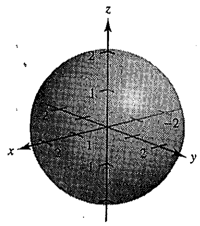
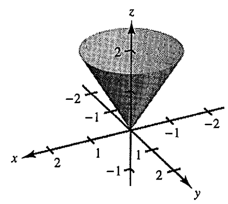
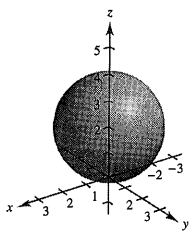
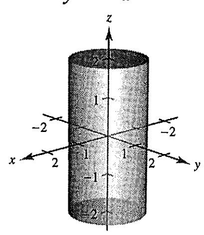
29. $(4, 0, \pi/2)$ **31.** $(4\sqrt{2}, 2\pi/3, \pi/4)$ **33.** $(4, \pi/6, \pi/6)$
35. $(\sqrt{6}, \sqrt{2}, 2\sqrt{2})$ **37.** $(0, 0, 12)$ **39.** $(\frac{5}{2}, \frac{5}{2}, -5\sqrt{2}/2)$
41. $\rho = 3 \csc \phi \csc \theta$ **43.** $\rho = 6$
45. $\rho = 3 \csc \phi$ **47.** $\tan^2 \phi = 2$
49. $x^2 + y^2 + z^2 = 4$ **51.** $3x^2 + 3y^2 - z^2 = 0$

53. $x^2 + y^2 + (z - 2)^2 = 4$ **55.** $x^2 + y^2 = 1$

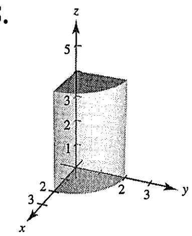



57. $(4, \pi/4, \pi/2)$ **59.** $(4\sqrt{2}, \pi/2, \pi/4)$
61. $(2\sqrt{13}, -\pi/6, \arccos[3/\sqrt{13}])$ **63.** $(13, \pi, \arccos[5/13])$
65. $(10, \pi/6, 0)$ **67.** $(36, \pi, 0)$
69. $(3\sqrt{3}, -\pi/6, 3)$ **71.** $(4, 7\pi/6, 4\sqrt{3})$

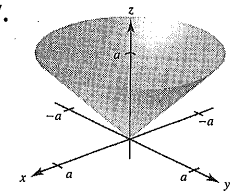
	Rectangular	*Cylindrical*	*Spherical*
73.	(4, 6, 3)	(7.211, 0.983, 3)	(7.810, 0.983, 1.177)
75.	(4.698, 1.710, 8)	$(5, \pi/9, 8)$	(9.434, 0.349, 0.559)
77.	(−7.071, 12.247, 14.142)	(14.142, 2.094, 14.142)	$(20, 2\pi/3, \pi/4)$
79.	(3, −2, 2)	(3.606, −0.588, 2)	(4.123, −0.588, 1.064)
81.	$(\frac{5}{2}, \frac{4}{3}, -\frac{3}{2})$	(2.833, 0.490, −1.5)	(3.206, 0.490, 2.058)
83.	(−3.536, 3.536, −5)	$(5, 3\pi/4, -5)$	(7.071, 2.356, 2.356)
85.	(2.804, −2.095, 6)	(−3.5, 2.5, 6)	(6.946, 5.642, 0.528)

87. d **88.** e **89.** c **90.** a **91.** f **92.** b
93. Rectangular to cylindrical:
$r^2 = x^2 + y^2, \tan \theta = y/x, z = z$
Cylindrical to rectangular:
$x = r \cos \theta, y = r \sin \theta, z = z$
95. Rectangular to spherical:
$\rho^2 = x^2 + y^2 + z^2, \tan \theta = y/x, \phi = \arccos(z/\sqrt{x^2 + y^2 + z^2})$
Spherical to rectangular:
$x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta, z = \rho \cos \phi$
97. (a) $r^2 + z^2 = 16$ (b) $\rho = 4$
99. (a) $r^2 + (z - 1)^2 = 1$ (b) $\rho = 2 \cos \phi$
101. (a) $r = 4 \sin \theta$ (b) $\rho = 4 \sin \theta/\sin \phi = 4 \sin \theta \csc \phi$
103. (a) $r^2 = 9/(\cos^2 \theta - \sin^2 \theta)$
(b) $\rho^2 = 9 \csc^2 \phi/(\cos^2 \theta - \sin^2 \theta)$

105.



107.



