problem 7, morning session, 1951

Section 9.7 *(page 658)*

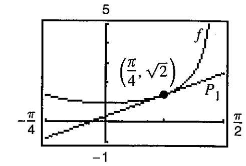
1. d **2.** c **3.** a **4.** b

5. $P_1 = \frac{1}{16}x + \frac{1}{4}$



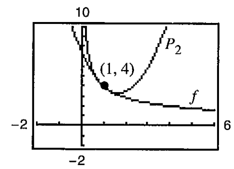
P_1 is the first-degree Taylor polynomial for f at 4.

7. $P_1 = \sqrt{2}x + \sqrt{2}(4 - \pi)/4$



P_1 is the first-degree Taylor polynomial for f at $\pi/4$.

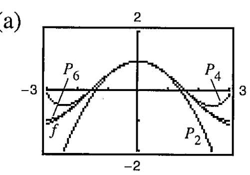
9.



x	0	0.8	0.9	1	1.1
$f(x)$	Error	4.4721	4.2164	4.0000	3.8139
$P_2(x)$	7.5000	4.4600	4.2150	4.0000	3.8150

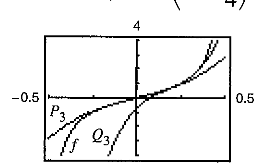
x	1.2	2
$f(x)$	3.6515	2.8284
$P_2(x)$	3.6600	3.5000

11. (a)



(b) $f^{(2)}(0) = -1$ $P_2^{(2)}(0) = -1$
$f^{(4)}(0) = 1$ $P_4^{(4)}(0) = 1$
$f^{(6)}(0) = -1$ $P_6^{(6)}(0) = -1$
(c) $f^{(n)}(0) = P_n^{(n)}(0)$

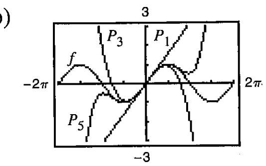
13. $1 + 4x + 8x^2 + \frac{32}{3}x^3 + \frac{32}{3}x^4$
15. $1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{48}x^3 + \frac{1}{384}x^4$ **17.** $x - \frac{1}{6}x^3 + \frac{1}{120}x^5$
19. $x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$ **21.** $1 - x + x^2 - x^3 + x^4 - x^5$
23. $1 + \frac{1}{2}x^2$ **25.** $2 - 2(x - 1) + 2(x - 1)^2 - 2(x - 1)^3$
27. $2 + \frac{1}{4}(x - 4) - \frac{1}{64}(x - 4)^2 + \frac{1}{512}(x - 4)^3$
29. $\ln 2 + \frac{1}{2}(x - 2) - \frac{1}{8}(x - 2)^2 + \frac{1}{24}(x - 2)^3 - \frac{1}{64}(x - 2)^4$
31. (a) $P_3(x) = \pi x + \frac{\pi^3}{3}x^3$
(b) $Q_3(x) = 1 + 2\pi\left(x - \frac{1}{4}\right) + 2\pi^2\left(x - \frac{1}{4}\right)^2 + \frac{8\pi^3}{3}\left(x - \frac{1}{4}\right)^3$



33. (a)

x	0	0.25	0.50	0.75	1.00
$\sin x$	0	0.2474	0.4794	0.6816	0.8415
$P_1(x)$	0	0.25	0.50	0.75	1.00
$P_3(x)$	0	0.2474	0.4792	0.6797	0.8333
$P_5(x)$	0	0.2474	0.4794	0.6817	0.8417

(b)



(c) As the distance increases, the polynomial approximation becomes less accurate.

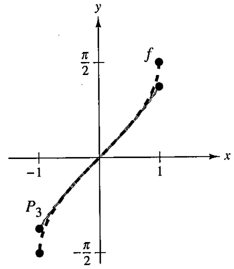
35. (a) $P_3(x) = x + \frac{1}{6}x^3$

(b)

x	-0.75	-0.50	-0.25	0	0.25
$f(x)$	-0.848	-0.524	-0.253	0	0.253
$P_3(x)$	-0.820	-0.521	-0.253	0	0.253

x	0.50	0.75
$f(x)$	0.524	0.848
$P_3(x)$	0.521	0.820

(c)



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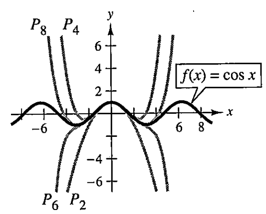
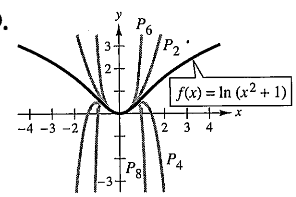


39.



