AB Calculus – Chapter P (Day 1) – Functions, Function Properties, and their Graphs

Evaluating a Function:

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x+2) =$

3. $f(x+h) =$

Use the graph $f(x)$ to answer the following.

4. $f(0) =$

$f(4) =$

$f(-1) =$

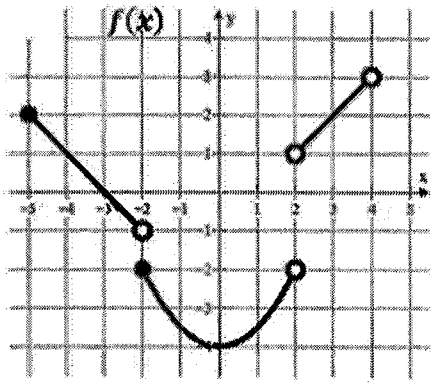
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$ when $x =$?

$f(x) = -3$ when $x =$?



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

5. slope $= 3$ and $(4, -2)$

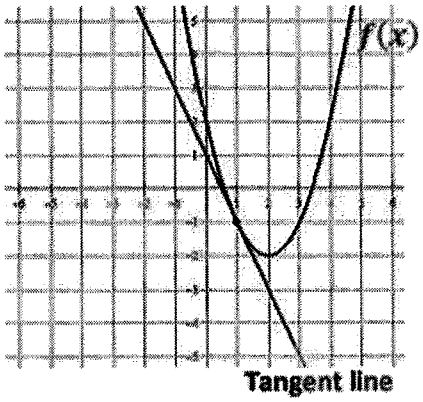
6. $m = -\frac{3}{2}$ and $f(-5) = 7$

7. $f(4) = -8$ and $f(-3) = 12$

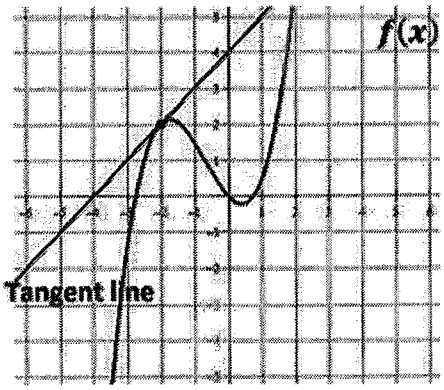
(2)

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$

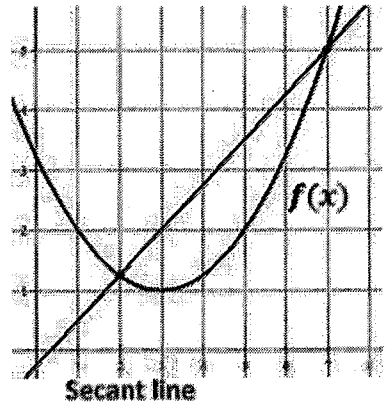


9. The line tangent to $f(x)$ at $x = -2$



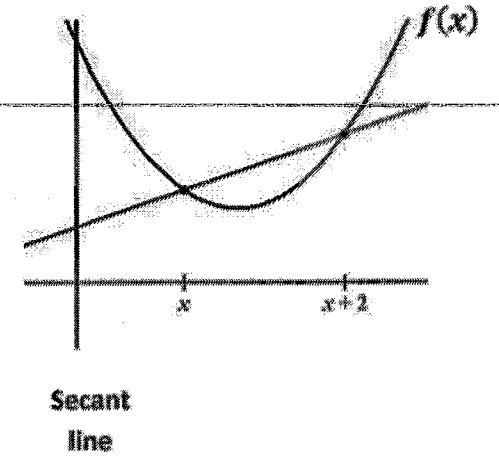
MULTIPLE CHOICE! Remember slope $= \frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?



A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$

11. Which choice represents the slope of the secant line shown?



A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

Find all Vertical Asymptotes, Horizontal Asymptotes, Holes, and x-intercepts (for Rational Functions)

I. **To Find Vertical Asymptotes:** Set Denominator Factors equal to zero and solve for x. (Make sure factors do not cancel with numerator)

II. **To Find Holes in graph:** Identify factors that cancels out between numerator and denominator. Set factor equal to zero and solve for x. To find the point (ordered pair), find y-value using the original function graph.