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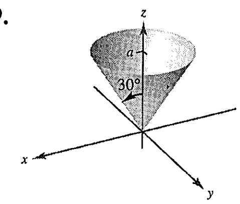
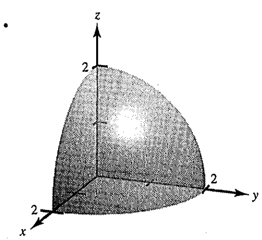


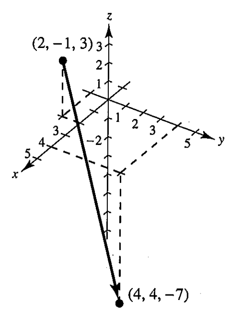
111.



113. Rectangular: $0 \le x \le 10$, $0 \le y \le 10$, $0 \le z \le 10$ **115.** Spherical: $4 \le \rho \le 6$
117. Cylindrical: $r^2 + z^2 \le 9, r \le 3 \cos \theta, 0 \le \theta \le \pi$
119. False. $\theta = c$ represents a vertical half-plane.
121. False. See page 821. **123.** Ellipse

Review Exercises for Chapter 11 (page 827)

1. (a) $\mathbf{u} = 3\mathbf{i} - \mathbf{j}$, $\mathbf{v} = 4\mathbf{i} + 2\mathbf{j}$ (b) $2\sqrt{5}$ (c) $10\mathbf{i}$
3. $\mathbf{v} = \langle -4, 4\sqrt{3} \rangle$ **5.** $(-5, 4, 0)$
7. Above the *xy*-plane and to the right of the *xz*-plane *or* below the *xy*-plane and to the left of the *xz*-plane
9. $(x - 3)^2 + (y + 2)^2 + (z - 6)^2 = \frac{225}{4}$
11. $(x - 2)^2 + (y - 3)^2 + z^2 = 9$
Center: (2, 3, 0)
Radius: 3
13.



$\mathbf{u} = \langle 2, 5, -10 \rangle$
15. Collinear
17. $(1/\sqrt{38})\langle 2, 3, 5 \rangle$
19. (a) $\mathbf{u} = \langle -1, 4, 0 \rangle, \mathbf{v} = \langle -3, 0, 6 \rangle$ (b) 3 (c) 45
21. Orthogonal **23.** $\theta = \arccos\left(\frac{\sqrt{2} + \sqrt{6}}{4}\right) = 15°$ **25.** π
27. Answers will vary. Example: $\langle -6, 5, 0 \rangle, \langle 6, -5, 0 \rangle$
29. $\mathbf{u} \cdot \mathbf{u} = 14 = \|\mathbf{u}\|^2$ **31.** $\langle -\frac{15}{14}, \frac{5}{7}, -\frac{5}{14} \rangle$
33. $(1/\sqrt{5})(-2\mathbf{i} - \mathbf{j})$ or $(1/\sqrt{5})(2\mathbf{i} + \mathbf{j})$
35. 4 **37.** $\sqrt{285}$ **39.** $100 \sec 20° \approx 106.4$ lb
41. (a) $x = 3 + 6t, y = 11t, z = 2 + 4t$
(b) $(x - 3)/6 = y/11 = (z - 2)/4$

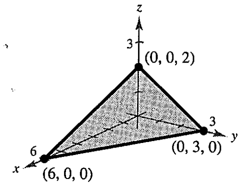
43. (a) $x = 1, y = 2 + t, z = 3$ (b) None

45. (a) $x = t, y = -1 + t, z = 1$ (b) $x = y + 1, z = 1$

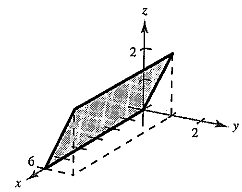
47. $27x + 4y + 32z + 33 = 0$ **49.** $x + 2y = 1$

51. $\frac{8}{7}$ **53.** $\sqrt{35}/7$

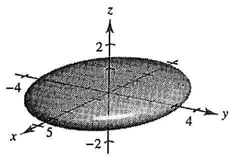
55. Plane



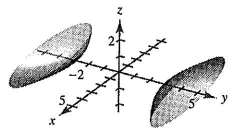
57. Plane



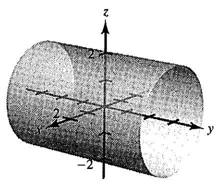
59. Ellipsoid



61. Hyperboloid of two sheets



63. Cylinder



65. Let $y = 2\sqrt{x}$ and revolve around the x-axis.

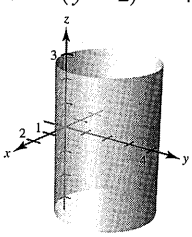
67. (a) $(4, 3\pi/4, 2)$ (b) $(2\sqrt{5}, 3\pi/4, \arccos[\sqrt{5}/5])$

