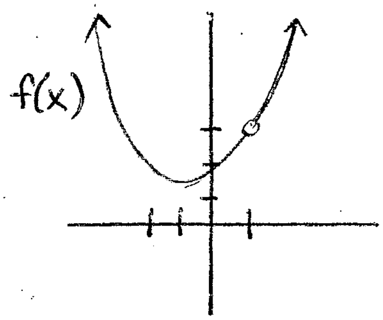
Ch. 1.2 Notes on Limits

Limit: y-value that a function or graph approaches as the x-value gets closer to a given constant



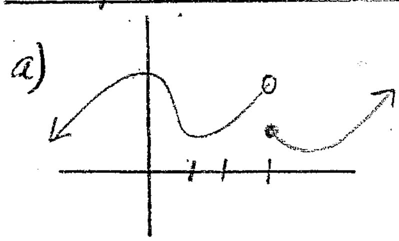
"The limit of f(x) as x approaches 1 is 3."

Notation: $\lim_{x \to 1} f(x) = 3$

* In order for a limit to exist, the graph must approach the same y-value from both directions.

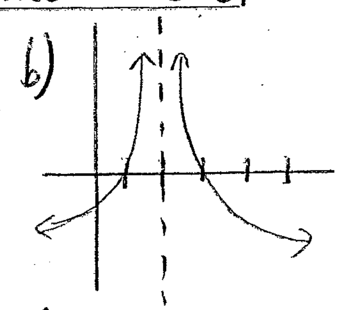
Examples where limit does not exist:

a)



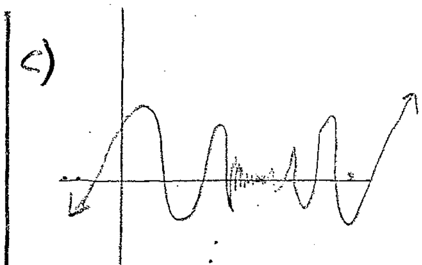
$\lim_{x \to 3} f(x) =$ ____
(____)

b)



$\lim_{x \to 2} f(x) =$ ____
(or ____)

c)



Graphs with oscillating behavio

ex: $f(x) = \sin\left(\frac{1}{x}\right)$

$\lim_{x \to 3} f(x) =$ ____

$\boxed{\text{Ex. 1}}$ Finding limit using calculator and table of values.

Find $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$

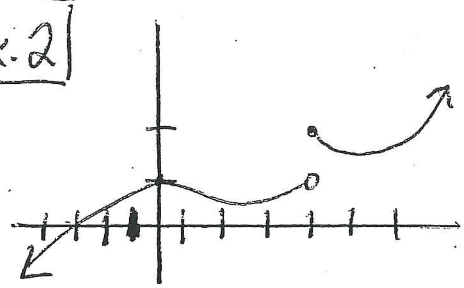
Steps:

1) Enter f(x) in "$Y_1 =$"
2) 2nd window (Tblset)
   ↳ Independent: $\boxed{\text{Ask}}$
3) 2nd Graph (Table)
4) Enter values of x approaching 1 from both sides: .9, .99, .999, 1.0001, 1.001, 1.01, 1.

X	0.9	0.99	0.999	1	1.0001	1.001	1.01	1.1
Y	2.71	2.97	2.997	und	3.0003	3.003	3.030	3.31

$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} =$ ___ since y-value approaches ___ from both sides the graph.

Ex. 2



a) $\lim_{x \to 0} f(x) =$

b) $\lim_{x \to -3} f(x) =$

c) $\lim_{x \to 4} f(x) =$

5) Sketch graph of function f with the given characteristics:

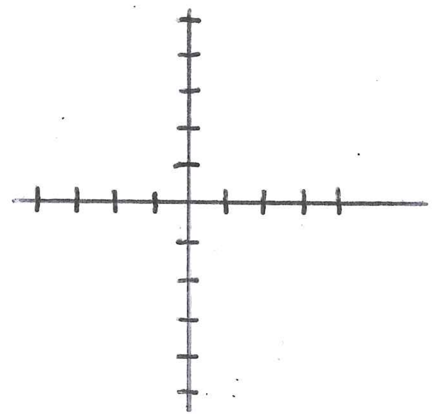
a) $f(-3) = 4$

b) $\lim_{x \to -3} f(x) = -2$

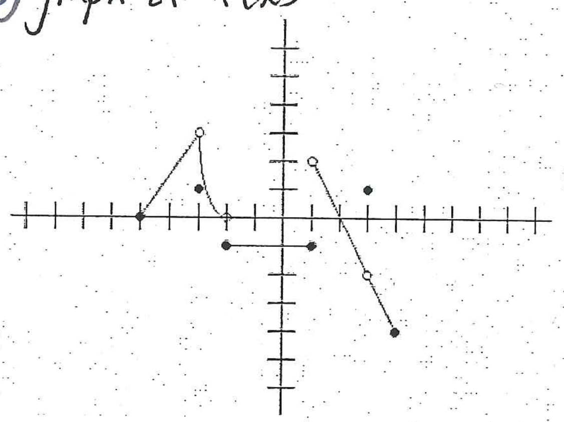
c) $\lim_{x \to 1} f(x) = \text{DNE}$

d) $f(3) = \text{DNE}$

e) $\lim_{x \to 3} f(x) = -1$



3) graph of f(x)



a) $f(-2) =$

b) $\lim_{x \to -2} f(x) =$

c) $\lim_{x \to -3} f(x) =$

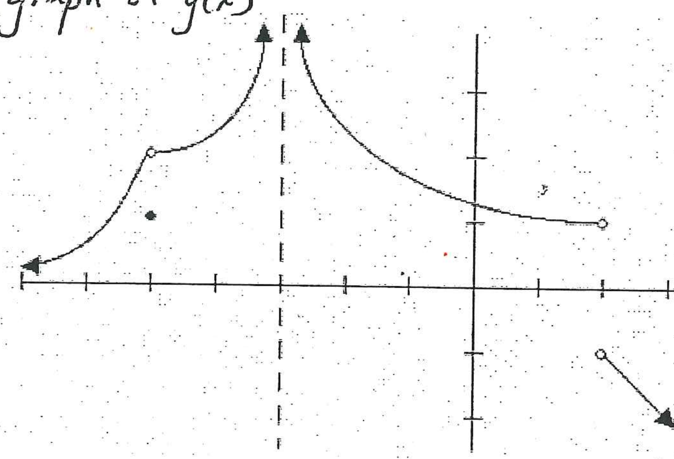
d) $\lim_{x \to 2} f(x) =$

e) $\lim_{x \to 3} f(x) =$

f) $f(3) =$

g) $\lim_{x \to -1.34} f(x) =$

4) graph of g(x)



a) $\lim_{x \to -5} g(x) =$

b) $\lim_{x \to -3} g(x) =$

c) $g(-5) =$

d) $\lim_{x \to 0} g(x) =$

e) $\lim_{x \to 2} g(x) =$

f) $g(2) =$

g) $\lim_{x \to 3} g(x) =$

Ch. 1.2 p. 54-55 1, 6, 9-21 all, 23, 25, 53-55, 63, 65, 66

6) $\lim_{x\to 4} \dfrac{[x/(x+1)]-(4/5)}{x-4}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
f(x)	0.041	0.0401	0.04	und	0.04	0.039	0.0392

$\lim_{x\to 4} f(x) = \boxed{0.04}$

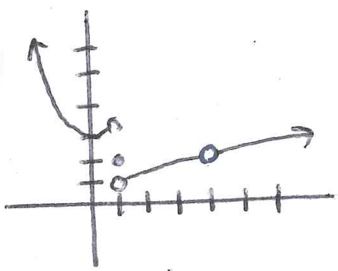
10) $\lim_{x\to 1} (x^2+2) = 3$

12) $\lim_{x\to 1} f(x) = 3$

14) $\lim_{x\to 3} \frac{1}{x-3} = \text{DNE}$

16) $\lim_{x\to 0} \sec x = 1$

18) $\lim_{x\to \pi/2} \tan x = \text{DNE}$

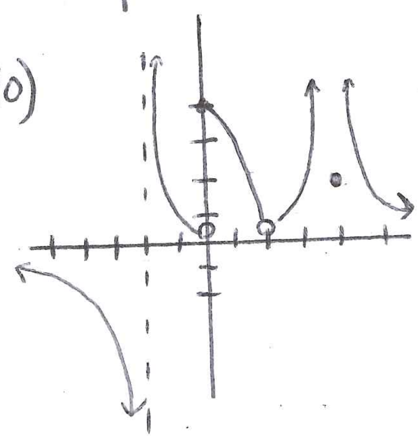
19) 

