AP Calculus Ch. 2.2 Notes: Position-Velocity-Acceleration(PVA)

Instantaneous velocity ,v(t), of the object is the derivative of the position function s(t) with respect to time

Acceleration, a(t), is the derivative of velocity with respect to time

AVERAGE rate of change of f(x) from a to b = slope of secant = $\dfrac{f(b)-f(a)}{b-a}$

INSTANTANEOUS rate of change of f(x) at x = c = slope of tangent = $f'(c)$

Speed = |velocity|

Displacement = how far you are from where you started
Distance = total amount you have traveled

Ex) If I travel 10 feet to the right and then turn around and travel 3 feet back to the left, my distance is 13 feet but my displacement is 7 feet.

Speed is **increasing** when velocity and acceleration have the **same** sign.
Speed is **decreasing** when velocity and acceleration have **opposite** signs.

Particle Motion

Particle motion (linear motion) describes the object moving along a line (usually along a horizontal line)

x(t) = Position function
v(t) = velocity function
a(t) = acceleration function

Positive velocity indicates ______________________________

Negative velocity indicates ______________________________

When v(t) = 0, this indicates ______________________________

A.P. Calculus PVA **Worksheet 2.2a**

A ball is thrown vertically upwards from the edge of a building and it eventually hits the ground next to the building. If the height of the ball at any given time, $t \geq 0$ (seconds), is $h(t) = -16t^2 + 64t + 80$ (feet), answer the following:

1. Sketch a diagram and label values at important places

2. How tall is the building?

3. When does the ball reach maximum height?

4. What is the maximum height?

5. How long does it take to hit the ground?

6. What was the initial velocity?

7. What is the velocity at $t = 1$ second? At $t = 2$ seconds?

8. What is the height at $t = 3$ seconds?

9. What is the <u>speed</u> when it hits the ground?

10. What is the acceleration at $t = 1$ second? At $t = 2$ seconds?

AP Calculus   PVA (Position-Velocity-Acceleration) Notes

Instantaneous velocity ,v(t), of the object is the derivative of the position function s(t) with respect to time

$v(t) = s'(t)$

Acceleration, a(t), is the derivative of velocity with respect to time

$a(t) = v'(t) = s''(t)$

AVERAGE rate of change of f(x) from a to b = slope of secant = $\dfrac{f(b)-f(a)}{b-a}$

INSTANTANEOUS rate of change of f(x) at x = c = slope of tangent = $f'(c)$

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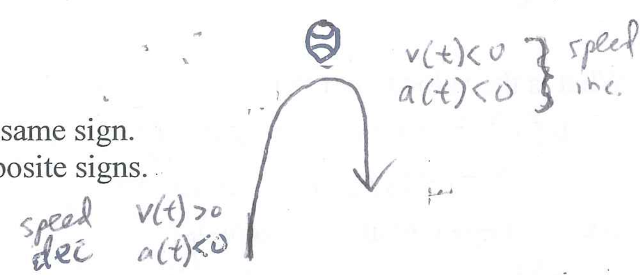


Particle Motion

Particle motion (linear motion) describes the object moving along a line (usually along a horizontal line)

x(t) = Position function
v(t) = velocity function
a(t) = acceleration function

Positive velocity indicates particle moving in positive direction (usually right)

Negative velocity indicates " " " negative direction (usually left)

When v(t) = 0, this indicates particle is at rest

A.P. Calculus PVA **Worksheet 2-2a**

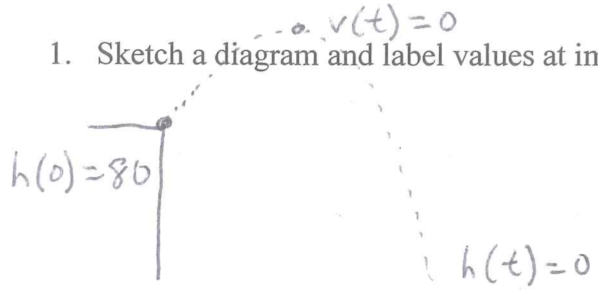
A ball is thrown vertically upwards from the edge of a building and it eventually hits the ground next to the building. If the height of the ball at any given time, $t \geq 0$ (seconds), is $h(t) = -16t^2 + 64t + 80$ (feet), answer the following:

$h(t) = -16t^2 + 64t + 80$

$v(t) = -32t + 64$

$a(t) = -32$

1. Sketch a diagram and label values at important places



2. How tall is the building?

$h(0) = 80$ ft.

3. When does the ball reach maximum height?

find t when $v(t) = 0$

$0 = -32t + 64$

$32t = 64$

$\boxed{t = 2 \text{ sec}}$

4. What is the maximum height?

find $h(2)$

$h(2) = -16(2)^2 + 64(2) + 80 = \boxed{144 \text{ ft}}$

5. How long does it take to hit the ground?

set $h(t) = 0$

$0 = -16t^2 + 64t + 80$

$0 = 16t^2 - 64t - 80$

$0 = t^2 - 4t - 5$

$0 = (t-5)(t+1)$

$t = 5$, $\cancel{t = -1}$ $\quad t = 5$ secs.

6. What was the initial velocity?

$v(0) = -32(0) + 64 = 64$

7. What is the velocity at $t = 1$ second? At $t = 2$ seconds?

$v(1) = -32 + 64 = 32$ ft/s

$v(2) = -32(2) + 64 = 0$ ft/s

8. What is the height at $t = 3$ seconds?

$h(3) = -16(3)^2 + 64(3) + 80 = 128$ ft.

9. What is the speed when it hits the ground?

$v(5) = -32(5) + 64 = -96 \Rightarrow 96$ ft/s

10. What is the acceleration at $t = 1$ second? At $t = 2$ seconds?

$a(1) = -32$ ft/s^2

$a(2) = -32$ ft/s^2

11) Avg. velocity $[0, 2]$

$\dfrac{h(2) - h(0)}{2 - 0} = \dfrac{144 - 80}{2 - 0} = 32$ ft/s

12) Avg. acceleration $[1, 2]$

$\dfrac{v(2) - v(1)}{2 - 1} = \dfrac{0 - 32}{1} = -32$ ft/s^2

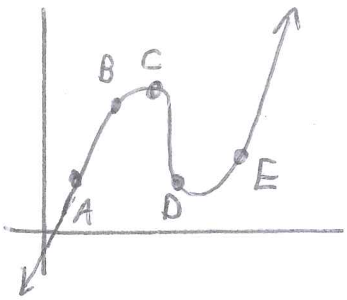
13) inc. or dec. speed at $t = 1$ sec.

$\left.\begin{array}{l} v(1) = 32 \text{ ft/s} \\ a(1) = -32 \text{ ft/s} \end{array}\right]$ dec. speed

2.2b Homework p.116-118 #67, 89-93 odd, 94-97

#99, 104, 106, 109, 113

67)



a) Greatest Avg. rate of change (slope steepest) between A and B

b) Avg. ROC between A and B is <u>greater</u> than instantaneous R.O.C. at B

89) Find Avg. ROC and compare with instaneous ROC at endpts.

Avg. ROC = slope between endpts. $f(t) = 2t + 7 \quad [1, 2]$

$f(1) = 9$

$f(2) = 11$

Avg. ROC $= \frac{11-9}{2-1} = \frac{2}{1} = \boxed{2}$ | $f'(t) = 2$

Avg. ROC $= f'(t) = 2$

91) $f(x) = \frac{-1}{x} \quad [1, 2]$

$f(1) = -1$

$f(2) = \frac{-1}{2}$

$m_{Avg} = \frac{\frac{-1}{2} - -1}{2-1} = \boxed{\frac{1}{2}}$

$f(x) = \frac{-1}{x} = -x^{-1}$

$f'(x) = x^{-2} = \frac{1}{x^2}$

$f'(1) = \boxed{1}$

$f'(2) = \boxed{\frac{1}{4}}$

~~93) $s(t) = -16t^2 + v_0 t + s_0$ → silver dollar dropped from 1362 ft tall building~~

