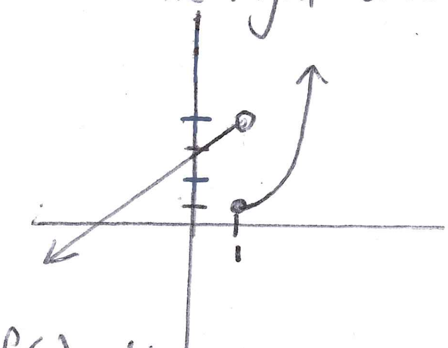
Ch. 1.4a Notes Continuity and One-sided Limits.

A. One-sided Limits - describes the function's behavior from the left or the right side of an x-value.

Ex. 1 $f(x) = \begin{cases} x^2, & x \geq 1 \\ x+3, & x < 1 \end{cases}$



a) Left-handed limit: $\lim_{x \to 1^-} f(x) = 4$

"The limit, (y-value that graph approaches), from the left side of $x=1$ is 4"

b) Right-handed limit: $\lim_{x \to 1^+} f(x) = 1$

"The limit, (y-value that graph approaches), from the right side of $x=1$ is 1"

* If $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$, then the limit of $f(x)$ as $x \to c$ exists.

B. Continuity - continuity exists if you can draw the graph without lifting your pencil.

3 Conditions for continuity: *Important*

1) $f(c)$ is defined → point exists

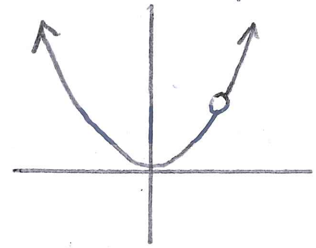
2) $\lim_{x \to c} f(x)$ exists $\left(\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)\right)$ → limit exists

3) $\lim_{x \to c} f(x) = f(c)$ → limit exists where point exists (conditions 1 and 2 agree)

* When checking for discontinuity, step through each of the continuity conditions in order. Stop once you reach a condition that fails.

C. Types of Discontinuity

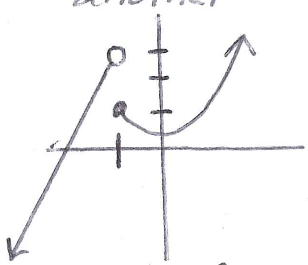
1. Removable Discontinuity (hole in graph) → a graph with removable discontinuity can be made continuous by filling in a single point.



*Removable discontinuity fails continuity condition #3

$\lim_{x \to c} f(x) \neq f(c)$

2. Nonremovable Discontinuity (step, jump discontinuity) - a discontinuity where the graph jumps from one connected piece of graph to another



*Nonremovable discontinuity fails continuity condition #2

2) $\lim_{x \to c} f(x)$ does not exist: $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

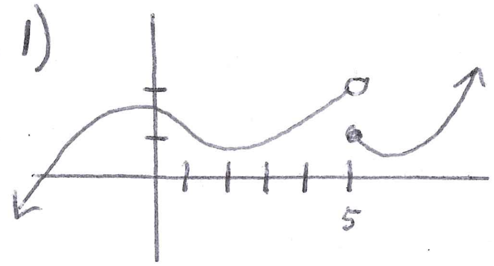
Since $\lim_{x \to -1^-} f(x) = 3$ and $\lim_{x \to -1^+} f(x) = 1$, $\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$

so $\lim_{x \to -1} f(x)$ does not exist

Class

Examples: Using continuity conditions, determine the reason why the following graphs are discontinuous. Then categorize each as removable or nonremovable discontinuity.

Ex. 1)

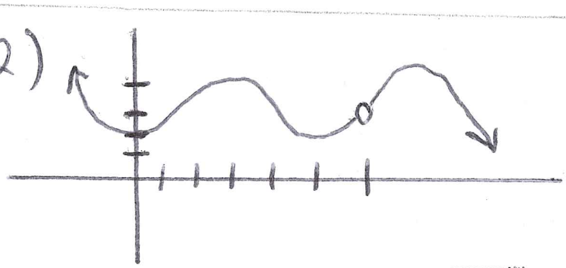


a) $f(5)=1$, defined ✓

b) $\lim_{x\to5^-} f(x)=2$, $\lim_{x\to5^+} f(x)=1$, so $\lim_{x\to5} f(x)$ DNE

* Since 2nd continuity condition fails, this graph has <u>nonremovable discontinuity</u>

Ex. 2)



