Ch. 11 Review Homework p. 827 2-54 evens

2) Find a) component form of u and v $\langle v_1 - u_1, v_2 - u_2 \rangle$
b) magnitude of v $\|v\| = \sqrt{(q_1-p_1)^2 + (q_2-p_2)^2}$
c) $2u + v$

$P = (-2,-1)$ $Q = (5,-1)$ $R = (2,4)$

a) $\overrightarrow{PQ} = \langle 5 - -2, -1 - -1 \rangle = \langle 7, 0 \rangle = \boxed{7i}$
$u = \overrightarrow{PQ}$
$v = \overrightarrow{PR} = \langle 2 - -2, 4 - -1 \rangle = \langle 4, 5 \rangle = \boxed{4i + 5j}$

b) $v = \overrightarrow{PR} = \|v\| = \sqrt{4^2 + 5^2} = \boxed{\sqrt{41}}$

c) $2u + v = 2(7i) + 4i + 5j = \boxed{18i + 5j}$

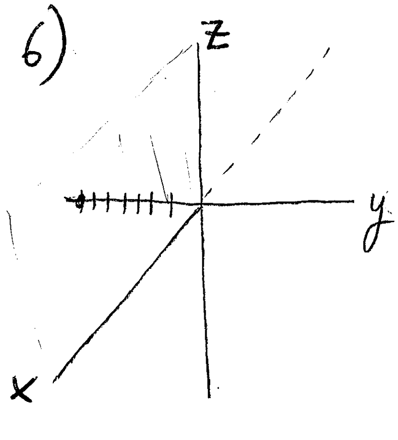
$*v = \|v\| u$

4) $\|v\| = \frac{1}{2}$ $\theta = 225°$ $v = \|v\| \langle \cos\theta, \sin\theta \rangle$

Find component form of v. $v = \|v\| \cos\theta i + \|v\| \sin\theta j$

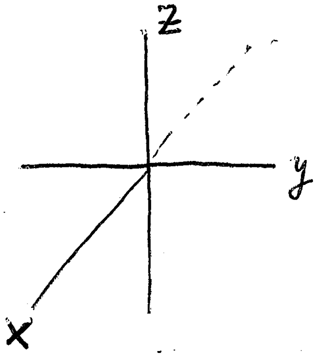
$v = \frac{1}{2} \cos 225° i + \frac{1}{2} \sin 225° j$

$= \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) i + \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) j = \boxed{-\frac{\sqrt{2}}{4} i - \frac{\sqrt{2}}{4} j}$

6) 

$\boxed{(0,-7,0)}$

8) $xy < 0$



2^{nd} or 4^{th} quadrant

(x, and y have opposite signs)

Ex. $(-2, 3, \_)$

$(4, -5, \_)$

z can be any #.

10) Standard equation of sphere $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$

Midpt: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Endpt: $(0,0,4)$ and $(4,6,0)$

Midpt (center) $= \left(\frac{0+4}{2}, \frac{0+6}{2}, \frac{4+0}{2}\right) = (2,3,2)$

Distance (radius) $= \sqrt{(2-0)^2+(3-0)^2+(2-4)^2} = \sqrt{4+9+4} = \sqrt{17}$

$$\boxed{\text{Equation: } (x-2)^2+(y-3)^2+(z-2)^2=17}$$

12) Complete square to write equation in standard form

$\left(\frac{b}{2}\right)^2$

$x^2+y^2+z^2-10x+6y-4z+34=0$

$x^2-10x+\underline{25}+y^2+6y+\underline{9}+z^2-4z+\underline{4} = -34+\underline{25}+\underline{9}+\underline{4}$

$(x-5)^2+(y+3)^2+(z-2)^2=4$

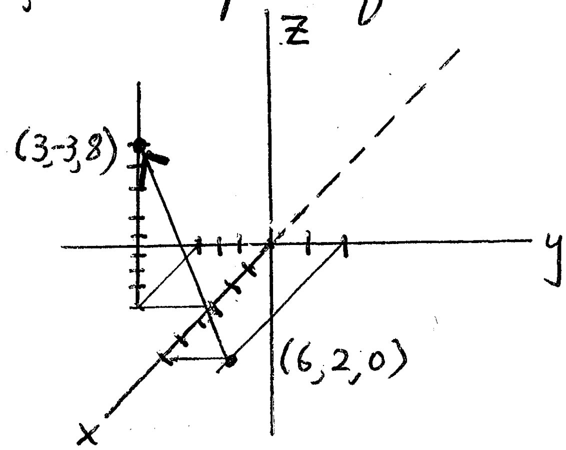
center: $(5,-3,2)$

radius: 2

14) Sketch directed line segment, find component form of vector.

