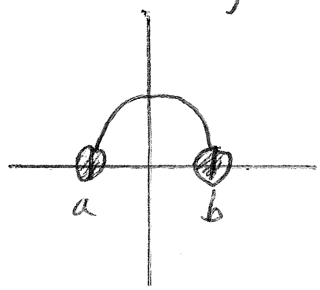
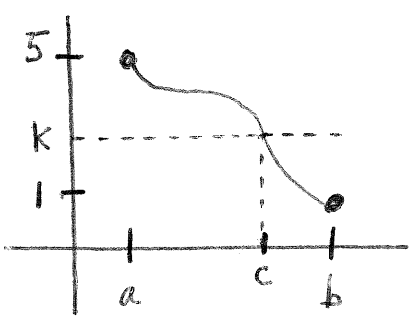
Ch. 1.4b Notes on Continuity continued

A. <u>Continuity on a closed interval</u>: If a function is continuous on an open interval (a,b) and $\lim_{x\to a^+} f(x) = f(a)$ and $\lim_{x\to b^-} f(x) = f(b)$, then function is continuous on closed interval $[a,b]$



B. <u>Intermediate Value Theorem (IVT)</u>

If f is continuous on a closed interval $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=k$.



*In other words, if a function is continuous, then it has to hit all the y-values between the endpoints.

$\boxed{\text{Ex.1}}$ Use IVT to show there is a zero $(y=0)$ in the interval $[0,1]$ for $f(x) = x^3+2x-1$.

Since $f(x)$ is a polynomial, function is continuous. Now, test endpoints:

$f(0) = 0^3+2(0)-1 = -1$

$f(1) = 1^3+2(1)-1 = 2$

Therefore, there must be a y-value of 0 between -1 and 2.

$\boxed{\text{Ex. 2}}$ Verify that IVT applies to $f(x) = \frac{x^2+x}{x-1}$ on interval $[5/2, 4]$ for $f(c)=6$. Then find c.

$f(x)$ is continuous on interval $[5/2, 4]$. $x \neq 1$, but the vertical asymptote lies outside the interval.

Test endpts: $f(5/2) = \frac{35}{6} \approx 5.8$
$f(4) = \frac{20}{3} \approx 6.7$

Therefore there must be an $f(c)=6$ between $x=5/2$ and $x=4$

set $f(x)=6$, solve for x

$$\frac{x^2+x}{x-1} = 6 \qquad x^2+x = 6x-6$$
$$x^2-5x+6=0$$
$$(x-2)(x-3)=0$$

$x = \cancel{2}, 3$

$c = 3$ ($x=2$ is outside the endpoints)

1.4b Homework p.80-81 # 55-63 odd, 75, 79, 83-89 odd, 90-92 all, 95, 104

57) $f(x)=\begin{cases} x^3, & x\le 2 \\ ax^2, & x>2 \end{cases}$

set $x^3=ax^2$, plug in $x=2$

$2^3=a(2)^2$

$\boxed{a=2}$

59) $f(x)=\begin{cases} 2 & x\le -1 \\ ax+b & -1<x<3 \\ -2 & x\ge 3 \end{cases}$

set $2=ax+b$ when $x=-1$

$2=a(-1)+b$

$b=2+a$

set $-2=ax+b$ when $x=3$

$-2=3a+b$

$-2=3a+(2+a)$

$-4=4a$

$\boxed{a=-1}$, $\boxed{b=1}$

75) Explain why f(x) has zero in given interval

$f(x)=\frac{1}{16}x^4-x^3+3 \quad [1,2]$

Since f(x) is a polynomial, f(x) is continuous.

Test endpts: $f(1)=\frac{33}{16}\approx 2.1$

$f(2)=-4$

By IVT, there must be at least 1 y-value of 0 between 2.1 and 4

79) use IVT and calculator to approximate zero in interval [0, 1]

$f(x)=x^3+x-1$ f(x) is continuous (polynomial)

Test endpts: $f(0)=-1$

$f(1)=1$

By IVT, there is at least one y-value of 1 between -1 and 1

*use calculator to estimate x-intercept

$\boxed{x\approx 0.6823}$

83) Verify IVT and find value of c.