III. **To Find Horizontal Asymptote:** Compare Degrees between Numerator (N) and Denominator (D)

a) If **N = D**, then horizontal asymptote is **y = (ratio of leading coefficients)**

b) If **N < D**, the horizontal asymptote is **y = 0**

c) If **N > D**, there is **no horizontal asymptote.**

9. $f(x) = \frac{x+2}{3-x}$

Holes: _____

Vertical Asymptotes: _____

Horizontal Asymptote: _____

x-intercept: _____

10. $f(x) = \frac{4x-4}{x^2-9}$

Holes: _____

Vertical Asymptotes: _____

Horizontal Asymptote: _____

x-intercept: _____

11. $f(x) = \frac{x^2-2x}{x^3-5x^2+6x}$

Holes: _____

Vertical Asymptotes: _____

Horizontal Asymptote: _____

x-intercept: _____

12. $f(x) = \frac{5x^2+2}{3x^2-12}$

Holes: _____

Vertical Asymptotes: _____

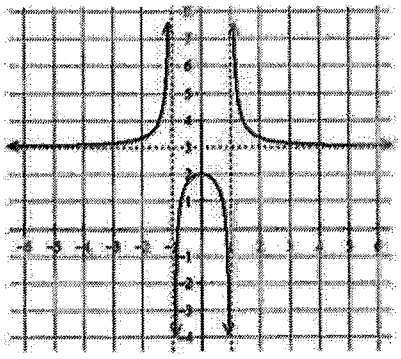
Horizontal Asymptote: _____

x-intercept: _____

(4)

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



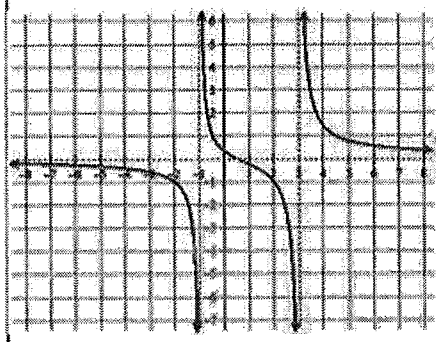
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



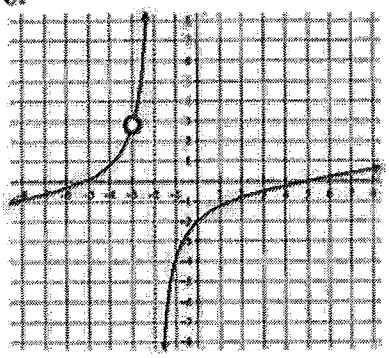
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

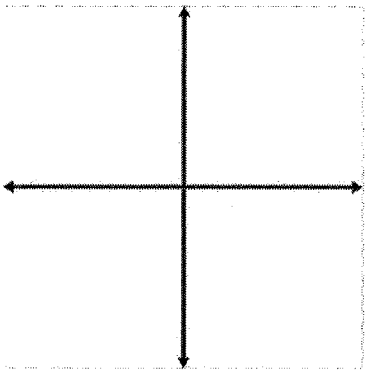
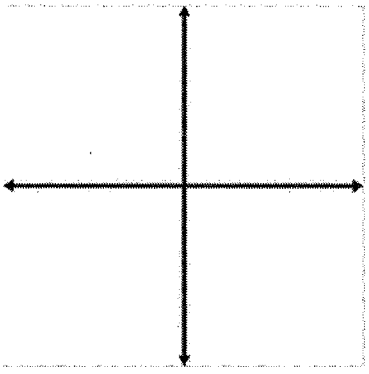
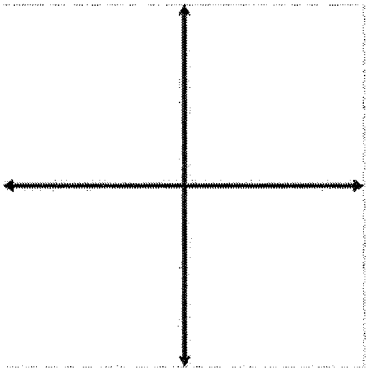
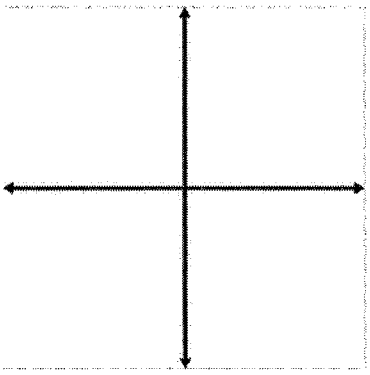
(C) I and III only

