Ch. 1.3b More Evaluating Limits Notes

A. Simplify using conjugate expression

*If there is a sum or difference of 2 terms in numerator, multiply numerator and denominator by the conjugate.

$\boxed{\text{Ex. 1}}$ $\lim\limits_{x\to 4} \dfrac{6-\sqrt{x+32}}{x-4}$

$$\lim_{x\to 4} \frac{6-\sqrt{x+32}}{x-4} \cdot \frac{(6+\sqrt{x+32})}{(6+\sqrt{x+32})} = \lim_{x\to 4} \frac{36-(x+32)}{(x-4)(6+\sqrt{x+32})}$$

$$\lim_{x\to 4} \frac{\cancel{4-x}^{\,-1}}{\cancel{(x-4)}(6+\sqrt{x+32})} = \frac{-1}{6+\sqrt{36}} = \frac{-1}{6+6} = \boxed{\frac{-1}{12}}$$

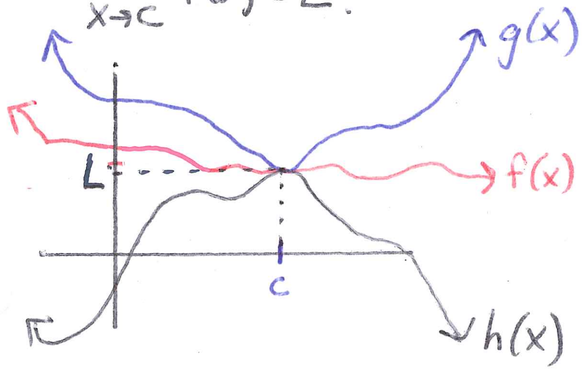
B. Simplify by finding common denominator

$\boxed{\text{Ex. 2}}$ $\lim\limits_{x\to 0} \dfrac{\frac{1}{x+4}-\frac{1}{4}}{x}$

$$\lim_{x\to 0} \frac{\frac{4-(x+4)}{4(x+4)}}{x}$$

$$\lim_{x\to 0} \frac{-\cancel{x}}{4(x+4)(\cancel{x})} = \frac{-1}{4(0+4)} = \boxed{\frac{-1}{16}}$$

C. <u>Squeeze Theorem</u>: If $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, and if $\lim\limits_{x\to c} h(x) = L = \lim\limits_{x\to c} g(x)$, then $\lim\limits_{x\to c} f(x) = L$.



$\boxed{\text{Ex. 3}}$ Let $h(x)=1$, $g(x)=x^2+1$.
If $h(x) \le f(x) \le g(x)$, find $\lim\limits_{x\to 0} f(x)$

Since $\lim\limits_{x\to 0} h(x) = 1$ and $\lim\limits_{x\to 0} g(x) = 1$, by squeeze theorem, $\lim\limits_{x\to 0} f(x) = 1$

Ch. 1.3b Homework p.68-69 #53-65 odds, 83, 84, 85, 87, 95, 96, 98, 105, 113.

55) $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{(\sqrt{x+5}+3)}{(\sqrt{x+5}+3)} = \lim_{x\to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$

$\lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x+5}+3)} = \frac{1}{\sqrt{4+5}+3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$

57) $\lim_{x\to 0} \frac{\frac{1}{3+x}-\frac{1}{3}}{x} = \lim_{x\to 0} \frac{\frac{3-(3+x)}{3(x+3)}}{x} = \lim_{x\to 0} \frac{\frac{-\cancel{x}}{3(x+3)}}{\cancel{x}} = \frac{-1}{3(0+3)}$

$= \boxed{\frac{-1}{9}}$

61) $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{(\cancel{x^2}+2x\Delta x+\Delta x^2-\cancel{2x}-2\Delta x+\cancel{1}-\cancel{x^2}+\cancel{2x}-\cancel{1})}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\cancel{\Delta x}(2x+\Delta x-2)}{\cancel{\Delta x}} = \boxed{2x-2}$

84) $f(x)=\sqrt{x}$ find $\lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x} \cdot \frac{(\sqrt{x+\Delta x}+\sqrt{x})}{(\sqrt{x+\Delta x}+\sqrt{x})} = \lim_{\Delta x\to 0} \frac{\cancel{x}+\cancel{\Delta x}-\cancel{x}}{\cancel{\Delta x}(\sqrt{x+\Delta x}+\sqrt{x})} = \frac{1}{\sqrt{x+0}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$

85) $f(x)=\frac{4}{x}$ $\lim_{\Delta x\to 0} \frac{\frac{4}{x+\Delta x}-\frac{4}{x}}{\Delta x}$ $\lim_{\Delta x\to 0} \frac{\frac{4x-4(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{\cancel{4x}-\cancel{4x}-4\cancel{\Delta x}}{(\cancel{\Delta x})(x)(x+\Delta x)} = \lim_{\Delta x\to 0} \frac{-4}{x(x+\Delta x)} = \frac{-4}{x(x+0)} = \boxed{\frac{-4}{x^2}}$

Ch. 1.3b HW continued:

95) Two functions that agree at all but one point will have limits that agree even if one function has a different point defined/undefined

Ex. $\lim_{x \to 0} x + 1 = 1$ and $\lim_{x \to 0} \frac{x(x+1)}{x} = 1$

96)

98) If a function f is squeezed between 2 functions, h and g, such that $h(x) \leq f(x) \leq g(x)$ and $h(x), g(x)$ have same limit L as $x \to c$, then $\lim_{x \to c} f(x) = L$

113) $\lim_{x \to 0} \frac{|x|}{x} = 1$ False