41. 2.7083 **43.** 0.7419 **45.** $R_4 \le 2.03 \times 10^{-5}$; 0.000001
47. $R_3 \le 7.82 \times 10^{-3}$; 0.00085 **49.** 3 **51.** 5
53. $n = 9$; $\ln(1.5) \approx 0.4055$ **55.** $-0.3936 < x < 0$
57. $-0.9467 < x < 0.9467$
59. The graphs of the approximating polynomial P and the elementary function f both pass through the point $(c, f(c))$, and the slope of the graph of P is the same as the slope of the graph of f at the point $(c, f(c))$. If P is of degree n, then the first n derivatives of f and P agree at c. This allows the graph of P to resemble the graph of f near the point $(c, f(c))$.
61. See "Definitions of nth Taylor Polynomial and nth Maclaurin Polynomial" on page 638.
63. As the degree of the polynomial increases, the graph of the Taylor polynomial becomes a better and better approximation of the function within the interval of convergence. Therefore, the accuracy is increased.
65. (a) $f(x) \approx P_4(x) = 1 + x + (1/2)x^2 + (1/6)x^3 + (1/24)x^4$
$g(x) \approx Q_5(x) = x + x^2 + (1/2)x^3 + (1/6)x^4 + (1/24)x^5$
$Q_5(x) = xP_4(x)$
(b) $g(x) \approx P_6(x) = x^2 - x^4/3! + x^6/5!$
(c) $g(x) \approx P_4(x) = 1 - x^2/3! + x^4/5!$
67. (a) $Q_2(x) = -1 + (\pi^2/32)(x + 2)^2$
(b) $R_2(x) = -1 + (\pi^2/32)(x - 6)^2$

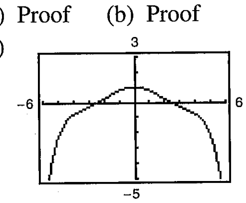
(c) No. Horizontal translations of the result in part (a) are possible only at $x = -2 + 8n$ (where n is an integer) because the period of f is 8.
69. Proof
71. As you move away from $x = c$, the Taylor polynomial becomes less and less accurate.

Section 9.8 *(page 654)*

1. 0 **3.** 2 **5.** $R = 1$ **7.** $R = \frac{1}{4}$ **9.** $R = \infty$
11. $(-4, 4)$ **13.** $(-1, 1]$ **15.** $(-\infty, \infty)$ **17.** $x = 0$
19. $(-6, 6)$ **21.** $(-5, 13]$ **23.** $(0, 2]$ **25.** $(0, 6)$
27. $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **29.** $(-\infty, \infty)$ **31.** $(-1, 1)$ **33.** $x = 3$
35. $R = c$ **37.** $(-k, k)$ **39.** $(-1, 1)$
41. $\displaystyle\sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!}$ **43.** $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!}$
45. (a) $(-3, 3)$ (b) $(-3, 3)$ (c) $(-3, 3)$ (d) $[-3, 3)$
47. (a) $(0, 2]$ (b) $(0, 2)$ (c) $(0, 2)$ (d) $[0, 2]$
49. A series of the form

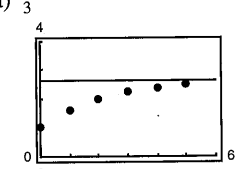
$$\sum_{n=0}^{\infty} a_n(x - c)^n = a_0 + a_1(x - c) + a_2(x - c)^2 + \cdots + a_n(x - c)^n + \cdots$$

is called a power series centered at c, where c is a constant.
51. The interval of convergence of a power series is the set of all values of x for which the power series converges.
53. You differentiate and integrate the power series term by term. The radius of convergence remains the same. However, the interval of convergence might change.
55. Many answers possible.
(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{x}{2}\right)^n$ Geometric: $\left|\frac{x}{2}\right| < 1 \Rightarrow |x| < 2$
(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$ converges for $-1 < x \le 1$
(c) $\displaystyle\sum_{n=1}^{\infty} (2x + 1)^n$ Geometric:
$|2x + 1| < 1 \Rightarrow -1 < x < 0$
(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(x - 2)^n}{n4^n}$ converges for $-2 \le x < 6$
57. (a) For $f(x)$: $(-\infty, \infty)$; For $g(x)$: $(-\infty, \infty)$
(b) Proof (c) Proof (d) $f(x) = \sin x$; $g(x) = \cos x$
59–63. Proofs
65. (a) Proof (b) Proof
(c)