(D) II and III only

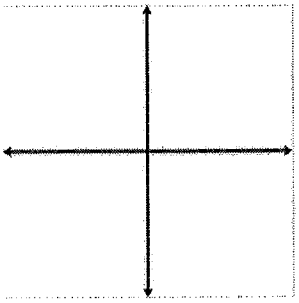
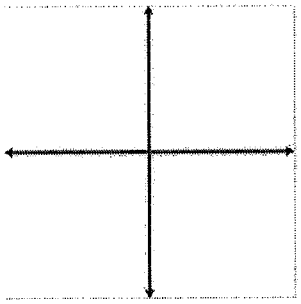
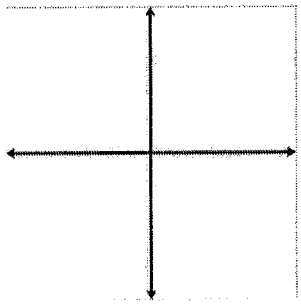
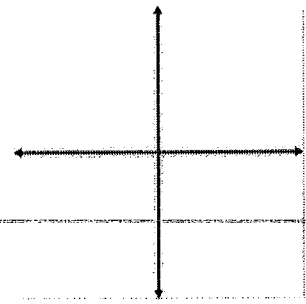
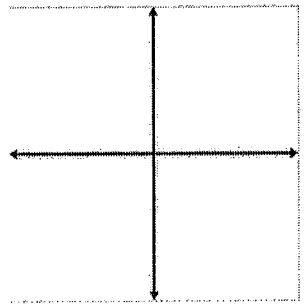
(E) I, II and III

AB Calculus – Chapter P (Day 2) – Parent Graphs of Families of Functions and Transformations

Review of Parent Functions and Graphs

Parent Function	Graph	Characteristics
Linear Function		Domain: _____ Range: _____ End Behavior: As $x \to$ _____, $f(x) \to$ _____ As $x \to$ _____, $f(x) \to$ _____
Quadratic Function		Domain: _____ Range: _____ End Behavior: As $x \to$ _____, $f(x) \to$ _____ As $x \to$ _____, $f(x) \to$ _____
Cubic Function		Domain: _____ Range: _____ End Behavior: As $x \to$ _____, $f(x) \to$ _____ As $x \to$ _____, $f(x) \to$ _____
Reciprocal (Rational Function)		Domain: _____ Range: _____ End Behavior: As $x \to$ _____, $f(x) \to$ _____ As $x \to$ _____, $f(x) \to$ _____

(6)

Parent Function	Graph	Characteristics
Square Root Function		Domain: _____ Range: _____ End Behavior: As $x \to$ _____, $f(x) \to$ _____ As $x \to$ _____, $f(x) \to$ _____
Cube Root Function		Domain: _____ Range: _____ End Behavior: As $x \to$ _____, $f(x) \to$ _____ As $x \to$ _____, $f(x) \to$ _____
Exponential Function		Domain: _____ Range: _____ End Behavior: As $x \to$ _____, $f(x) \to$ _____ As $x \to$ _____, $f(x) \to$ _____
Logarithmic Function		Domain: _____ Range: _____ End Behavior: As $x \to$ _____, $f(x) \to$ _____ As $x \to$ _____, $f(x) \to$ _____
Absolute Value Function		Domain: _____ Range: _____ End Behavior: As $x \to$ _____, $f(x) \to$ _____ As $x \to$ _____, $f(x) \to$ _____

Transformations of Parent Functions *Given parent function $f(x)$, the family of functions can be represented by the transformed function $f(x) = -\boldsymbol{A}f(x-\boldsymbol{h})+\boldsymbol{k}$

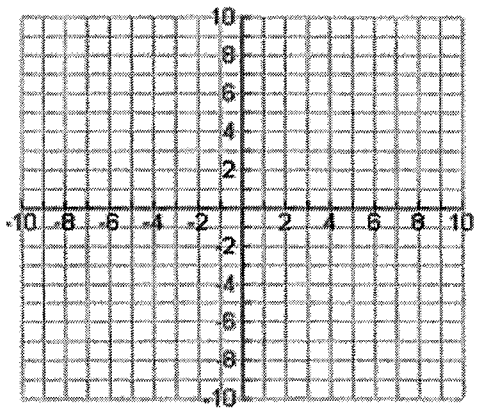
Transformations Review

$f(x+h)$ or $f(x-h)$	
$f(x)+k$ or $f(x)-k$	
$A * f(x)$	
$-f(x)$	

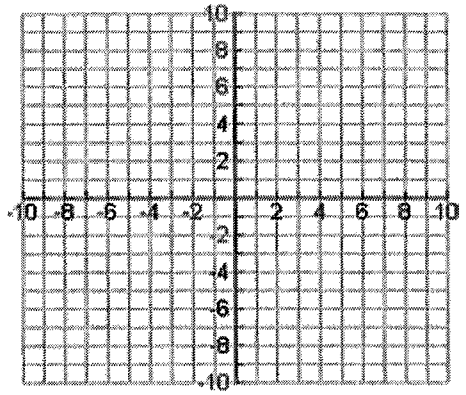
For the following functions, **i)** identify parent function, **ii)** list the transformations