Initial pt. $(6,2,0)$

Terminal pt. $(3,-3,8)$



$V = \langle 3-6, -3-2, 8-0 \rangle$

$\boxed{V = \langle -3, -5, 8 \rangle}$

16) Use vectors to determine if points are collinear

$(5,-4,7)$, $(8,-5,5)$, $(11,6,3)$

[* Two vectors u and v are parallel if there is a scalar c such that $u = cv$]

$V = \langle 8-5, -5+4, 5-7 \rangle = \langle 3, -1, -2 \rangle$

$U = \langle 11-8, 6+5, 3-5 \rangle = \langle 3, 11, 2 \rangle$

points are not collinear Since $u \neq cv$

18) Find vector v of magnitude 8 in direction $\langle 6,-3,2 \rangle$

$$\frac{\langle v_1, v_2, v_3 \rangle}{\|v\|} \rightarrow 8 \cdot \frac{\langle 6,-3,2 \rangle}{\sqrt{49}} = \frac{8}{7}\langle 6,-3,2 \rangle$$

$$= \boxed{\left\langle \frac{48}{7}, \frac{-24}{7}, \frac{16}{7} \right\rangle}$$

20) $u = \overrightarrow{PQ}$ $v = \overrightarrow{PR}$

$P = (2,-1,3)$ $Q = (0,5,1)$ $R = (5,5,0)$

a) $u = \langle -2, 6, -2 \rangle$ $v = \langle 3, 6, -3 \rangle$ $u = -2i + 6j - 2k$ $v = 3i + 6j - 3k$

b) $u \cdot v = -2(3) + 6(6) - 2(-3) = 36$

c) $v \cdot v = 3(3) + 6(6) - 3(-3) = 54$

[*vectors are orthogonal if $u \cdot v = 0$*]

22) Determine if u and v are orthogonal, parallel, neither

$u = \langle -4, 3, -6 \rangle \quad v = \langle 16, -12, 24 \rangle$

Since $v = -4u$, vectors are parallel

24) Find angle θ between the vectors

$u = \langle 4, -1, 5 \rangle \quad v = \langle 3, 2, -2 \rangle$

Since $u \cdot v = 4(3) + (-1)(2) + 5(-2) = 0$, the vectors are normal (orthogonal). Therefore, $\theta = \pi/2$

26) $u = \langle 1, 0, -3 \rangle \quad v = \langle 2, -2, 1 \rangle$

*Angle between 2 vectors $\cos\theta = \dfrac{u \cdot v}{\|u\| \cdot \|v\|}$

$u \cdot v = 2 + 0 - 3 = -1$

$\|u\| = \sqrt{10} \quad \|v\| = \sqrt{9} = 3$

$\cos\theta = \dfrac{-1}{3\sqrt{10}} \quad \theta = \cos^{-1}\left(\dfrac{-1}{3\sqrt{10}}\right) \approx \boxed{83.9^\circ}$

28) $W = F \cdot \overrightarrow{PQ} \quad W = \|\text{proj}_{PQ} F\| \|\overrightarrow{PQ}\| = \cos\theta \|F\| \|\overrightarrow{PQ}\|$

$W = 75 \cdot 8 \cdot \cos 30$

$W = \cos 30 \cdot 75 \cdot 8 = 300\sqrt{3}$ ft. lb

30) $u = \langle 3, -2, 1 \rangle \quad v = \langle 2, -4, -3 \rangle$

$\cos\theta = \dfrac{u \cdot v}{\|u\| \|v\|} = \dfrac{6 + 8 - 3}{\sqrt{14} \cdot \sqrt{29}} \quad \theta = \cos^{-1}\left(\dfrac{11}{\sqrt{14} \cdot \sqrt{29}}\right) \approx \boxed{56.9^\circ}$

Force · vector

32) $W = w \cdot u$

$W = \langle -1, 2, 2 \rangle \cdot \langle 3, -2, 1 \rangle = -3 - 4 + 2 = \boxed{-5}$

34) * cross product

$u = \langle 3, -2, 1 \rangle \quad v = \langle 2, -4, -3 \rangle \quad w = \langle -1, 2, 2 \rangle$