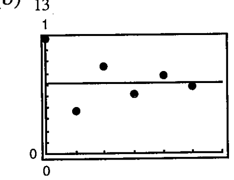


(d) 0.92

67. (a) $\frac{8}{3}$



(b) $\frac{8}{13}$

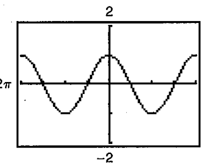


(c) The alternating series converges more rapidly. The partial sums of the series of positive terms approach the sum from below. The partial sums of the alternating series alternate sides of the horizontal line representing the sum.

(d)

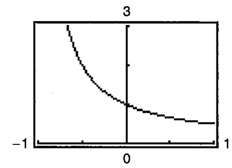
M	10	100	1000	10,000
N	5	14	24	35

69.



$f(x) = \cos x$

71.



$f(x) = 1/(1 + x)$

73. False. Let $a_n = (-1)^n/(n2^n)$. **75.** True **77.** Proof

79. (a) $(-1, 1)$ (b) $f(x) = (c_0 + c_1x + c_2x^2)/(1 - x^3)$

81. Proof

Section 9.9 *(page 662)*

1. $\sum_{n=0}^{\infty} \frac{x^n}{4^{n+1}}$ **3.** $\sum_{n=0}^{\infty} \frac{4}{3}\left(\frac{-x}{3}\right)^n$

5. $\sum_{n=0}^{\infty} \frac{(x-1)^n}{2^{n+1}}$ **7.** $\sum_{n=0}^{\infty} (3x)^n$ **9.** $-\frac{5}{9}\sum_{n=0}^{\infty}\left[\frac{2}{9}(x+3)\right]^n$

$(-1, 3)$ $\left(-\frac{1}{3}, \frac{1}{3}\right)$ $\left(-\frac{15}{2}, \frac{3}{2}\right)$

11. $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{n+1} x^n}{4^{n+1}}$ **13.** $\sum_{n=0}^{\infty}\left[\frac{1}{(-3)^n} - 1\right]x^n$

$\left(-\frac{4}{3}, \frac{4}{3}\right)$ $(-1, 1)$

15. $\sum_{n=0}^{\infty} x^n[1 + (-1)^n] = 2\sum_{n=0}^{\infty} x^{2n}$ **17.** $2\sum_{n=0}^{\infty} x^{2n}$

$(-1, 1)$ $(-1, 1)$

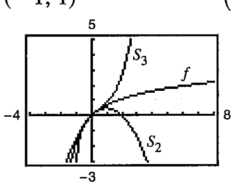
19. $\sum_{n=1}^{\infty} n(-1)^n x^{n-1}$ **21.** $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$

$(-1, 1)$ $(-1, 1]$

23. $\sum_{n=0}^{\infty} (-1)^n x^{2n}$ **25.** $\sum_{n=0}^{\infty} (-1)^n (2x)^{2n}$

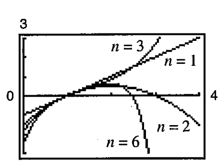
$(-1, 1)$ $\left(-\frac{1}{2}, \frac{1}{2}\right)$

27.



x	0.0	0.2	0.4	0.6	0.8	1.0
S_2	0.000	0.180	0.320	0.420	0.480	0.500
$\ln(x + 1)$	0.000	0.182	0.336	0.470	0.588	0.693
S_3	0.000	0.183	0.341	0.492	0.651	0.833

29. (a)



(b) $\ln x, 0 < x \le 2, R = 1$

(c) -0.6931

(d) $\ln(0.5)$; The error is approximately 0.

31. 0.245 **33.** 0.125 **35.** $\sum_{n=1}^{\infty} nx^{n-1}, \ -1 < x < 1$

37. $\sum_{n=0}^{\infty} (2n + 1)x^n, \ -1 < x < 1$

39. $E(n) = 2$. Because the probability of obtaining a head on a single toss is $\frac{1}{2}$, it is expected that, on average, a head will be obtained in two tosses.