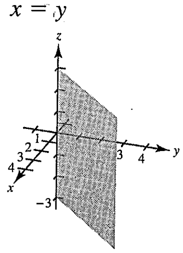
69. $(50\sqrt{5}, -\pi/6, \arccos[1/\sqrt{5}])$

71. $(25\sqrt{2}/2, -\pi/4, -25\sqrt{2}/2)$

73. (a) $r^2 \cos 2\theta = 2z$ (b) $\rho = 2 \sec 2\theta \cos \phi \csc^2 \phi$

75. $x^2 + (y - 2)^2 = 4$ **77.** $x = y$





P.S. Problem Solving (page 829)

1–3. Proofs

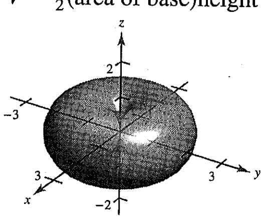
5. (a) $3\sqrt{2}/2 \approx 2.12$ (b) $\sqrt{5} \approx 2.24$

7. (a) $\pi/2$ (b) $\frac{1}{2}(\pi ab)k^2$

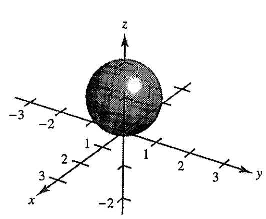
(c) $V = \frac{1}{2}(\pi ab)k^2$

$V = \frac{1}{2}$(area of base)height

9. (a)



(b)



11. Proof

13. (a) Tension: $2\sqrt{3}/3 \approx 1.1547$ lb

Magnitude of $\mathbf{u}$: $\sqrt{3}/3 \approx 0.5774$ lb

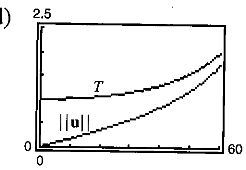
(b) $T = \sec \theta$; $\|\mathbf{u}\| = \tan \theta$; Domain: $0 \le \theta \le 90°0$

(c)

θ	0°	10°	20°	30°	40°
T	1	1.0154	1.0642	1.1547	1.3054
$\|\mathbf{u}\|$	0	0.1763	0.3640	0.5774	0.8391

θ	50°	60°
T	1.5557	2
$\|\mathbf{u}\|$	1.1918	1.7321

(d)



(e) Both are increasing functions.

(f) $\lim_{\theta \to \pi/2^-} T = \infty$ and $\lim_{\theta \to \pi/2^-} \|\mathbf{u}\| = \infty$

Yes. As θ increases, both T and $\|\mathbf{u}\|$ increase.

15. $\langle 0, 0, \cos \alpha \sin \beta - \cos \beta \sin \alpha \rangle$; Proof

17. $$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|\mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})|}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|}{\|\mathbf{u} \times \mathbf{v}\|}$$

19. Proof

Chapter 12

Section 12.1 (page 837)

1. $(-\infty, 0) \cup (0, \infty)$ **3.** $(0, \infty)$ **5.** $[0, \infty)$ **7.** $(-\infty, \infty)$

9. (a) $\frac{1}{2}\mathbf{i}$ (b) $\mathbf{j}$ (c) $\frac{1}{2}(s + 1)^2\mathbf{i} - s\mathbf{j}$ (d) $\frac{1}{2}\Delta t(\Delta t + 4)\mathbf{i} - \Delta t\mathbf{j}$

11. (a) $\ln 2\mathbf{i} + \dfrac{1}{2}\mathbf{j} + 6\mathbf{k}$ (b) Not possible

(c) $\ln(t - 4)\mathbf{i} + \dfrac{1}{t - 4}\mathbf{j} + 3(t - 4)\mathbf{k}$

(d) $\ln(1 + \Delta t)\mathbf{i} - \dfrac{\Delta t}{1 + \Delta t}\mathbf{j} + 3\Delta t\mathbf{k}$

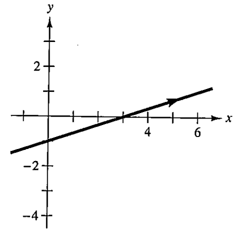
13. $\sqrt{1 + t^2}$ **15.** $t^2(5t - 1)$; The dot product is a scalar.

17. b **18.** c **19.** d **20.** a

21. (a) $(-20, 0, 0)$ (b) $(10, 20, 10)$

(c) $(0, 0, 20)$ (d) $(20, 0, 0)$

23.



