Ch. 3.5 Notes Limits at Infinity (End behavior)

A. Checking for horizontal asymptotes: (H.A.)

Method 1:

If $f(x) = \frac{p(x)}{q(x)}$, compare degrees between numerator and denominator

- If denominator degree > numerator degree, H.A. is $y=0$
- If denominator degree = numerator degree, H.A. is $y=\frac{a}{b}$ (leading coefficients)
- If numerator degree > denominator degree then no Horizontal Asymptote: limit at infinity is either $+\infty$ or $-\infty$.

Method 2:

Divide every term by variable with highest degree

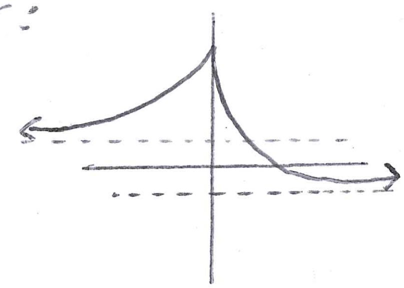
Ex.1 $f(x)=\frac{3x^2+2}{4x^3-5x}$ $\lim\limits_{x\to\infty}\frac{3x^2+2}{4x^3-5x} \to \lim\limits_{x\to\infty}\frac{\frac{3x^2}{x^3}+\frac{2}{x^3}}{\frac{4x^3}{x^3}-\frac{5x}{x^3}} = \frac{0}{4} = \boxed{0}$

Ex.2 $f(x)=\frac{2x^3+x^2}{3x^2-2x}$ $\lim\limits_{x\to\infty}\frac{\frac{2x^3}{x^3}+\frac{x^2}{x^3}}{\frac{3x^2}{x^3}-\frac{2x}{x^3}} = \boxed{+\infty}$

Ex.3 $f(x)=\frac{4x^3-2x^2}{5x^3-3x+9}$ $\lim\limits_{x\to\infty}\frac{\frac{4x^3}{x^3}-\frac{2x^2}{x^3}}{\frac{5x^3}{x^3}-\frac{3x}{x^3}+\frac{9}{x^3}} = \boxed{\frac{4}{5}}$

* Horizontal Asymptote is a description of end behavior, not a boundary that the graph cannot cross. A function can never cross a vertical asymptote, but it might cross horizontal asymptote.

B. Finding H.A. with radicals in denominator:



Ex. 4) Find H.A. for $y = \dfrac{-2x+6}{\sqrt{5x^2+1}}$

* Test $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$

* Divide each term by variable with degree of highest order

$$\lim_{x\to\infty} \frac{-2x+6}{\sqrt{5x^2+1}} = \lim_{x\to\infty} \frac{\frac{-2x}{x}+\frac{6}{x}}{\sqrt{\frac{5x^2}{x^2}+\frac{1}{x^2}}} = \frac{-2+0}{\sqrt{5+0}} = \boxed{\frac{-2}{\sqrt{5}}}$$

$$\lim_{x\to-\infty} \frac{-2x+6}{\sqrt{5x^2+1}} = \lim_{x\to-\infty} \frac{\frac{-2x}{x}+\frac{6}{x}}{-\sqrt{\frac{5x^2}{x^2}+\frac{1}{x^2}}} = \frac{-2+0}{-\sqrt{5+0}} = \boxed{\frac{2}{\sqrt{5}}}$$

*Need to add a negative sign when evaluating limit for $x\to-\infty$

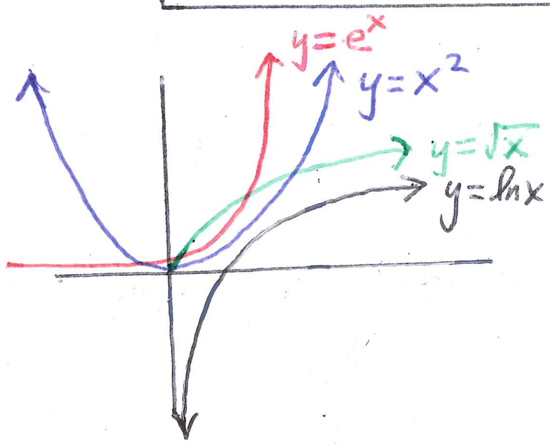
2 horizontal asymptotes

$y = \frac{2}{\sqrt{5}}$, $y = \frac{-2}{\sqrt{5}}$

C. Comparative Growth Rates

* Different families of functions grow at different rates as x approaches $+\infty$.

$$\boxed{\text{Logs} < \text{Radicals} < \text{Polynomial} < \text{Exponential}}$$



Ex. 5 $\lim\limits_{x\to\infty} \dfrac{\sqrt{50000x+10000}}{x^2} = \dfrac{\text{radical}}{\text{polynomial}} = \boxed{0}$

Ex. 6 $\lim\limits_{x\to\infty} \dfrac{-e^{2x}}{10000x^4+x^5} = \dfrac{\text{exponential}}{\text{polynomial}} = \boxed{-\infty}$

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19) a) $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^2-4} = \lim_{x\to\infty} \frac{\overset{\to 0}{\frac{5}{x^2}} - \overset{\to 0}{\frac{2x^{3/2}}{x^2}}}{\underset{3}{\frac{3x^2}{x^2}} - \overset{\to 0}{\frac{4}{x^2}}} = \frac{0}{3} = \boxed{0}$

b) $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^{3/2}-4} = \lim_{x\to\infty} \frac{\frac{5}{x^{3/2}} - \frac{2x^{3/2}}{x^{3/2}}}{\frac{3x^{3/2}}{x^{3/2}} - \frac{4}{x^{3/2}}} = \frac{0-2}{3-0} = \boxed{\frac{-2}{3}}$

c) $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x-4} = \lim_{x\to\infty} \frac{\frac{5}{x^{3/2}} - \frac{2x^{3/2}}{x^{3/2}}}{\frac{3x}{x^{3/2}} - \frac{4}{x^{3/2}}} = \frac{0-2}{0-0} = \frac{-2}{0} \Rightarrow \boxed{-\infty}$

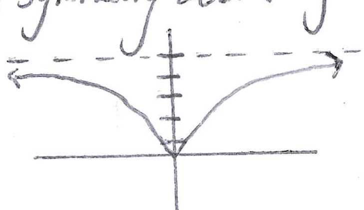
25) $\lim_{x\to-\infty} \frac{5x^2}{x+3}$ $\quad \frac{5(-\infty)^2}{(-\infty)+3} = \frac{+}{-} = \boxed{-\infty}$

27) $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2-x}}$ $\quad \lim_{x\to-\infty} \frac{\frac{x}{x}}{-\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}} = \frac{1}{-\sqrt{1-0}} = \boxed{-1}$

29) $\lim_{x\to-\infty} \frac{2x+1}{\sqrt{x^2-x}} = \lim_{x\to-\infty} \frac{\frac{2\cancel{x}}{\cancel{x}} + \frac{1}{x}}{-\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}} = \frac{2+0}{-\sqrt{1-0}} = \boxed{-2}$

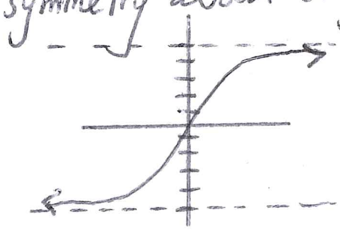
54) $\lim_{x\to\infty} f(x) = 5$. Find $\lim_{x\to-\infty} f(x)$ for each condition

a) symmetry about y-axis



$\lim_{x\to-\infty} f(x) = 5$

b) symmetry about origin



$\lim_{x\to-\infty} f(x) = -5$