41. Because $\frac{1}{1 + x} = \frac{1}{1 - (-x)}$, substitute $(-x)$ into the geometric series.

43. Because $\frac{5}{1 + x} = 5\left(\frac{1}{1 - (-x)}\right)$, substitute $(-x)$ into the geometric series and then multiply the series by 5.

45. Proof **47.** (a) Proof (b) 3.14

49. $\ln\frac{3}{2} \approx 0.4055$; See Exercise 21.

51. $\ln\frac{7}{5} \approx 0.3365$; See Exercise 49.

53. $\arctan\frac{1}{2} \approx 0.4636$; See Exercise 52.

55. The series in Exercise 52 converges to its sum at a lower rate because its terms approach 0 at a much lower rate.

57. The series converges on the interval $(-5, 3)$ and perhaps also at one or both endpoints.

59. $\sqrt{3}\pi/6$ **61.** $S_1 = 0.3183098862, \ 1/\pi \approx 0.3183098862$

Section 9.10 *(page 673)*

1. $\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$ **3.** $\frac{\sqrt{2}}{2}\sum_{n=0}^{\infty} \frac{(-1)^{n(n+1)/2}}{n!}\left(x - \frac{\pi}{4}\right)^n$

5. $\sum_{n=0}^{\infty} (-1)^n (x - 1)^n$ **7.** $\sum_{n=0}^{\infty} \frac{(-1)^n (x - 1)^{n+1}}{n + 1}$

9. $\sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n+1}}{(2n + 1)!}$ **11.** $1 + x^2/2! + 5x^4/4! + \cdots$

13–15. Proofs **17.** $\sum_{n=0}^{\infty} (-1)^n (n + 1)x^n$

19. $1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n - 1)x^n}{2^n n!}$

21. $\frac{1}{2}\left[1 + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n - 1)x^{2n}}{2^{3n} n!}\right]$

23. $1 + \frac{x}{2} + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \cdots (2n - 3)x^n}{2^n n!}$

25. $1 + \frac{x^2}{2} + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \cdots (2n - 3)x^{2n}}{2^n n!}$

27. $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$ **29.** $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$ **31.** $\sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n+1}}{(2n + 1)!}$

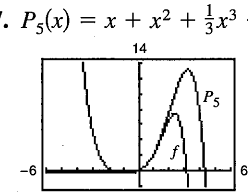
33. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$ **35.** $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{(2n)!}$

37. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n + 1)!}$ **39.** $\frac{1}{2}\left[1 + \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}\right]$

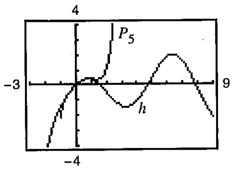
41. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n + 1)!}$ **43.** $\begin{cases} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n + 1)!}, & x \ne 0 \\ 1, & x = 0 \end{cases}$

45. Proof

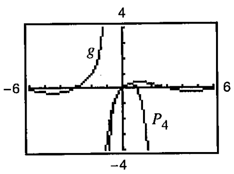
47. $P_5(x) = x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5$



49. $P_5(x) = x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{3}{40}x^5$



51. $P_4(x) = x - x^2 + \frac{5}{6}x^3 - \frac{5}{6}x^4$

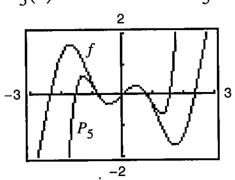


53. $\sum_{n=0}^{\infty} \frac{(-1)^{(n+1)}x^{2n+3}}{(2n+3)(n+1)!}$ **55.** 0.6931 **57.** 7.3891

59. 0 **61.** 1 **63.** 0.8075 **65.** 0.9461 **67.** 0.4872

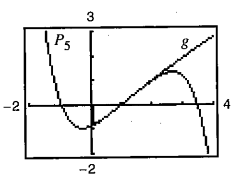
69. 0.2010 **71.** 0.7040 **73.** 0.3412

75. $P_5(x) = x - 2x^3 + \frac{2}{3}x^5$



$\left[-\frac{3}{4}, \frac{3}{4}\right]$

77. $P_5(x) = (x-1) - \frac{1}{24}(x-1)^3 + \frac{1}{24}(x-1)^4 - \frac{71}{1920}(x-1)^5$



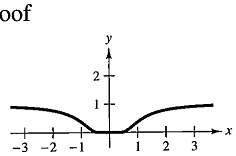
$\left[\frac{1}{4}, 2\right]$

79. See "Guidelines for Finding a Taylor Series" on page 668.

81. (a) Replace x with $(-x)$. (b) Replace x with $3x$.
(c) Multiply series by x.