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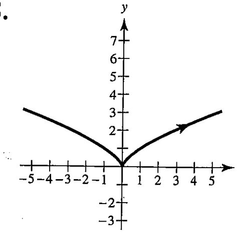
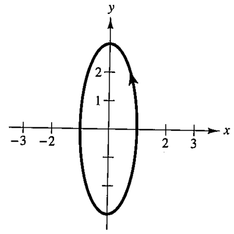
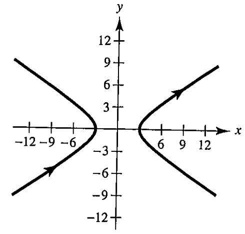


27.



29.



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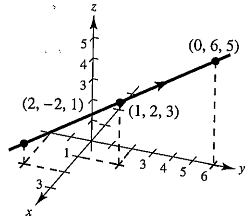

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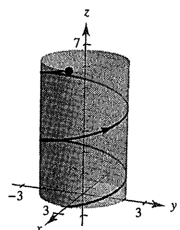

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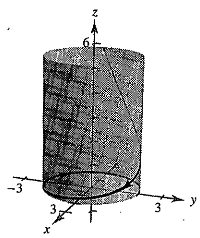

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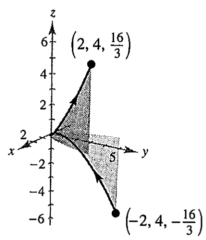
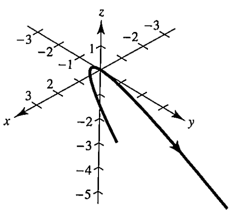
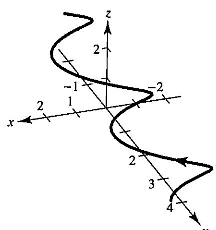


39.



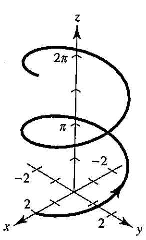
Parabola

41.



Helix

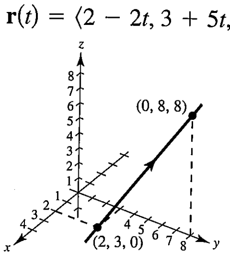
43.



(a) The helix is translated two units back on the x-axis.
(b) The height of the helix increases at a greater rate.
(c) The orientation of the graph is reversed.
(d) The axis of the helix is the x-axis.
(e) The radius of the helix is increased from 2 to 6.

45–51. Answers will vary.

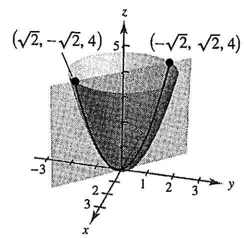
53. $\mathbf{r}(t) = \langle 2 - 2t, 3 + 5t, 8t \rangle$



55. $\mathbf{r}_1(t) = t\mathbf{i},\quad 0 \le t \le 4$
$\mathbf{r}_2(t) = (4 - 4t)\mathbf{i} + 6t\mathbf{j},\quad 0 \le t \le 1$
$\mathbf{r}_3(t) = (6 - t)\mathbf{j},\quad 0 \le t \le 6$

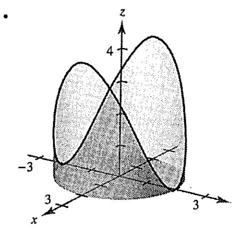
57. $\mathbf{r}_1(t) = t\mathbf{i} + t^2\mathbf{j},\quad 0 \le t \le 2$
$\mathbf{r}_2(t) = (2 - t)\mathbf{i} + 4\mathbf{j},\quad 0 \le t \le 2$
$\mathbf{r}_3(t) = (4 - t)\mathbf{j},\quad 0 \le t \le 4$

59.



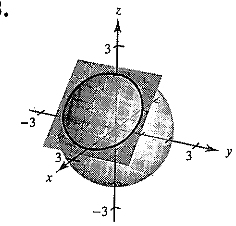
$\mathbf{r}(t) = t\mathbf{i} - t\mathbf{j} + 2t^2\mathbf{k}$

61.



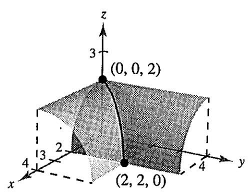
$\mathbf{r}(t) = 2\sin t\mathbf{i} + 2\cos t\mathbf{j} + 4\sin^2 t\mathbf{k}$

63.



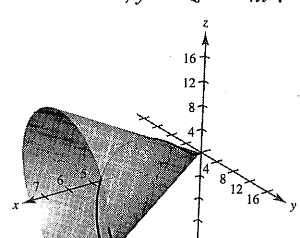
$\mathbf{r}(t) = (1 + \sin t)\mathbf{i} + \sqrt{2}\cos t\mathbf{j} + (1 - \sin t)\mathbf{k}$ and
$\mathbf{r}(t) = (1 + \sin t)\mathbf{i} - \sqrt{2}\cos t\mathbf{j} + (1 - \sin t)\mathbf{k}$

65.