$\boxed{\text{Show that } u \times v = -(v \times u)}$ $\qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$u \times v = \begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \\ 3 & -2 & 1 \\ 2 & -4 & -3 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ -4 & -3 \end{vmatrix} i + \begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} j + \begin{vmatrix} 3 & -2 \\ 2 & 4 \end{vmatrix} k$$

$$= 10i - (-11)j + -8k$$

$$= \boxed{10i + 11j - 8k}$$

$$v \times u \begin{vmatrix} i & j & k \\ 2 & -4 & -3 \\ 3 & -2 & 1 \end{vmatrix} = \begin{vmatrix} -4 & -3 \\ -2 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & -4 \\ 3 & -2 \end{vmatrix} k$$

$$= \boxed{-10j - 11j + 8k}$$

36) Show that $u \times (v + w) = (u \times v) + (u \times w)$

$$\langle 3, -2, 1 \rangle \times \langle 1, -2, -1 \rangle = \begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \\ 3 & -2 & 1 \\ 1 & -2 & -1 \end{vmatrix} = \boxed{4i + 4j - 4k}$$

$u \times v = 10i + 11j - 8k$

$u \times w = -6i - 7j + 4k$

$+$

$\boxed{4i + 4j - 4k}$

38) Area of triangle $= \frac{1}{2}\|v \times w\|$ adjacent sides

$v = \langle 2, -4, -3 \rangle \quad w = \langle -1, 2, 2 \rangle$

$$v \times w = \begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \\ 2 & -4 & -3 \\ -1 & 2 & 2 \end{vmatrix} = \begin{vmatrix} -4 & -3 \\ 2 & 2 \end{vmatrix} i - \begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} j + \begin{vmatrix} 2 & -4 \\ -1 & 2 \end{vmatrix} k$$

$$= -2i - 1j + 0k$$

$$\|v \times W\| = \sqrt{4+1} = \sqrt{5}$$

$$\text{Area} = \frac{1}{2}\|v \times w\| = \frac{1}{2}(\sqrt{5}) = \boxed{\frac{\sqrt{5}}{2}}$$

40) Volume $V = |u \cdot (v \times w)|$

$u = \langle 2, 1, 0 \rangle$
$v = \langle 0, 2, 1 \rangle$
$w = \langle 0, -1, 2 \rangle$

$$V = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} 2 - \begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} 1 + \begin{vmatrix} 0 & 2 \\ 0 & -1 \end{vmatrix} 0$$

$$= 5(2) - 0 + 0 = \boxed{10}$$

42) $(-1, 4, 3) \quad (8, 10, 5) \quad \boxed{v = \langle 9, 6, 2 \rangle}$

a) parametric equations

$x = x_1 + v_1 t$
$y = y_1 + v_2 t$
$z = z_1 + v_3 t$

$$\boxed{\begin{aligned} x &= -1 + 6t \\ y &= 4 + 6t \\ z &= 3 + 2t \end{aligned}}$$

b) symmetric equations

$$\frac{x - x_1}{v_1} = \frac{x - x_2}{v_2} = \frac{x - x_3}{v_3}$$

$$\boxed{\frac{x+1}{9} = \frac{y-4}{6} = \frac{z-3}{2}}$$

44) line passing pt $(1,2,3)$ and parallel to $x=y=z$

a) $x=1+t \quad y=2+t \quad z=3+t \qquad v=\langle 1,1,1\rangle$

b) $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$

46) line passing $(0,1,4)$ and $\perp$ to $u=\langle 2,-5,1\rangle$
$v=\langle -3,1,4\rangle$

$$u\times v=\begin{vmatrix} i & j & k \\ 2 & -5 & 1 \\ -3 & 1 & 4 \end{vmatrix}=-21i-11j-13k$$

Direction numbers : 21, 11, 13

a) $x=0+21t \quad y=1+11t \quad z=4+13t$

b) $\frac{x+0}{21}=\frac{y-1}{11}=\frac{z-4}{13}$

48) Find equation of plane $(-2,3,1)$ $\perp$ to $n=3i-j+k$

$$*\; v_1(x-x_1)+v_2(y-y_1)+v_3(z-z_1)=0$$

$$\boxed{3(x+2)-1(y-3)+1(z-1)=0}$$

$$3x+6-1y+3+z-1=0$$

$$\boxed{3x-y+z+8=0}$$

50) plane pass $Q(5,1,3)$ and $P(2,-2,1)$ and
$\perp$ to plane $2x+y-z=4$ $\quad n=\langle 2,1,-1\rangle$

$$V=\langle 3,3,2\rangle$$

$$V\times n=\begin{vmatrix} i & j & k \\ 3 & 3 & 2 \\ 2 & 1 & -1 \end{vmatrix}=\langle -5,7,-3\rangle$$

$$\boxed{\begin{gathered} -5(x-5)+7(y-1)-3(z-3)=0 \\ -5x+7y-3z+27=0 \end{gathered}}$$