$f(x) = x^2 + x - 1 \quad [0, 5] \quad f(c) = 11$

$f(x)$ is polynomial, so continuous function.

Test endpts: $f(0) = -1$, $f(5) = 29$] By IVT, there is at least 1 y-value of 11 ($f(c) = 11$) between -1 and 29.

* To find c, set $f(x) = 11$, solve for x

$x^2 + x - 1 = 11$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = 3, \cancel{x = -4}$

$\boxed{c = 3}$ c = −4 is outside interval

$\boxed{f(3) = 11}$

85) $f(x) = x^3 - x^2 + x - 2 \quad [0, 3] \quad f(c) = 4$

$f(x)$ is polynomial, therefore continuous function. Test endpts:

$f(0) = -2$, $f(3) = 19$ | By IVT, there is at least one y-value of 4 ($f(c) = 4$) between -2 and 19

* $x^3 - x^2 + x - 2 = 4$

$x^3 - x^2 + x - 6 = 0$

synthetic division

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & \downarrow & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & \boxed{0} \end{array}$$

$(x-2)(x^2 + x + 3) = 0$

$x = 2$

$\boxed{c = 2, \ f(2) = 4}$

87) How is continuity failing? (Determine specific continuity condition)

a) 2^{nd} condition: $\lim_{x \to c} f(x) = DNE$

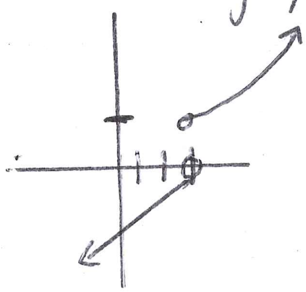
b) 1^{st} condition: $f(c)$ undefined

c) 3^{rd} condition: $\lim_{x \to c} f(x) \neq f(c)$

d) 2^{nd} condition: $\lim_{x \to c} f(x) = DNE$

89) Sketch graph with characteristics: $\lim_{x\to 3^+} f(x) = 1$

$\lim_{x\to 3^-} f(x) = 0$



$*$ $f(x)$ is not continuous at $x = 3$

b/c $\lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x)$, $\lim_{x\to 3} f(x) = \text{DNE}$

90) If f and g are continuous, then $f(x) + g(x)$ are continuous

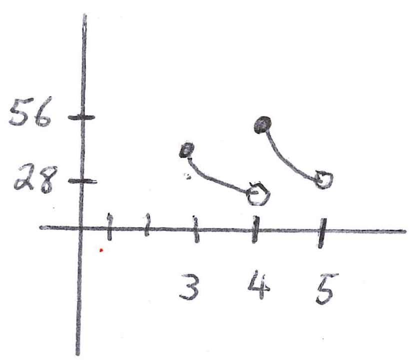
But $\frac{f(x)}{g(x)}$ not necessarily : ex: $\frac{x^2-1}{x}$

92) True

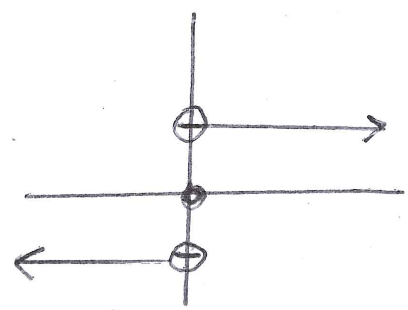
95) $\lim_{t\to 4^-} f(t) = 28$ $\lim_{t\to 4^+} f(t) = 56$

amount of chlorine at end of day 3

amount of chlorine at beginning of day 4



104) $\text{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$



a) $\lim_{x\to 0^-} \text{sgn}(x) = -1$

b) $\lim_{x\to 0^+} \text{sgn}(x) = 1$

c) $\lim_{x\to 0} \text{sgn}(x) = \text{DNE}$