$\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + \sqrt{4 - t^2}\mathbf{k}$

67. Let $x = t$, $y = 2t\cos t$, and $z = 2t\sin t$. Then
$$y^2 + z^2 = (2t\cos t)^2 + (2t\sin t)^2 = 4t^2\cos^2 t + 4t^2\sin^2 t = 4t^2(\cos^2 t + \sin^2 t) = 4t^2.$$
Since $x = t$, $y^2 + z^2 = 4x^2$.

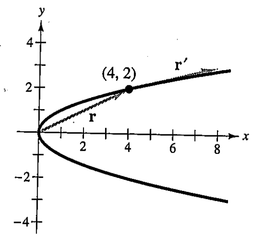


69. $2\mathbf{i} + 2\mathbf{j} + \frac{1}{2}\mathbf{k}$ **71.** 0 **73.** Limit does not exist.
75. $(-\infty, 0), (0, \infty)$ **77.** $[-1, 1]$
79. $(-\pi/2 + n\pi, \pi/2 + n\pi)$, n is an integer.
81. A function of the form $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$ (plane) or $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ (space) is a vector-valued function, where the component functions f, g, and h are real-valued functions of the parameter t.
83. (a) $\mathbf{s}(t) = t^2\mathbf{i} + (t - 3)\mathbf{j} + (t + 3)\mathbf{k}$
(b) $\mathbf{s}(t) = (t^2 - 2)\mathbf{i} + (t - 3)\mathbf{j} + t\mathbf{k}$
(c) $\mathbf{s}(t) = t^2\mathbf{i} + (t + 2)\mathbf{j} + t\mathbf{k}$
85–87. Proofs **89.** True

91. False; although $\mathbf{r}(4) = \mathbf{u}(2) = \langle 4, 16 \rangle$, the particles do not collide because they reach this point at different times.

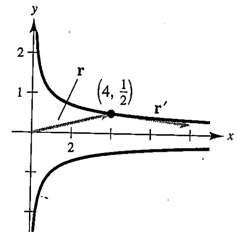
Section 12.2 (page 846)

1. $\mathbf{r}(2) = 4\mathbf{i} + 2\mathbf{j}$
$\mathbf{r}'(2) = 4\mathbf{i} + \mathbf{j}$



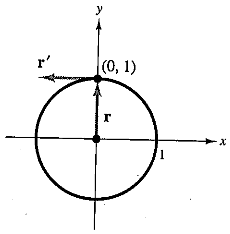
$\mathbf{r}'(t_0)$ is tangent to the curve at t_0.

3. $\mathbf{r}(2) = 4\mathbf{i} + \frac{1}{2}\mathbf{j}$
$\mathbf{r}'(2) = 4\mathbf{i} - \frac{1}{4}\mathbf{j}$



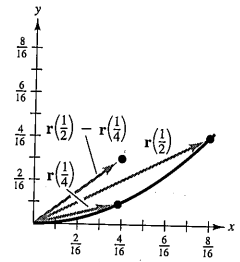
$\mathbf{r}'(t_0)$ is tangent to the curve at t_0.

5. $\mathbf{r}(\pi/2) = \mathbf{j}$
$\mathbf{r}'(\pi/2) = -\mathbf{i}$



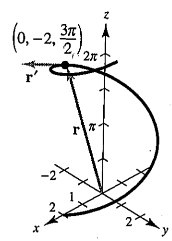
$\mathbf{r}'(t_0)$ is tangent to the curve at t_0.

7. (a) and (b)



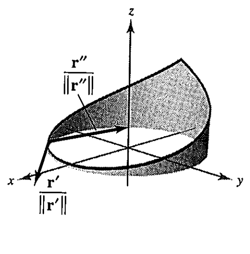
(c) The vector $\dfrac{\mathbf{r}(1/2) - \mathbf{r}(1/4)}{1/2 - 1/4}$ approximates the tangent vector $\mathbf{r}'(1/4)$.