52) Find distance b/t point $Q(3,-2,4)$ and plane $2x-5y+z=10$

$$* \; D=\frac{|\overrightarrow{PQ}\cdot n|}{\|n\|}$$

$n=\langle 2,-5,1\rangle$ $\quad \boxed{p.799}$

point on plane $P(5,0,0)$

$\overrightarrow{PQ}=\langle -2,-2,4\rangle$ $\quad D=\dfrac{\langle -2,-2,4\rangle\cdot\langle 2-5,1\rangle}{\sqrt{30}}=\dfrac{10}{\sqrt{10}}=\boxed{\dfrac{\sqrt{30}}{3}}$

54) Find distance b/t $Q(-5,1,3)$ and line $x=1+t,\ y=3-2t,\ z=5-t$

$u=\langle 1,-2,-1\rangle$ $\quad P=(1,3,5)$ (in space) $\quad D=\dfrac{\|\overrightarrow{PQ}\times u\|}{\|u\|}$

$\overrightarrow{PQ}=\langle -6,-2,-2\rangle$

$$\overrightarrow{PQ}\times u=\begin{vmatrix} i & j & k \\ -6 & -2 & -2 \\ 1 & -2 & -1 \end{vmatrix}=\langle -2,-8,14\rangle$$

$$D=\frac{\sqrt{264}}{\sqrt{6}}=\boxed{2\sqrt{11}}$$

44) line passing pt $(1,2,3)$ and parallel to $x=y=z$

a) $x = 1+t \quad y = 2+t \quad z = 3+t \qquad v = \langle 1,1,1 \rangle$

b) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

46) line passing $(0,1,4)$ and $\perp$ to $u = \langle 2,-5,1 \rangle$
$v = \langle -3,1,4 \rangle$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & -5 & 1 \\ -3 & 1 & 4 \end{vmatrix} = -21i - 11j - 13k$$

Direction numbers : 21, 11, 13

a) $x = 0 + 21t \quad y = 1 + 11t \quad z = 4 + 13t$

b) $\frac{x+0}{21} = \frac{y-1}{11} = \frac{z-4}{13}$

48) Find equation of plane $(-2,3,1)$ $\perp$ to $n = 3i - j + k$

* $v_1(x-x_1) + v_2(y-y_1) + v_3(z-z_1) = 0$

$$\boxed{3(x+2) - 1(y-3) + 1(z-1) = 0}$$

$3x + 6 - 1y + 3 + z - 1 = 0$

$$\boxed{3x - y + z + 8 = 0}$$

50) plane pass $\overset{Q}{(5,1,3)}$ and $\overset{P}{(2,-2,1)}$ and $\perp$ to plane $2x+y-z=4$ $\quad n=\langle 2,1,-1\rangle$

$$V=\langle 3,3,2\rangle$$

$$V\times n=\begin{vmatrix} i & j & k \\ 3 & 3 & 2 \\ 2 & 1 & -1 \end{vmatrix}=\langle -5,7,-3\rangle$$

$$\boxed{\begin{array}{c} -5(x-5)+7(y-1)-3(z-3)=0 \\ -5x+7y-3z+27=0 \end{array}}$$

52) Find distance b/t point $\overset{Q}{(3,-2,4)}$ and plane $2x-5y+z=10$

$$* \; D=\frac{|\vec{PQ}\cdot n|}{\|n\|}$$

$n=\langle 2,-5,1\rangle$ $\boxed{\text{p. 799}}$

point on plane $\overset{P}{(5,0,0)}$

$$\vec{PQ}=\langle -2,-2,4\rangle \qquad D=\frac{\langle -2,-2,4\rangle\cdot\langle 2-5,1\rangle}{\sqrt{30}}=\frac{10}{\sqrt{10}}=\boxed{\frac{\sqrt{30}}{3}}$$

54) Find distance b/t $\overset{Q}{(-5,1,3)}$ and line $x=1+t,\ y=3-2t,\ z=5-t$ (in space)

$u=\langle 1,-2,-1\rangle$ $\quad P=(1,3,5)$ $\qquad D=\dfrac{\|\vec{PQ}\times u\|}{\|u\|}$

$$\vec{PQ}=\langle -6,-2,-2\rangle$$

$$\vec{PQ}\times u=\begin{vmatrix} i & j & k \\ -6 & -2 & -2 \\ 1 & -2 & -1 \end{vmatrix}=\langle -2,-8,14\rangle$$

$$D=\frac{\sqrt{264}}{\sqrt{6}}=\boxed{2\sqrt{11}}$$