iii) find domain and range **iv)** sketch the graph

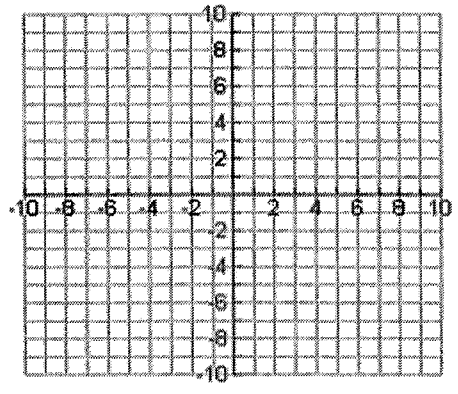
1) $f(x) = 2(x-3)^2+1$



2) $f(x) = -\sqrt{x+3}+1$

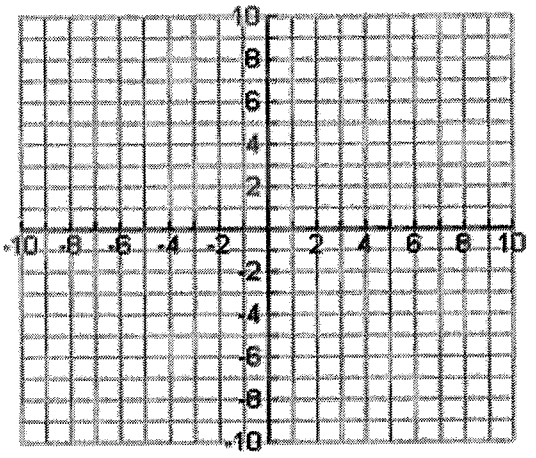


3) $f(x) = |x+4|-1$

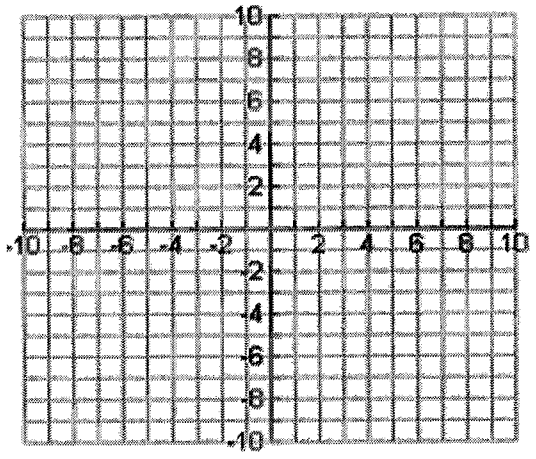


(8)