9. $\mathbf{r}\left(\dfrac{3\pi}{2}\right) = -2\mathbf{j} + \left(\dfrac{3\pi}{2}\right)\mathbf{k}$
$\mathbf{r}'\left(\dfrac{3\pi}{2}\right) = 2\mathbf{i} + \mathbf{k}$

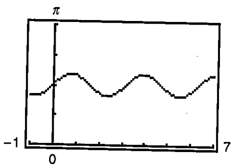


11. $6\mathbf{i} - 14t\mathbf{j} + 3t^2\mathbf{k}$ **13.** $-3a \sin t \cos^2 t\mathbf{i} + 3a \sin^2 t \cos t\mathbf{j}$
15. $-e^{-t}\mathbf{i}$ **17.** $\langle \sin t + t \cos t, \cos t - t \sin t, 1 \rangle$
19. (a) $6t\mathbf{i} + \mathbf{j}$ (b) $18t^3 + t$
21. (a) $-4 \cos t\mathbf{i} - 4 \sin t\mathbf{j}$ (b) 0
23. (a) $\mathbf{i} + t\mathbf{k}$ (b) $t^3/2 + t$
25. (a) $\langle \cos t - t \sin t, \sin t + t \cos t, 0 \rangle$ (b) t
27. $\dfrac{\mathbf{r}'(-1/4)}{\|\mathbf{r}'(-1/4)\|} = \dfrac{1}{\sqrt{4\pi^2 + 1}}(\sqrt{2}\pi\mathbf{i} + \sqrt{2}\pi\mathbf{j} - \mathbf{k})$

$\dfrac{\mathbf{r}''(-1/4)}{\|\mathbf{r}''(-1/4)\|} = \dfrac{1}{2\sqrt{\pi^4 + 4}}(-\sqrt{2}\pi^2\mathbf{i} + \sqrt{2}\pi^2\mathbf{j} + 4\mathbf{k})$

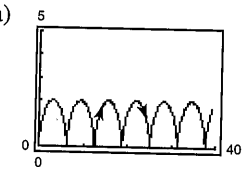


29. $(-\infty, 0), (0, \infty)$ **31.** $(n\pi/2, (n + 1)\pi/2)$
33. $(-\infty, \infty)$ **35.** $(-\infty, 0), (0, \infty)$
37. $(-\pi/2 + n\pi, \pi/2 + n\pi)$, n is an integer.
39. (a) $\mathbf{i} + 3\mathbf{j} + 2t\mathbf{k}$ (b) $2\mathbf{k}$ (c) $8t + 9t^2 + 5t^4$
(d) $-\mathbf{i} + (9 - 2t)\mathbf{j} + (6t - 3t^2)\mathbf{k}$
(e) $8t^3\mathbf{i} + (12t^2 - 4t^3)\mathbf{j} + (3t^2 - 24t)\mathbf{k}$
(f) $(10 + 2t^2)/\sqrt{10 + t^2}$
41. (a) $7t^6$ (b) $12t^5\mathbf{i} - 5t^4\mathbf{j}$
43. $\theta(t) = \arccos\left(\dfrac{-7 \sin t \cos t}{\sqrt{9 \sin^2 t + 16 \cos^2 t}\sqrt{9 \cos^2 t + 16 \sin^2 t}}\right)$



Maximum: $\theta\left(\dfrac{\pi}{4}\right) = \theta\left(\dfrac{5\pi}{4}\right) \approx 1.855$
Minimum: $\theta\left(\dfrac{3\pi}{4}\right) = \theta\left(\dfrac{7\pi}{4}\right) \approx 1.287$
Orthogonal: $\dfrac{n\pi}{2}$, n is an integer

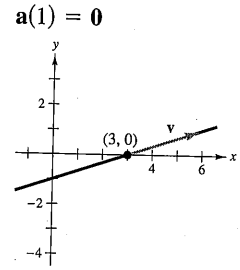
45. $\mathbf{r}'(t) = 3\mathbf{i} - 2t\mathbf{j}$ **47.** $\mathbf{r}'(t) = 2t\mathbf{i} + 2\mathbf{k}$
49. $t^2\mathbf{i} + t\mathbf{j} + t\mathbf{k} + \mathbf{C}$ **51.** $\ln t\mathbf{i} + t\mathbf{j} - \frac{2}{5}t^{5/2}\mathbf{k} + \mathbf{C}$
53. $(t^2 - t)\mathbf{i} + t^4\mathbf{j} + 2t^{3/2}\mathbf{k} + \mathbf{C}$
55. $\tan t\mathbf{i} + \arctan t\mathbf{j} + \mathbf{C}$ **57.** $4\mathbf{i} + \frac{1}{2}\mathbf{j} - \mathbf{k}$
59. $a\mathbf{i} + a\mathbf{j} + (\pi/2)\mathbf{k}$ **61.** $2\mathbf{i} + (e^2 - 1)\mathbf{j} - (e^2 + 1)\mathbf{k}$
63. $2e^{2t}\mathbf{i} + 3(e^t - 1)\mathbf{j}$ **65.** $600\sqrt{3}t\mathbf{i} + (-16t^2 + 600t)\mathbf{j}$
67. $((2 - e^{-t^2})/2)\mathbf{i} + (e^{-t} - 2)\mathbf{j} + (t + 1)\mathbf{k}$
69. See "Definition of the Derivative of a Vector-Valued Function" and Figure 12.8 on page 840.
71. The three components of $\mathbf{u}$ are increasing functions of t at $t = t_0$.
73–79. Proofs
81. (a)