83. Proof

85. (a)



(b) Proof

(c) $\sum_{n=0}^{\infty} 0x^n = 0 \neq f(x)$

87. Proof **89.** 10 **91.** -0.0390625 **93.** $\sum_{n=0}^{\infty} \binom{k}{n} x^n$

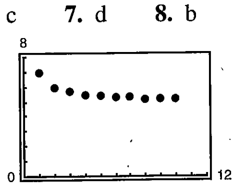
95. Proof

Review Exercises for Chapter 9 *(page 676)*

1. 5, 25, 125, 625, 3125 **3.** $-\frac{1}{4}, \frac{1}{16}, -\frac{1}{64}, \frac{1}{256}, -\frac{1}{1024}$ **5.** a

6. c **7.** d **8.** b

9.



Converges to 5

11. Converges to 5 **13.** Diverges **15.** Converges to 0

17. Converges to 0 **19.** $a_n = 5n - 2$ **21.** $a_n = \frac{1}{(n! + 1)}$

23. (a)

n	1	2	3	4
A_n	$8100.00	$8201.25	$8303.77	$8407.56

n	5	6	7	8
A_n	$8512.66	$8619.07	$8726.80	$8835.89

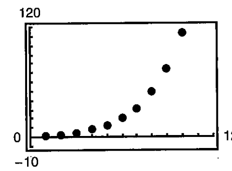
(b) $13,148.96

25. 3, 4.5, 5.5, 6.25, 6.85

27. (a)

n	5	10	15	20	25
S_n	13.2	113.3	873.8	6648.5	50,500.3

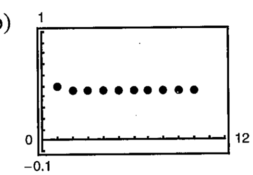
(b)



29. (a)

n	5	10	15	20	25
S_n	0.4597	0.4597	0.4597	0.4597	0.4597

(b)



31. $\frac{5}{3}$ **33.** 5.5 **35.** (a) $\sum_{n=0}^{\infty} (0.09)(0.01)^n$ (b) $\frac{1}{11}$

37. Diverges **39.** Diverges **41.** $45\frac{1}{3}$ m **43.** Diverges

45. Converges **47.** Diverges **49.** Diverges

51. Converges **53.** Diverges **55.** Converges

57. Converges **59.** Diverges **61.** Diverges

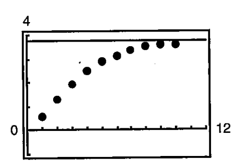
63. Converges **65.** Diverges

67. (a) Proof

(b)

n	5	10	15	20	25
S_n	2.8752	3.6366	3.7377	3.7488	3.7499

(c)



(d) 3.75

69. $P_3(x) = 1 - 2x + 2x^2 - \frac{4}{3}x^3$

71. $P_3(x) = 1 - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3$ **73.** 3 terms

75. $(-10, 10)$ **77.** $[1, 3]$ **79.** Converges only at $x = 2$

81. (a) $(-5, 5)$ (b) $(-5, 5)$ (c) $(-5, 5)$ (d) $[-5, 5)$

83. Proof **85.** $\sum_{n=0}^{\infty} \frac{2}{3}\left(\frac{x}{3}\right)^n$ **87.** $\sum_{n=0}^{\infty} 2\left(\frac{x-1}{3}\right)^n$; $(-2, 4)$

89. $\ln \frac{5}{4} \approx 0.2231$ **91.** $e^{1/2} \approx 1.6487$

93. $\cos \frac{2}{3} \approx 0.7859$ **95.** $\frac{\sqrt{2}}{2} \sum_{n=0}^{\infty} \frac{(-1)^{n(n+1)/2}}{n!}\left(x - \frac{3\pi}{4}\right)^n$

97. $\sum_{n=0}^{\infty} \frac{(x \ln 3)^n}{n!}$ **99.** $-\sum_{n=0}^{\infty} (x+1)^n$