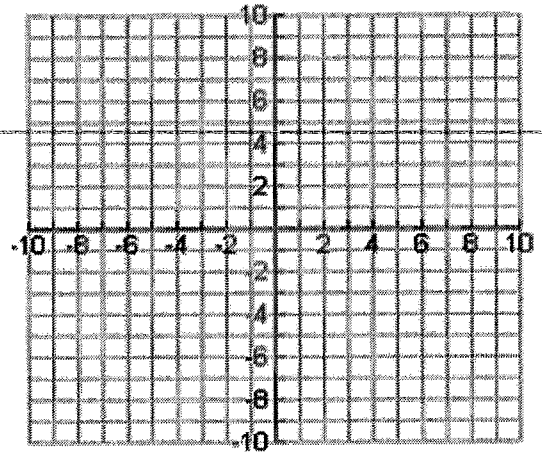
4) $f(x) = \dfrac{2}{x+1} - 3$



5) $f(x) = e^{x+2} - 3$



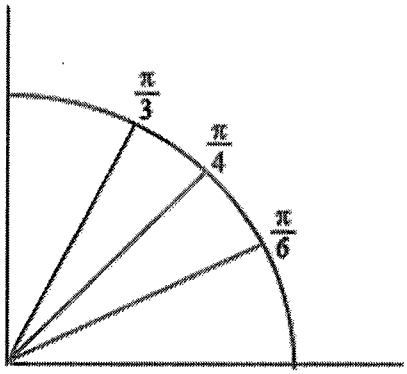
6) $f(x) = \ln(x-3) + 2$

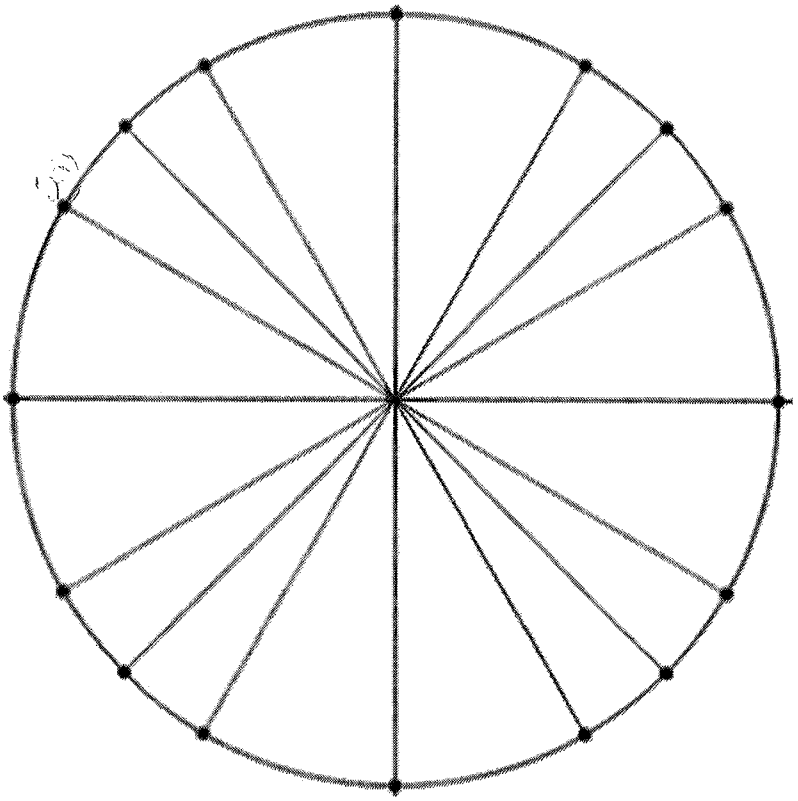


AB Calculus Ch. P Notes (Day 3) - Trig Review, Exponential Function Review, and Log Function Review

Trig Unit Circle Review

Quadrant I

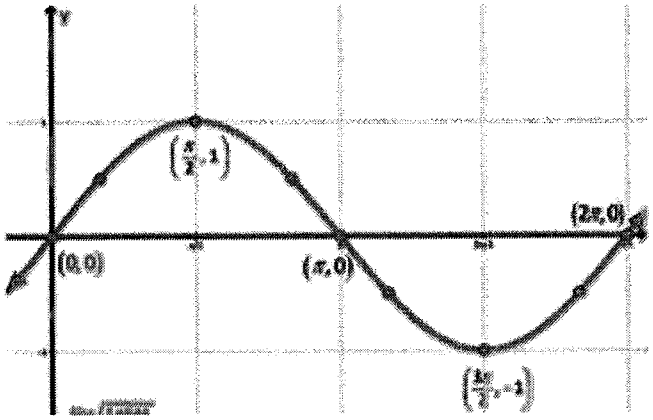




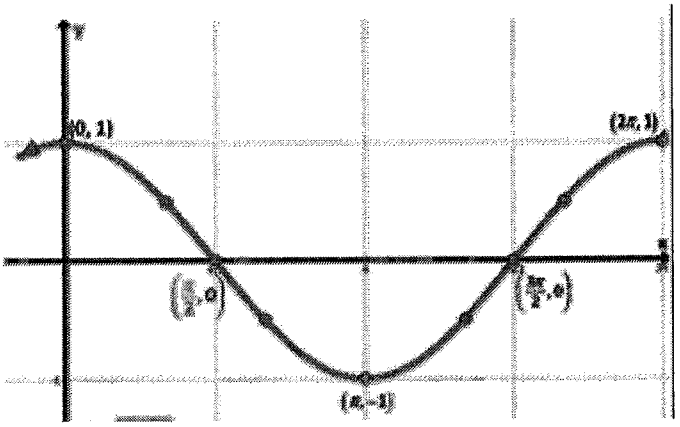
Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.		
31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

(10)