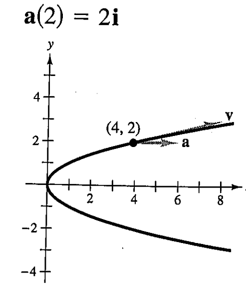
(b) The maximum of $\|\mathbf{r}'\|$ is 2; the minimum of $\|\mathbf{r}'\|$ is 0. The maximum and the minimum of $\|\mathbf{r}''\|$ is 1.
83. True
85. False: Let $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k}$, then $d/dt[\|\mathbf{r}(t)\|] = 0$, but $\|\mathbf{r}'(t)\| = 1$.
87. Proof

Section 12.3 (page 854)

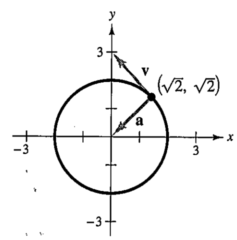
1. $\mathbf{v}(1) = 3\mathbf{i} + \mathbf{j}$
$\mathbf{a}(1) = \mathbf{0}$



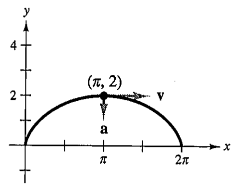
3. $\mathbf{v}(2) = 4\mathbf{i} + \mathbf{j}$
$\mathbf{a}(2) = 2\mathbf{i}$



5. $\mathbf{v}(\pi/4) = -\sqrt{2}\mathbf{i} + \sqrt{2}\mathbf{j}$
$\mathbf{a}(\pi/4) = -\sqrt{2}\mathbf{i} - \sqrt{2}\mathbf{j}$



7. $\mathbf{v}(\pi) = 2\mathbf{i}$
$\mathbf{a}(\pi) = -\mathbf{j}$



9. $\mathbf{v}(t) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
$\|\mathbf{v}(t)\| = \sqrt{14}$
$\mathbf{a}(t) = \mathbf{0}$

11. $\mathbf{v}(t) = \mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$
$\|\mathbf{v}(t)\| = \sqrt{1 + 5t^2}$
$\mathbf{a}(t) = 2\mathbf{j} + \mathbf{k}$

13. $\mathbf{v}(t) = \mathbf{i} + \mathbf{j} - \left(t/\sqrt{9 - t^2}\right)\mathbf{k}$
$\|\mathbf{v}(t)\| = \sqrt{(18 - t^2)/(9 - t^2)}$
$\mathbf{a}(t) = \left(-9/(9 - t^2)^{3/2}\right)\mathbf{k}$

15. $\mathbf{v}(t) = 4\mathbf{i} - 3\sin t\mathbf{j} + 3\cos t\mathbf{k}$
$\|\mathbf{v}(t)\| = 5$
$\mathbf{a}(t) = -3\cos t\mathbf{j} - 3\sin t\mathbf{k}$

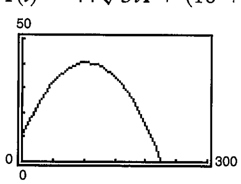
17. (a) $x = 1 + t$
$y = -1 - 2t$
$z = \frac{1}{4} + \frac{3}{4}t$
(b) $(1.100, -1.200, 0.325)$

19. $\mathbf{v}(t) = t(\mathbf{i} + \mathbf{j} + \mathbf{k})$
$\mathbf{r}(t) = (t^2/2)(\mathbf{i} + \mathbf{j} + \mathbf{k})$
$\mathbf{r}(2) = 2(\mathbf{i} + \mathbf{j} + \mathbf{k})$

21. $\mathbf{v}(t) = \left(t^2/2 + \frac{9}{2}\right)\mathbf{j} + \left(t^2/2 - \frac{1}{2}\right)\mathbf{k}$
$\mathbf{r}(t) = \left(t^3/6 + \frac{9}{2}t - \frac{14}{3}\right)\mathbf{j} + \left(t^3/6 - \frac{1}{2}t + \frac{1}{3}\right)\mathbf{k}$
$\mathbf{r}(2) = \frac{17}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$

23. The velocity of an object involves both magnitude and direction of motion, whereas speed involves only magnitude.

25. $\mathbf{r}(t) = 44\sqrt{3}t\mathbf{i} + (10 + 44t - 16t^2)\mathbf{j}$

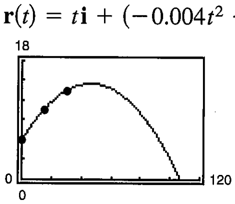


27. $v_0 = 40\sqrt{6}$ ft/sec; 78 ft

29. Proof

31. (a) $y = -0.004x^2 + 0.37x + 6$
$\mathbf{r}(t) = t\mathbf{i} + (-0.004t^2 + 0.37t + 6)\mathbf{j}$

