Ch. 1.5 Notes: Limits approaching infinity (V.A.'s)

<u>Infinite Limits</u>: a limit where the function increases or decreases without bound as x approaches c

$\lim_{x \to c} f(x) = \pm\infty$

* If the limit as x approaches c from either right or left is $\pm\infty$, then $x = c$ is a vertical asymptote.

Rational functions: $y = \frac{f(x)}{g(x)}$ ← If $g(x)$ doesn't cancel, then it's an asymptote!

$\boxed{\text{Ex. 1}}$ Find all vertical asymptotes of $f(x) = \frac{x^2-3x+2}{x^2-4}$

$f(x) = \frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x+2)}$ vertical asymptote: $x = -2$

$\boxed{\text{Ex. 2}}$ Determine $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$ for $f(x) = \frac{x+1}{x-2}$

a) $\lim_{x \to 2^-} \frac{x+1}{x-2}$ →

<u>Steps</u>: (One-sided limits)
1) plug in x-value
2) If undefined, simplify and re-plug in value
3) If still undefined, plug decimal values, determine signs of numerator/denominator, decide if $+\infty/-\infty$

↓

$\lim_{x \to 2^-} \frac{x+1}{x-2} = \frac{1.9+1}{1.9-2} = \frac{+}{-} = \boxed{-\infty}$

b) $\lim_{x \to 2^+} \frac{x+1}{x-2} = \frac{2.1+1}{2.1-2} = \frac{+}{+} = \boxed{+\infty}$

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Ch. 1.5 Homework p. 88-90 #1-15 odd, 19, 21, 23, 25, 29-41 odd, 49, 51, 53-57, 60, 62, 63, 67-69

5) $f(x) = \frac{1}{x^2-9} = \frac{1}{(x+3)(x-3)}$

$\lim_{x\to -3^+} \frac{1}{x^2-9} = \frac{1}{(x+3)(x-3)} \rightarrow \frac{1}{(-2.9+3)(-2.9-3)} = \frac{1}{(+)(-)} = \frac{+}{-} = \boxed{-\infty}$

$\lim_{x\to -3^-} \frac{1}{(x+3)(x-3)} = \frac{1}{(-3.1+3)(-3.1-3)} = \frac{+}{(-)(-)} = \frac{+}{+} = \boxed{+\infty}$

7) $f(x) = \frac{x^2}{x^2-9} = \frac{x^2}{(x+3)(x-3)}$

$\lim_{x\to -3^+} \frac{x^2}{(x+3)(x-3)} = \frac{+}{(+)(-)} = \boxed{-\infty}$ $\quad \lim_{x\to -3^-} \frac{x^2}{(x+3)(x-3)} = \frac{+}{(-)(-)} = \boxed{+\infty}$

11) Find vertical asymptote $h(x) = \frac{x^2-2}{x^2-x-2} = \frac{(x^2-2)}{(x-2)(x+1)}$

$\lim_{x\to 2^-} \frac{x^2-2}{(x-2)(x+1)} = \frac{1.9^2-2}{(1.9-2)(1.9+1)} = \frac{+}{(-)(+)} = \frac{+}{-} = \boxed{-\infty}$

$\lim_{x\to 2^+} \frac{x^2-2}{(x-2)(x+1)} = \frac{2.1^2-2}{(2.1-2)(2.1+1)} = \frac{+}{(+)(+)} = \boxed{+\infty}$

$\lim_{x\to -1^+} \frac{x^2-2}{(x-2)(x+1)} = \frac{(-0.9)^2-2}{(-0.9-2)(-0.9+1)} = \frac{-}{(-)(+)} = \boxed{+\infty}$

$\lim_{x\to -1^-} \frac{(-1)^2-2}{(-1.1-2)(-1.1+1)} = \frac{-}{(-)(-)} = \frac{-}{+} = \boxed{-\infty}$