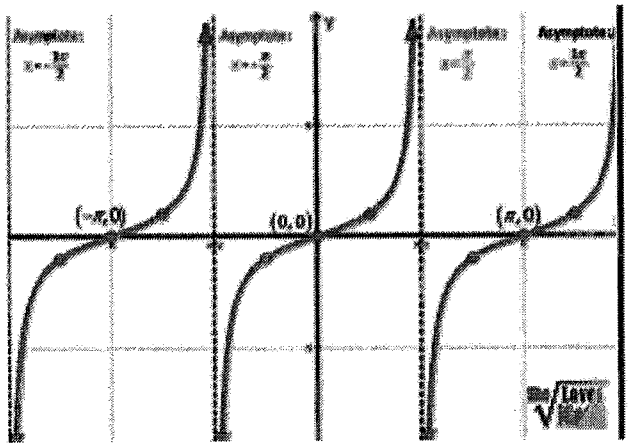
$y = \sin(x)$



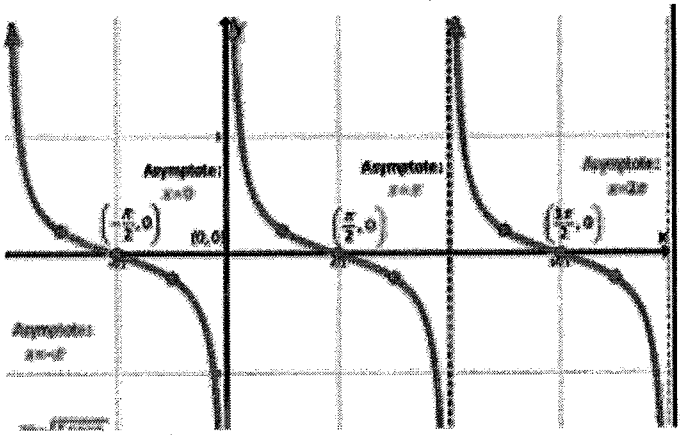
$y = \cos(x)$



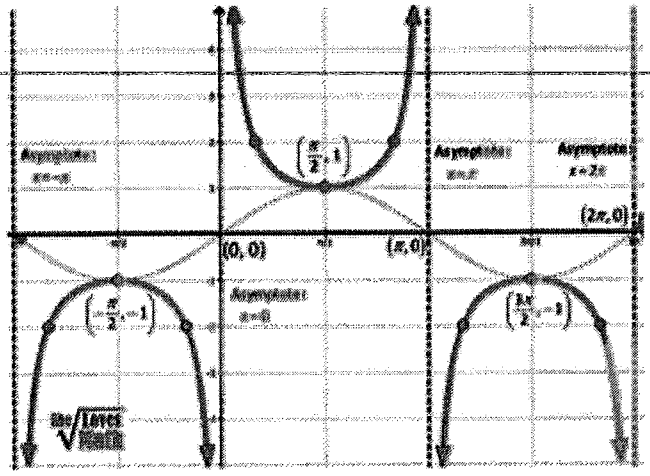
$y = \tan(x)$



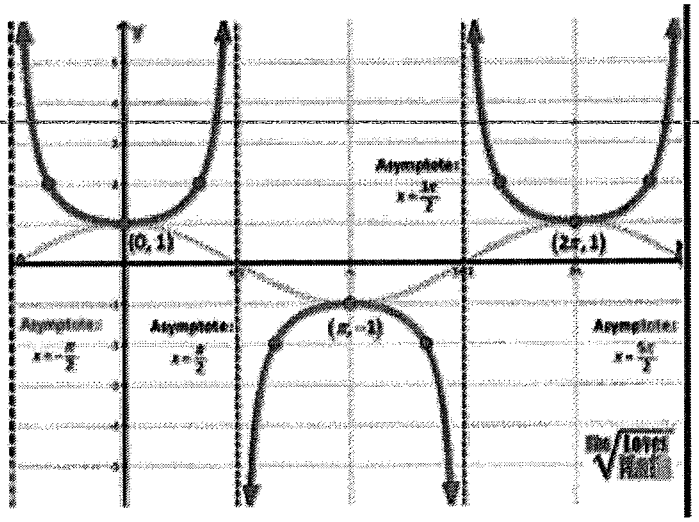
$y = \cot(x)$



$y = \csc(x)$



$y = \sec(x)$



Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$

Review of Exponential and Log properties:

If $b^x = b^y$ *then* $x = y$	$\ln(ab) = \ln a + \ln b$	$\ln x = log_e x$
$a^{m+n} = a^m * a^n$ $a^{m-n} = \frac{a^m}{a^n}$	$ln\left(\frac{a}{b}\right) = \ln a - \ln b$	• $\ln 1 = 0$ • $\ln e = 1$ • $\ln 0$ = *does not exist (vertical asymptote)*
$(a^m)^n = a^{m*n}$	$\ln a^n = n * \ln a$	*i) If* $\boldsymbol{y = b^x}$ *then* $\boldsymbol{log_b y = x}$ *ii) If* $\boldsymbol{log_b y = x}$ *then* $\boldsymbol{y = b^x}$

Solve the following:

52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

55) **Solve the following:**

a) $\log_{81}\sqrt{3} = x$	b) $\log_x 64^{\frac{3}{4}} = \frac{1}{2}$
c) $9 = 4 + \log_2(x + 3)$	d) $\frac{1}{3}\ln x = \ln 8$
e) $\log_b 8 = \log_b x + \log_b(x - 2)$	f) $4\ln(x + 3) = 12$
g) $e^{3x} = 6$	h) $7^{(x-4)} = 100$

AB Calculus – Ch. P.4 Miscellaneous Practice problems (Day 4)

