Ch. 1.3a Evaluating Limits Algebraically Notes:

Rules:

Suppose $\lim_{x \to c} f(x) = L$

1) $\lim_{x \to c} b = b$ 2) $\lim_{x \to c} b\,f(x) = bL$

I. To find limits for a function, first try to plug the argument (c) into the function. If the resulting value is a real number, then the value is the limit.

Ex. 1

a) $\lim_{x \to 2} (x^2 + 3x) = \boxed{10}$

b) $\lim_{x \to 2} 5 = \boxed{5}$

c) $\lim_{x \to -1} (3x^5 - 2x^2 + 7x + 4) = -3 - 2 - 7 + 4$
$= \boxed{-8}$

d) $\lim_{x \to \pi} x \cos x = \pi \cos \pi = \boxed{-\pi}$

II. Cancelling method:

*If you plug the argument (c) into the function and get a resulting value of $\frac{0}{0}$ (indeterminate form), then simplify further using factoring and cancelling

Steps:

1) plug in argument
2) $\frac{0}{0}$ means evaluate further
3) Try finding common factors to cancel
4) plug in argument into reduced function
5) Confirm resulting value is a real number

Ex. 2 $\lim_{x \to -2} \frac{x^2 + 5x + 6}{x + 2} = \frac{0}{0}$

$\lim_{x \to -2} \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}}$

$\lim_{x \to -2} x + 3 = -2 + 3 = \boxed{1}$

$\boxed{\text{Ex.3}}$ $\lim_{x \to 1} \frac{x^2+5x+6}{x-1} = \frac{12}{0} = \text{undefined}$

$\boxed{\text{Ex.4}}$ $\lim_{x \to 3} \frac{x^2-4}{x+2} = \frac{9-4}{3+2} = \frac{5}{5} = \boxed{1}$ $\quad \cancel{\lim_{x \to 3} \frac{(x+2)(x-2)}{(x+2)} = 3-2 = 1}$

$\boxed{\text{Ex.5}}$ $\lim_{x \to 2} \frac{x^2-4}{x+2} = \frac{4-4}{2+2} = \frac{0}{4} = \boxed{0}$

Ch. 1.3a Homework p.67-68 #1-25 odd, 37-51 odd

23) $f(x)=5-x \quad g(x)=x^3$

a) $\lim_{x\to 1} 5-x = 4$ b) $\lim_{x\to 4} x^3 = 4^3 = 64$

c) $\lim_{x\to 1} g(f(x)) = \lim_{x\to 1} g(5-x) = \lim_{x\to 1} (5-x)^3 = (5-1)^3 = 4^3 = 64$

25) $f(x)=4-x^2 \quad g(x)=\sqrt{x+1}$

a) $\lim_{x\to 1} 4-x^2 = 3$ b) $\lim_{x\to 3} \sqrt{x+1} = 2$

c) $\lim_{x\to 1} g(f(x)) = \lim_{x\to 1} g(4-x^2) = \lim_{x\to 1} \sqrt{4-x^2+1} = \sqrt{4-1+1} = 2$

41) write a simpler function: $g(x) = \frac{-2x^2+x}{x} = \frac{\cancel{x}(-2x+1)}{\cancel{x}} = -2x+1$

a) $\lim_{x\to 0} -2x+1 = 1$

b) $\lim_{x\to -1} -2x+1 = 3$

43) $g(x) = \frac{x^3-x}{x-1} = \frac{x(x^2-1)}{x-1} = \frac{x(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = x^2+x$

a) $\lim_{x\to 1} g(x) = \lim_{x\to 1} x^2+x = 2$

b) $\lim_{x\to -1} x^2+x = 1-1 = 0$

45) $\lim_{x\to -1} \frac{x^2-1}{x+1} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} = \boxed{-2}$

* Recall

$a^3-b^3=(a-b)(a^2+ab+b^2)$

47) $\lim_{x\to 2} \frac{x^3-8}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}} = \boxed{12}$

49) $\lim_{x\to5} \frac{x-5}{x^2-25} = \frac{0}{0}$

$\lim_{x\to5} \frac{\cancel{(x-5)}}{(x+5)\cancel{(x-5)}} = \boxed{\frac{1}{10}}$

51) $\lim_{x\to-3} \frac{x^2+x-6}{x^2-9} = \frac{0}{0}$

$\lim_{x\to-3} \frac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x-3)} = \frac{-5}{-6} = \boxed{\frac{5}{6}}$