Ch. P Day 1 Notes

1. Finding Intercepts:

a) x-intercepts : set $y=0$, solve for x (in numerator)

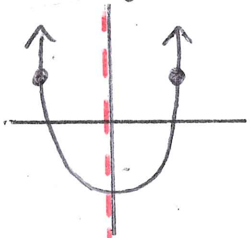
b) y-intercepts : set $x=0$, solve for y.

2. Determining Symmetry algebraically

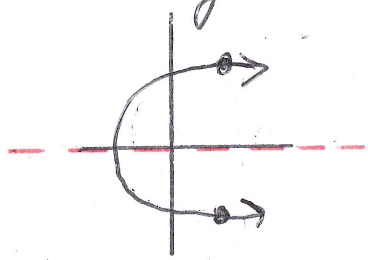
a) y-axis : replace x with "$-x$" and see if equation remains unchanged

b) x-axis : replace y with "$-y$" and see if equation remain unchanged

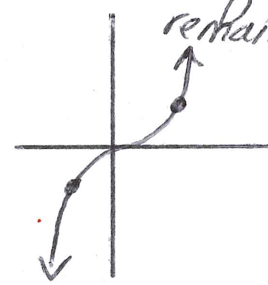
c) origin : replace x and y with "$-x$", "$-y$" and see if equation remain unchanged



y-axis symmetry



x-axis symmetry



origin symmetry

* Can graphs with x-axis symmetry be functions?

Ex. 1 Describe the symmetry of the following:

a) $y=x^2$ y-axis

b) $y=x^3$ origin

c) $y=x^2-x$ none

d) $x^2+y^2=25$ x-axis, y-axis, origin

3. Points of Intersection

To find this, solve an equation for one variable and substitute it into the other equation.

Ex. 2 Find points of intersection for the following:

$x^2+y^2=25$ and $2x+y=10$

(circle) (line)

$y=-2x+10 \qquad x^2+y^2=25$

$x^2+(-2x+10)^2=25$

$x^2+4x^2-40x+100=25$

$5x^2-40x+75=0$

$5(x^2-8x+15)=0$

$5(x-5)(x-3)=0$

$x=5, \; x=3$

$y=-2(5)+10=0$

$y=-2(3)+10=4$

points of intersection: (5,0) and (3,4)

Ex. 3 Find x and y-intercepts and discuss symmetry (determined algebraically)

$x=y^2+1$

* x-ints: set $y=0$
* y-ints: set $x=0$

$x=(0)^2+1$

$x=1$

x-ints: (1,0)

$0=y^2+1$

$y^2=-1$

y-ints: none

$x=(-y)^2+1$

$x=y^2+1$

This graph has x-axis symmetry (equation remains unchanged when replaced with $-y$)

Ch. P Day 2 Notes

1. Review formulas

a) slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{rise}}{\text{run}} = \dfrac{\text{change in } y}{\text{change in } x}$

b) linear equation

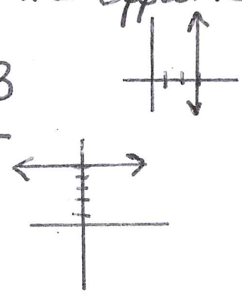
* point-slope: $y - y_1 = m(x - x_1)$
* slope-intercept: $y = mx + b$

parallel lines: slopes of lines are equal ($m_1 = m_2$)

perpendicular lines: slopes of lines are opposite reciprocal of each other ($m_1 = -\frac{1}{m_2}$)

vertical lines: example: $x = 3$

horizontal lines: example: $y = 5$



2. Solving Inequalities

Steps:

a) Rewrite as equation

b) Find critical values by setting both numerator and denominator equal to 0

c) Make sign line using critical values

d) Test values on sign line using the inequality.