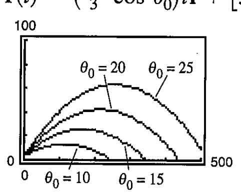
(b)



(c) 14.56 ft
(d) Initial velocity: 67.4 ft/sec; $\theta \approx 20.14°$

33. (a) $\mathbf{r}(t) = \left(\frac{440}{3}\cos\theta_0\right)t\mathbf{i} + \left[3 + \left(\frac{440}{3}\sin\theta_0\right)t - 16t^2\right]\mathbf{j}$

(b)



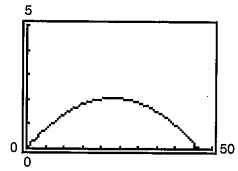
The minimum angle appears to be $\theta_0 = 20°$.

(c) $\theta_0 \approx 19.38°$

35. (a) $v_0 = 28.78$ ft/sec; $\theta = 58.28°$ (b) $v_0 \approx 32$ ft/sec

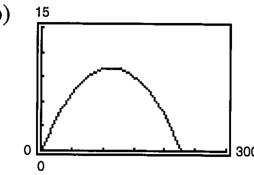
37. 1.91°

39. (a)



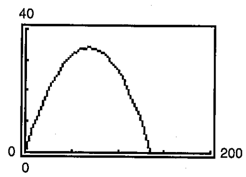
Maximum height: 2.1 ft
Range: 46.6 ft

(b)



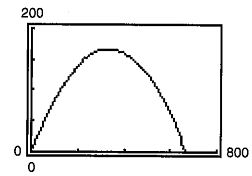
Maximum height: 10.0 ft
Range: 227.8 ft

(c)



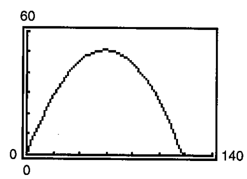
Maximum height: 34.0 ft
Range: 136.1 ft

(d)



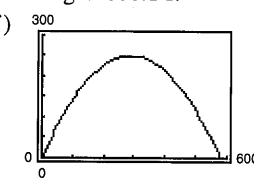
Maximum height: 166.5 ft
Range: 666.1 ft

(e)



Maximum height: 51.0 ft
Range: 117.9 ft

(f)



Maximum height: 249.8 ft
Range: 576.9 ft

41. Maximum height: 129.1 m
Range: 886.3 m

43. $\mathbf{v}(t) = b\omega[(1 - \cos\omega t)\mathbf{i} + \sin\omega t\mathbf{j}]$
$\mathbf{a}(t) = b\omega^2(\sin\omega t\mathbf{i} + \cos\omega t\mathbf{j})$
(a) $\|\mathbf{v}(t)\| = 0$ when $\omega t = 0, 2\pi, 4\pi, \ldots$
(b) $\|\mathbf{v}(t)\|$ is maximum when $\omega t = \pi, 3\pi, \ldots$

45. $\mathbf{v}(t) = -b\omega\sin\omega t\mathbf{i} + b\omega\cos\omega t\mathbf{j}$
$\mathbf{v}(t) \cdot \mathbf{r}(t) = 0$

47. $\mathbf{a}(t) = -b\omega^2(\cos\omega t\mathbf{i} + \sin\omega t\mathbf{j}) = -\omega^2\mathbf{r}(t)$

49. $8\sqrt{10}$ ft/sec

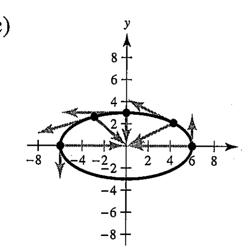
51–53. Proofs

55. (a) $\mathbf{v}(t) = -6\sin t\mathbf{i} + 3\cos t\mathbf{j}$
$\|\mathbf{v}(t)\| = 3\sqrt{3\sin^2 t + 1}$
$\mathbf{a}(t) = -6\cos t\mathbf{i} - 3\sin t\mathbf{j}$

(b)

t	0	$\pi/4$	$\pi/2$	$2\pi/3$	π
Speed	3	$3\sqrt{10}/2$	6	$3\sqrt{13}/2$	3

(c)



(d) The speed is increasing when the angle between **v** and **a** is in the interval $[0, \pi/2)$, and decreasing when the angle is in the interval $(\pi/2, \pi]$.

57. False; acceleration is the derivative of the velocity.

59. Proof