a) $f(1) = 2$

b) $\lim_{x\to 1} f(x) = \text{DNE}$

c) $f(4) = 2$

d) $\lim_{x\to 4} f(x) = 2$

20) 

a) $f(-2) = \text{DNE}$

b) $\lim_{x\to -2} f(x) = \text{DNE}$

c) $f(0) = 4$

d) $\lim_{x\to 0} f(x) = \text{DNE}$

e) $f(2) = \text{DNE}$

f) $\lim_{x\to 2} f(x) = 1/2$

g) $f(4) = 2$

h) $\lim_{x\to 4} f(x) = \text{DNE}$

21) Identify values of c where $\lim_{x\to c} f(c)$ exists

$\lim_{x\to c} f(x)$ exists for all $c \neq -3$

BC 22) $\lim_{x\to c} f(x)$ exists for all $c \neq -2, 0$

23) Sketch graph of f: $f(x) = \begin{cases} x^2 & , x \leq 2 \\ 8-2x & , 2 < x < 4 \\ 4 & , x \geq 4 \end{cases}$

Identify values of c where $\lim_{x \to c} f(x)$ exists

$y = x^2$

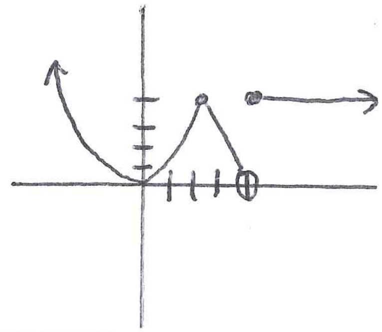
x	y
2	4
1	1
0	0
-1	1

$y = 8 - 2x$

x	y
2	4
3	2
4	0

$y = 4$

x	y
4	4
5	4
6	4



$\boxed{\lim_{x \to c} f(x) \text{ exists for for all } c \neq 4}$

BC

24) $f(x) = \begin{cases} \sin x & , x < 0 \\ 1 - \cos x & , 0 \leq x \leq \pi \\ \cos x & , x > \pi \end{cases}$

$y = \sin x$

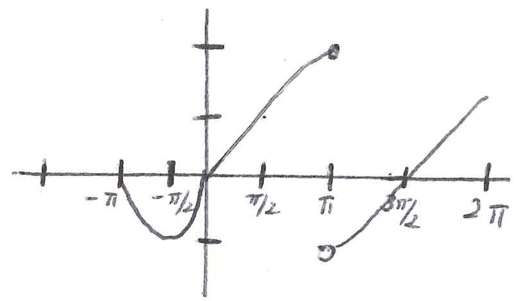
x	y
0	0
$-\pi/2$	-1
$-\pi$	0

$y = 1 - \cos x$

x	y
0	0
$\pi/2$	1
π	2

$y = \cos x$

x	y
π	-1
$3\pi/2$	0
2π	1



$\boxed{\lim_{x \to c} f(x) \text{ exists for all values of } c \neq \pi}$

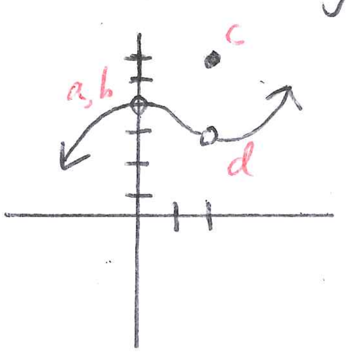
25) Sketch graph of function f with given characteristics

a) $f(0)$ undefined

b) $\lim_{x \to 0} f(x) = 4$

c) $f(2) = 6$

d) $\lim_{x \to 2} f(x) = 3$



53) $\lim_{x \to 8} f(x) = 25$ "The limit of the function as x approaches 8 is 25"

54) If $f(2) = 4$, no conclusion about $\lim_{x \to 2} f(x)$ can be reached without further information.

55) No conclusion can be reached. More information needed.

63) False

65) If $f(c)=L$, then $\lim_{x\to c} f(c)=L$ False

counterexample

$f(3)=4$

$\lim_{x\to 3} f(x)=6$

66) If $\lim_{x\to c} f(x)=L$, then $f(c)=L$ False

BC: #37-48

38) -1

40) 29/3

42) -1

44) 2

46) 0

48) 0