1) For the piecewise function $p(x) = \begin{cases} \sqrt{x+4}, x \leq 5 \\ (x-5)^2, x > 5 \end{cases}$ find each of the following:

a) $p(-3)$ b) $p(0)$ c) $p(5)$ d) $p(10)$

2) Simplify the following given that all angles are between 0 and π:

a) $\sin\left(2\arccos\frac{\sqrt{2}}{2}\right)$ b) $\cos\left(\arccos 0 + \arcsin\frac{1}{2}\right)$

3) Rewrite each of the following as an algebraic expression with no trig functions involved. (Hint: draw triangles and use Pythagorean Theorem.)

a) $\sin(\arccos 2x)$ b) $\cot(\arcsin x)$ c) $\sin(\arctan 3x)$

4) Solve the following (use identities where necessary) given that $0 \leq x < 2\pi$

a) $\cos^2 x - \cos x + 1 = \sin^2 x$ b) $\sin x \tan x = \sin x$ c) $\sin x = \cos 2x - 1$

d) $\sin 4x = \frac{1}{2}$

e) $\cot^2 x - \csc x = 1$

f) $\sec^2 x + 2\sec x = 0$

g) $\sin x = \cos x$

h) $\sin 2x = \cos x$

i) $\cot x \cos^2 x = 2\cot x$

5) Write the equation of each line in point-slope form:

a. given a point and the slope: $(1, -2)$ $m = 3$

b. given two points: $(1, -3)$ $(-7, 1)$

c. given the point $(-1, -2)$ and is perpendicular to the line $y - 2 = 3(x + 1)$.

d. given the point $(-1, -2)$ and is parallel to the line $3x + 2y = 1$.

6)

Find the equation of the line in slope-intercept form, passing through the following points.

a. $(-3,6)$ and $(-1,2)$ b. $(-7,1)$ and $(3,-4)$ c. $\left(-2,\frac{2}{3}\right)$ and $\left(\frac{1}{2},1\right)$

7)

Write equations of the line through the given point a) parallel and b) normal to the given line.

a. $(5,-3)$, $x+y=4$ b. $(-6,2)$, $5x+2y=7$ c. $(-3,-4)$, $y=-2$

8)

Find an equation of the line containing $(4,-2)$ and parallel to the line containing $(-1,4)$ and $(2,3)$. Put your answer in general form.

9)

Find k if the lines $3x-5y=9$ and $2x+ky=11$ are a) parallel and b) perpendicular.

10)

Condense and write as a single logarithm.

a. $\log_4 3+5\log_4 x$

b. $\log 3-5\log x$

c. $\ln 2+4\ln x-3\ln y-\ln 8$

d. $2\ln 4-\frac{1}{2}\ln x+\ln y-3\ln 2$

11)

Expand each logarithmic expression.

a. $\log_7 \frac{5x}{y^3}$

b. $\log \frac{x^3}{9y^3}$

c. $\ln 27\sqrt[3]{a}$

d. $\ln \frac{3x^2+1}{2x^3-3x^3}$

AP Calc AB **Selected Ch. P Review problems (Summer Packet)**

1) Write the equation of the line with the following characteristics

a) passes through (3, -4) and (5, 2)

b) is a horizontal line with a y-intercept at -4	c) is a vertical line that passes through (7, -8)

d) has an x-intercept at 5 and a y-intercept at -3

e) is parallel to the line $3x + 4y = 7$, passes through the point (-6, 4) and is written in point-slope form

f) is perpendicular to the line $5x - 3y = 0$, passes through the point $\left(\frac{3}{4}, \frac{7}{8}\right)$ and is written in point-slope form

(18)

6) Find the value of $\frac{f(x+h)-f(x)}{h}$ for each of the following functions:

a) $f(x) = 3x + 7$

b) $f(x) = 3x^2 - 2x + 1$

c) $f(x) = \frac{6}{x}$

24) State horizontal asymptote(s), vertical asymptote(s) and hole(s) for each of the following:

a) $y = \frac{2x^2-7x-4}{6x^2+7x+2}$

b) $y = \frac{5x^2+20x}{x^3-3x^2-28x}$

Key

AP Calculus AB

Selected Ch. P Review problems from Summer Packet

1) Write the equation of the line with the following characteristics

a) passes through (3, -4) and (5, 2)

