Ch. 2.3 Notes Product and Quotient Rules

Product Rule: formula used to find the derivatives of products of two or more functions

$$* \quad \frac{d}{dx}\left[f(x)\cdot g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

"f prime g plus f g prime"

$\boxed{\text{Ex. 1}}$ $y = \underbrace{(3x-2x^2)}_{f(x)}\underbrace{(5+4x)}_{g(x)}$ Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \underbrace{(3-4x)}_{f'}\underbrace{(5+4x)}_{g} + \underbrace{(3x-2x^2)}_{f}\underbrace{(4)}_{g'}$$

Quotient Rule: formula for finding derivative of function that is the quotient of two other functions.

$$* \quad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$\boxed{\text{Ex. 2}}$ $y = \frac{3x-2x^2}{5+4x}$ Find y'

$$y' = \frac{\overbrace{(3-4x)}^{f'}\overbrace{(5+4x)}^{g} - \overbrace{(3x-2x^2)}^{f}\overbrace{(4)}^{g'}}{\underbrace{(5+4x)^2}_{g^2}}$$

Higher order derivatives

$\boxed{\text{Ex. 3}}$ $y = 2x^5 + x^4 - 3x^3 - 8x^2 + 10x - 12$. Find y''''

$$y' = 10x^4 + 4x^3 - 9x^2 - 16x + 10$$
$$y'' = 40x^3 + 12x^2 - 18x - 16$$
$$y''' = 120x^2 + 24x - 18$$
$$y'''' = 240x + 24$$

* Notations

Notations for 1st derivative: $f'(x)$, $g'(x)$, y', $\frac{dy}{dx}$

Notation for 2nd derivative: $f''(x)$, $y''(x)$, y'', $\frac{d^2y}{dx^2}$

*Note: This means "2nd derivative", NOT "square the 1st derivative"

Notation for 3rd derivative: $f'''(x)$, y''', $\frac{d^3y}{dx^3}$

Ch. 2.3 Homework p. 126-129 #13, 15, 19-33 odd, 69, 73, 77, 81, 87, 93, 99-103 odd, 105-108, 115, 118, 129-133 odd

19) $y = \frac{x^2+2x}{3} = \frac{x^2}{3} + \frac{2}{3}x = \frac{1}{3}x^2 + \frac{2}{3}x$ $\quad \boxed{y' = \frac{2}{3}x + \frac{2}{3}}$

21) $y = \frac{7}{3x^3} = \frac{7}{3}x^{-3}$ $\quad y' = -3 \cdot \frac{7}{3}x^{-4} = \boxed{\frac{-7}{x^4}}$

23) $y = \frac{4x^{3/2}}{x^1} = 4x^{3/2-1} = 4x^{1/2}$ $\quad y' = \frac{1}{2} \cdot 4x^{-1/2} = \boxed{\frac{2}{x^{1/2}}}$

27) $f(x) = x\left(1 - \frac{4}{x+3}\right) = x - \frac{4x}{x+3}$ $\quad f'(x) = 1 - \frac{4(x+3)-(4x)(1)}{(x+3)^2}$

$= \boxed{1 - \frac{12}{(x+3)^2}}$

29) $f(x) = \frac{2x+5}{\sqrt{x}} = \frac{2x}{x^{1/2}} + \frac{5}{x^{1/2}} = 2x^{1/2} + 5x^{-1/2}$

$f'(x) = x^{-1/2} - \frac{5}{2}x^{-3/2} = \boxed{\frac{1}{x^{1/2}} - \frac{5}{2x^{3/2}}}$

33) $f(x) = \frac{2 - 1/x}{x - 3} \cdot \frac{x}{x} = \frac{2x-1}{x^2-3x}$ $\quad f'(x) = \frac{(2)(x^2-3x)-(2x-1)(2x-3)}{(x^2-3x)^2}$

69) Find equation of tangent line: $f(x) = \frac{8}{x^2+4}$ $(2,1)$

$f'(x) = \frac{(0)(x^2+4) - 8(2x)}{(x^2+4)^2}$

$f'(x) = \frac{-16x}{(x^2+4)^2}$

$f'(2) = \frac{-16(2)}{(2^2+4)^2} = \frac{-32}{64} = \frac{-1}{2}$

point: $(2,1)$

slope: $m = -1/2$

$\boxed{y - 1 = \frac{-1}{2}(x-2)}$

73) Determine where function has horizontal tangent line

$f(x) = \frac{x^2}{x-1}$ * set $f'(x) = 0$

$$f'(x) = \frac{(2x)(x-1) - (x^2)(1)}{(x-1)^2} = \frac{2x^2-2x-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

* when $f'(x) = 0$, set just the numerator of $f'(x) = 0$:

$\frac{x(x-2)}{(x-1)^2} = 0$ when $x(x-2) = 0$ $f'(x) = 0$ when $x = 0, x = 2$

$f(x) = \frac{x^2}{x-1}$ $f(0) = \frac{0}{-1} = 0$

$f(2) = \frac{4}{2-1} = 4$

$\boxed{(0,0) \text{ and } (2,4)}$

77) Find equation of tangent line to $f(x) = \frac{x+1}{x-1}$, parallel to line $2y + x = 6$

* set $f'(x) =$ slope of line

$$f'(x) = \frac{(1)(x-1) - (x+1)(1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

line: $2y + x = 6$

$2y = -x + 6$

$y = -\frac{1}{2}x + 3$ so slope $= -\frac{1}{2}$

set $\frac{-2}{(x-1)^2} = \frac{-1}{2}$ $(x-1)^2 = 4$

$x - 1 = \pm 2$

$x = -1, 3$ $f(-1) = 0$, $f(3) = 2$

slope: $m = \frac{-1}{2}$ slope: $m = \frac{-1}{2}$

$$\boxed{\begin{aligned} y - 0 &= \tfrac{-1}{2}(x+1) \\ y - 2 &= \tfrac{-1}{2}(x-3) \end{aligned}}$$