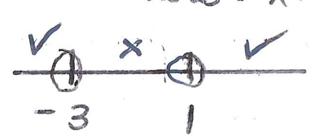
$\boxed{\text{Ex. 1}}$ Solve: $\dfrac{x+5}{x+3} < \dfrac{x+1}{x-1}$

$\dfrac{x+5}{x+3} = \dfrac{x+1}{x-1} \rightarrow \dfrac{x+5}{x+3} - \dfrac{x+1}{x-1} = 0$

$\dfrac{(x+5)(x-1)-(x+1)(x+3)}{(x+3)(x-1)} = 0$

$\dfrac{x^2+4x-5-x^2-4x+3}{(x+3)(x-1)} = 0 \rightarrow \dfrac{-2}{(x+3)(x-1)} = 0$

critical values: $x = -3, x = 1$



$\rightarrow \dfrac{x+5}{x+3} < \dfrac{x+1}{x-1}$

$\boxed{(-\infty, -3) \cup (1, \infty)}$

Ch. P Day 3 Notes

Domain: set of all values of x where function is defined.

*Restrictions vary depending on function type: rational, radical, trig functions.

Range: set of all values of y that have been mapped to value of x in domain of function

*Usually we'll have to look at graph to determine range or know information about parent graph and corresponding transformation.

Transformations: $y = -Af(Bx + C) + D$ ← shift up/down

reflection over x-axis

vertical stretch/compress

shift left/right

1. **Composition of functions**

*Determine domain before simplifying

$\boxed{\text{Ex.1}}$ $f(x) = 3x^2 \rightarrow$ Domain: $(-\infty, \infty)$

$g(x) = \sqrt{x-2} \rightarrow$ Domain: $[2, \infty)$

Determine $f(g(x))$ and domain:

$f(g(x)) = 3(\sqrt{x-2})^2 = 3(x-2)$ $\boxed{\text{Domain: } [2, \infty)}$

$\boxed{\text{Ex.2}}$ Find $g(f(x))$ and domain:

$g(f(x)) = \sqrt{3x^2 - 2} \rightarrow 3x^2 - 2 > 0$, $3x^2 = 2$ $x = \pm\sqrt{\frac{2}{3}}$

$+ \quad -\sqrt{2/3} \quad - \quad \sqrt{2/3} \quad +$

Domain: $(-\infty, -\sqrt{2/3}) \cup (\sqrt{\tfrac{2}{3}}, \infty)$

2) Test for symmetry

Even functions: $f(-x) = f(x)$ (symmetry about y-axis)

Odd functions: $f(-x) = -f(x)$ (symmetry about origin)

$\boxed{\text{Ex.3}}$ Test symmetry for

a) $f(x) = x^2 - 5$ $\quad f(-x) = (-x)^2 - 5$
$= x^2 - 5$ (even)

b) $f(x) = x - 2x^3$ $\quad f(-x) = (-x) - 2(-x)^3$
$= -x + 2x^3$

$f(-x) = -f(x)$
odd function

$-f(x) = -(x - 2x^3)$
$= -x + 2x^3$ ✓

3) Piecewise Functions

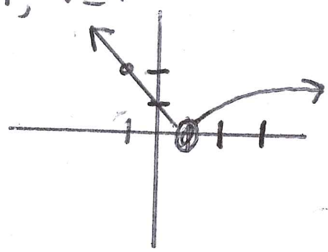
$\boxed{\text{Ex.4}}$ Graph $f(x) = \begin{cases} 1 - x, & x < 1 \\ \sqrt{x-1}, & x \geq 1 \end{cases}$, discuss domain and range

$y = 1 - x$

x	y
1	0
0	1
-1	2

$y = \sqrt{x-1}$

x	y
1	0
2	1
3	$\sqrt{2}$



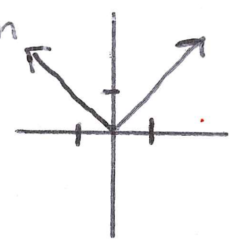
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Absolute Value Functions

* Can be rewritten as piecewise function

$y = |x| \rightarrow y = \begin{cases} x, & x > 0 \\ -x, & x \leq 0 \end{cases}$



$\boxed{\text{Ex.5}}$ $y = |2x + 6|$ $\quad y = \begin{cases} 2x + 6, & x > -3 \\ -(2x+6), & x \leq -3 \end{cases}$

$2x + 6 = 0$
$2x = -6$
$x = -3$