* $m = \frac{y_2 - y_1}{x_2 - x_1}$

point-slope form: $y - y_1 = m(x - x_1)$

$m = \frac{2-(-4)}{5-3} = \frac{6}{2} = 3$

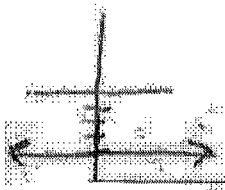
point: (5, 2)

slope: $m = 3$

$y - y_1 = m(x - x_1)$

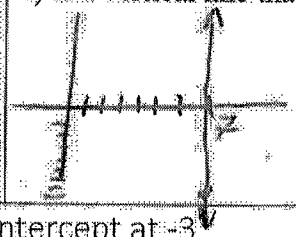
$y - 2 = 3(x - 5)$

b) is a horizontal line with a y-intercept at -4



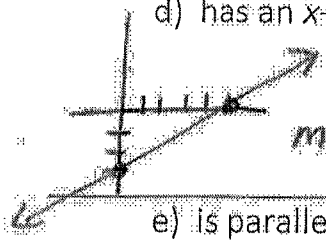
$y = -4$

c) is a vertical line that passes through (7, -8)



$x = 7$

d) has an x-intercept at 5 and a y-intercept at -3



points: (5, 0) and (0, -3)

$m = \frac{-3-0}{0-5} = \frac{-3}{-5} = \frac{3}{5}$

point: (5, 0)

slope: $m = 3/5$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{3}{5}(x - 5)$

e) is parallel to the line $3x + 4y = 7$, passes through the point (-6, 4) and is written in point-slope form

* Find slope of line $3x + 4y = 7$

$4y = -3x + 7$

$y = \frac{-3}{4}x + \frac{7}{4}$

$m = -3/4$

point: (-6, 4)

slope: $m = -3/4$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{-3}{4}(x + 6)$

f) is perpendicular to the line $5x - 3y = 0$, passes through the point $\left(\frac{3}{4}, \frac{7}{8}\right)$ and is written in point-slope form

* find slope of line $5x - 3y = 0$

$-3y = -5x$

$y = \frac{-5x}{-3} \rightarrow y = \frac{5}{3}x$

$m_1 = 5/3$ $\quad m_2 = \frac{-3}{5}$

point $\left(\frac{3}{4}, \frac{7}{8}\right)$

slope: $m = -3/5$

$y - y_1 = m(x - x_1)$

$y - \frac{7}{8} = \frac{-3}{5}\left(x - \frac{3}{4}\right)$

(20)

6) Find the value of $\frac{f(x+h)-f(x)}{h}$ for each of the following functions:

a) $f(x) = 3x + 7$

$f(\ ) = 3(\ ) + 7$

$f(x+h) = 3(x+h) + 7$

$\frac{3(x+h)+7-(3x+7)}{h}$

$\frac{3x+3h+7-3x-7}{h}$

$\frac{3h}{h} \rightarrow \boxed{3}$

b) $f(x) = 3x^2 - 2x + 1$

$f(\ ) = 3(\ )^2 - 2(\ ) + 1$

$f(x+h) = 3(x+h)^2 - 2(x+h) + 1$

$\frac{3(x+h)^2 - 2(x+h) + 1 - (3x^2 - 2x + 1)}{h}$

c) $f(x) = \frac{6}{x}$

$f(\ ) = \frac{6}{(\ )}$

$f(x+h) = \frac{6}{(x+h)}$

$\frac{\frac{6}{x+h} - \frac{6}{x}}{h}$

$\frac{\frac{6(x)}{x+h} - \frac{6}{x}(x+h)}{\frac{6x}{x(x+h)} - \frac{6(x+h)}{x(x+h)}}$

$\frac{\frac{6x-6x-6h}{x(x+h)}}{h}$

$\frac{6x-6(x+h)}{x(x+h)}$

$\frac{\frac{-6h}{x(x+h)}}{h} \cdot \frac{1}{h}$

$\boxed{\frac{-6}{x(x+h)}}$

24) State horizontal asymptote(s), vertical asymptote(s) and hole(s) for each of the following:

a) $y = \frac{2x^2-7x-4}{6x^2+7x+2} = \frac{(2x+1)(x-4)}{(3x+2)(2x+1)}$

x-int: $x = 4 \rightarrow (4, 0)$

VA: $3x+2=0 \quad x = -\frac{2}{3}$

hole: $2x+1=0 \quad x = -\frac{1}{2} \rightarrow (-\frac{1}{2}, -9)$

H.A: $y = \frac{2}{6} \rightarrow y = \frac{1}{3}$

b) $y = \frac{5x^2+20x}{x^3-3x^2-28x} \quad \frac{5x(x+4)}{x(x-7)(x+4)}$

x-int: none

VA: $x-7=0 \rightarrow x=7$

hole: $x=0$ and $x+4=0 \rightarrow x=-4$

$(0, \frac{+5}{7})$ and $(-4, \frac{-5}{11})$

H.A: $y=0$ ← * denominator has the higher degree