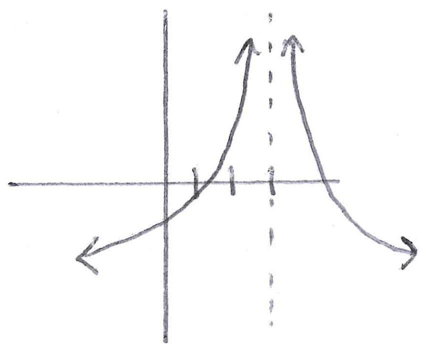
a) $f(6)$ undefined

b) $\lim_{x\to6} f(x)=3$

c) $\lim_{x\to6} f(x) \neq f(6)$

* Since 3rd continuity condition fails, this graph has <u>removable discontinuity</u>

Ex.3)

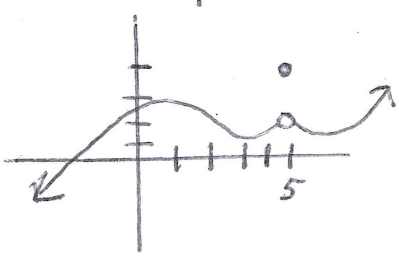


a) $f(3)$ undefined

b) $\lim_{x\to3} f(x) = \text{DNE}/+\infty$

* Since 2nd condition fails, <u>nonremovable discontinuity</u>

Ex.4)



a) $f(5)=4$

b) $\lim_{x\to5} f(x)=2$

c) $\lim_{x\to5} f(x)\neq f(5)$

* Since 3rd condition fails, <u>removable discontinuity</u>

Ex.5) Find point (x-value) of discontinuity for $f(x)=\frac{x^2-9}{x-3}$
Determine if removable/nonremovable discontinuity

$f(x)=\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$

Condit

Conditions:

a) $f(3)$ undefined

b) $\lim_{x\to3} f(x)=6$

c) $\lim_{x\to3} f(x)\neq f(3)$

Since condition #3 fails, function has removable discontinuity.

*Set $f(3)=6$ to make graph continuous

Ch. 1.4a Homework p.78-80 #1-17 odd, 25-33 odd
37-47 odd

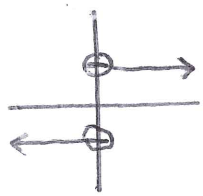
7) $\lim_{x\to 5^+} \frac{x-5}{x^2-25} \quad \lim_{x\to 5^+} \frac{\cancel{(x-5)}}{(x+5)\cancel{(x-5)}} = \boxed{\frac{1}{10}}$

*when evaluating one-sided limits algebraically, treat them the same as previous limit problems: (plug in, simplify, reduce, re-plug in)

* ONLY when the limit doesn't exist, then evaluate one-sided limits further by determing if $+\infty$ or $-\infty$.

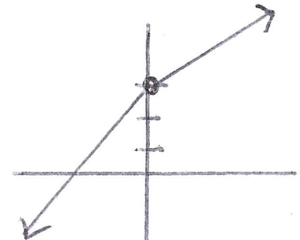
9) $\lim_{x\to -3^-} \frac{x}{\sqrt{x^2-9}} = \frac{0}{0} \to \frac{-3.01}{\sqrt{(-3.01)^2-9}} = \frac{-}{\sqrt{+}} = - \quad \lim_{x\to -3^-} \frac{x}{\sqrt{x^2-9}} = \boxed{-\infty}$

11) $\lim_{x\to 0^-} \frac{|x|}{x}$



$\lim_{x\to 0^-} \frac{|x|}{x} = \boxed{-1}$

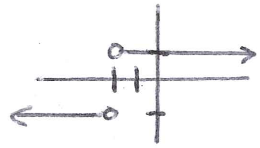
31) $f(x) = \begin{cases} 3-x, & x \le 0 \\ 3+\frac{1}{2}x, & x > 0 \end{cases} \quad [-1, 4]$



Since $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 3$, $f(x)$ is continuous on $[-1, 4]$

37) $f(x) = \frac{x}{x^2-x} = \frac{\cancel{x}}{\cancel{x}(x-1)}$ Removable discontinuity at $x=0$ (hole)
Nonremovable discontinuity at $x=1$ (vertical asymptote)

43) $f(x) = \frac{|x+2|}{x+2}$



Nonremovable discontinuity at $x=-2$

47) $f(x) = \begin{cases} \frac{1}{2}x+1, & x \le 2 \\ 3-x, & x > 2 \end{cases}$

a) $f(2) = 2$

b) $\lim_{x\to 2^-} f(x) = 2$, $\lim_{x\to 2^+} f(x) = 1$

Since $\lim_{x\to 2^-} f(x) \ne \lim_{x\to 2^+} f(x)$, $\lim_{x\to 2} f(x) = DNE$

Nonremovable discontinuity at $x=2$.