~~a) $v_0 = 0$ $s_0 = 1362$ $s(t) = -16t^2 + 1362$ $v(t) = -32t$~~

height of object at given time

initial velocity

initial position

93) $s(t) = -16t^2 + V_0 t + S_0$ → silver dollar dropped from 1362 ft tall building

a) $V_0 = 0$ $S_0 = 1362$ $s(t) = -16t^2 + 1362$ $V(t) = -32t$

b) Find <u>avg. velocity</u> → find slope between endpts: $\frac{s(2) - s(1)}{2 - 1}$ $[1, 2]$

$s(2) = 1298$
$s(1) = 1346$

avg. velocity $= \frac{1298 - 1346}{2 - 1} = \boxed{-48 \text{ ft/s}}$

c) $v(t) = s'(t) = -32t$

$v(1) = -32$ ft/s
$v(2) = -64$ ft/s

d) Find time for coin to reach ground: set $s(t) = 0$

$0 = -16t^2 + 1362$ $t^2 = \frac{1362}{16}$ $\boxed{t \approx 9.23 \text{ secs.}}$

e) Find velocity of coin at impact

$v(9.23) = -32(9.23) \approx \boxed{-295.242 \text{ ft/s}}$

94) Ball thrown down from 220 ft building, initial velocity $= -22$ ft/s

ⓐ Find velocity at 3 secs. ⓑ Find velocity after falling 108 ft.

$s(t) = -16t^2 + V_0 t + S_0$
$s(t) = -16t^2 - 22t + 220$
$v(t) = -32t - 22$

ⓐ $v(3) = -32(3) - 22 = -118$ ft/s

b) height is 112 ft (after falling 108 ft)

$-16t^2 - 22t + 220 = 112$
$-16t^2 - 22t + 108 = 0$
$-2(8t^2 + 11t - 54) = 0$
$-2(8t + 27)(t - 2)$ $t = 2, \cancel{-\frac{27}{8}}$
$v(2) = -32(2) - 22 = \boxed{-86 \text{ ft/s}}$

95) $s(t) = -4.9t^2 + V_0 t + S_0$ $\quad V_0 = 120\text{m/s} \quad S_0 = 0$

ⓐ Find velocity after 5 secs, ⓑ find velocity after 10 secs.

$s(t) = -4.9t^2 + 120t$

$v(t) = -9.8t + 120$

$v(5) = 71\text{ m/s} \qquad v(10) = 22\text{m/s}$

96) stone dropped from top of building into pool of water at ground level: How high is building if splash seen 6.8 secs. after stone is dropped.

$s(t) = -4.9t^2 + V_0 t + S_0 \qquad V_0 = 0 \qquad \text{Find } S_0$

$\downarrow$

$0 = -4.9(6.8)^2 + 0t + S_0$

$\boxed{226.6\text{m} = S_0}$

113) find a, b such that f is <u>differentiable</u> everywhere.

$f(x) = \begin{cases} ax^3, & x \leq 2 \\ x^2 + b, & x > 2 \end{cases}$

* set equations equal
* set derivatives equal

$ax^3 = x^2 + b$ at $x = 2$

$a(2)^3 = 2^2 + b$

$8a = 4 + b$

$3ax^2 = 2x$ at $x = 2$

$3a(2)^2 = 2(2)$

$12a = 4$

$\underline{\underline{a = 1/3}}$

$8(1/3) = 4 + b$

$\frac{8}{3} - 4 = b$

$\underline{\underline{\frac{-4}{3} = b}}$