$\boxed{\text{V.A. at } x=2,\ x=-1}$

35) $\lim\limits_{x\to 3^+} \dfrac{x^2}{x^2-9} \to \dfrac{9}{0} \to \dfrac{x^2}{(x+3)(x-3)} \to \dfrac{(3.1)^2}{(3.1+3)(3.1-3)} = \dfrac{+}{(+)(+)} = \boxed{+\infty}$

37) $\lim\limits_{x\to -3^-} \dfrac{x^2+2x-3}{x^2+x-6} = \lim\limits_{x\to -3^-} \dfrac{\cancel{(x+3)}(x-1)}{\cancel{(x+3)}(x-2)} = \dfrac{-3-1}{-3-2} = \dfrac{-4}{-5} = \boxed{\dfrac{4}{5}}$

41) $\lim\limits_{x\to 0^-} \left(1+\dfrac{1}{x}\right) \to 1+\dfrac{1}{0} \to 1+\dfrac{1}{-0.1} = 1-10 = - \to \boxed{-\infty}$

54) Vertical Asymptote: The line $x=c$ is a vertical asymptote if the graph approaches $\pm\infty$ as x approaches c.

56) Not all rational functions have vertical asymptote:
(see #15: $g(t) = \dfrac{t-1}{t^2+1}$)

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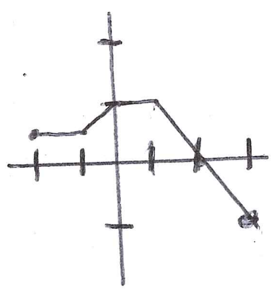
57) $f(x)$

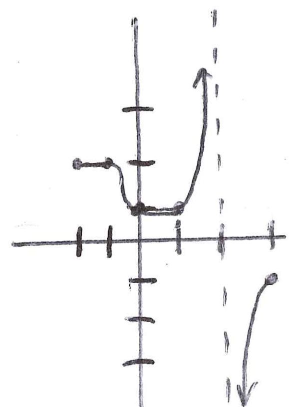


$f(x)$

x	$f(x)$
-2	$1/2$
-1	$1/2$
0	1
1	1
2	0
3	-1

$g(x) = \dfrac{1}{f(x)}$

x	$g(x)$
-2	$1/\tfrac{1}{2} = 2$
-1	$1/\tfrac{1}{2} = 2$
0	$1/1 = 1$
1	$1/1 = 1$
2	$1/0 =$ und.
3	$1/{-1} = -1$



60) $x\%$ of illegal drugs

$C = \dfrac{528x}{100-x} \quad 0 \le x < 100$

a) $x=25 \to C(25) = \$176$ million

b) $x=50 \to C(50) = \$528$ million

c) $x=75 \to C(75) = \$1584$ million

d) $\lim\limits_{x\to 100^-} \dfrac{528x}{100-x} = \dfrac{52800}{0} \to \dfrac{+}{+} = +\infty$

Seizing 100% of illegal drugs is not possible.

62) $r = \frac{2x}{\sqrt{625-x^2}}$ ft/sec.

a) Find r when $x=7$ $\qquad r = \frac{14}{\sqrt{625-49}} = \frac{7}{12}$ ft/s

b) Find r when $x=15$ $\qquad r = \frac{30}{\sqrt{625-15^2}} = \frac{3}{2}$ ft/s

c) $\lim_{x \to 25^-} \frac{2x}{\sqrt{625-x^2}} = \frac{50}{0} \to \frac{2(24.9)}{\sqrt{625-(24.9)^2}} = \frac{+}{+} = \boxed{+\infty}$

True/False

67) $f(x) = \frac{p(x)}{x-1}$ has vertical asymptote at $x=1$

*False $f(x) = \frac{x\cancel{(x-1)}}{\cancel{(x-1)}}$

68) All Rational function has vertical asymptote

*False : $f(x) = \frac{1}{x^2+1}$

69) All polynomial Functions have no vertical asymptote: True