Ch. 2.5 Notes Implicit Differentiation

Explicit equations: Equations where x's and y's are separated on different sides of the equation. (example: $y = 9x^2 + 4\sqrt{x} + 8$)
(solved for y)

Implicit equations: Equations where x's and y's are mixed together on same side(s) of the equation
(not solved for y)
(example: $y^2 = xy - x^2$)

Explicit Differentiation	Implicit Differentiation
$y = 3x^2 - 9x^3 + 5$	$y^2 - 5x = 4$
$\boxed{\frac{dy}{dx} = 6x - 27x^2}$	$2y\left(\frac{dy}{dx}\right) - 5 = 0$ $\quad \boxed{\frac{dy}{dx} = \frac{5}{2y}}$

Steps:

1) Take derivative of each term with respect to x
2) If variable is y, find derivative and attach $\frac{dy}{dx}$ to the derivative
3) Move all terms containing $\frac{dy}{dx}$ to left side of equation.
4) Move all other terms to right side of equation.
5) Factor out $\frac{dy}{dx}$ on left side of equation
6) Solve for $\frac{dy}{dx}$

$\boxed{\text{Ex.1}}$ $x^2 - 2y^3 + 4y = 2$ Find $\frac{dy}{dx}$

$$2x - 6y^2\left(\frac{dy}{dx}\right) + 4\left(\frac{dy}{dx}\right) = 0$$

$$-6y^2\frac{dy}{dx} + 4\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx}(4 - 6y^2) = -2x$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{4 - 6y^2}}$$

Ex. 2

$3xy^3 - 2y = 7$ Find $\frac{dy}{dx}$ or y'

*product rule
$f'g + fg'$

$$\underbrace{3x}_{f}\underbrace{y^3}_{g} - 2y = 7$$

$$\underbrace{(3)}_{f'}\underbrace{(y^3)}_{g} + \underbrace{(3x)}_{f}\underbrace{(3y^2)\frac{dy}{dx}}_{g'} - 2\left(\frac{dy}{dx}\right) = 0$$

$$9xy^2\frac{dy}{dx} - 2\frac{dy}{dx} = -3y^3$$

$$\frac{dy}{dx}(9xy^2 - 2) = -3y^3$$

$$\boxed{\frac{dy}{dx} = \frac{-3y^3}{9xy^2 - 2}}$$

Ex. 3

Differentiate $y^2 = 5x$ with respect to t

$$2y\left(\frac{dy}{dt}\right) = 5\left(\frac{dy}{dt}\right)$$

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5) $x^3 - xy + y^2 = 4$

$3x^2 + (-1)y + (-x)\left(\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$

$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = -3x^2 + y$

$\frac{dy}{dx}(-x + 2y) = -3x^2 + y$

$\frac{dy}{dx} = \frac{y - 3x^2}{-x + 2y}$

9) $x^3 - 3x^2y + 2xy^2 = 12$

$3x^2 + (-6x)y + (-3x^2)\left(\frac{dy}{dx}\right) + 2y^2 + 2x(2y)\frac{dy}{dx} = 0$

$-3x^2\frac{dy}{dx} + 4xy\frac{dy}{dx} = -3x^2 + 6xy - 2y^2$

$\frac{dy}{dx}(-3x^2 + 4xy) = -3x^2 + 6xy - 2y^2$

$\frac{dy}{dx} = \frac{-3x^2 + 6xy - 2y^2}{-3x^2 + 4xy}$

17) $x^2 + y^2 = 16$

Explicitly

$y^2 = -x^2 + 16$

$y = \pm\sqrt{16 - x^2}$

$\frac{dy}{dx} = \pm\frac{1}{2}(16 - x^2)^{-1/2}(-2x)$

$= \frac{-x}{\pm\sqrt{16-x^2}} = \boxed{\frac{-x}{y}}$

Implicitly $x^2 + y^2 = 16$

$2x + 2y\left(\frac{dy}{dx}\right) = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y} = \boxed{\frac{-x}{y}}$

Evaluate derivative at given pt.

23) $y^2 = \frac{x^2-4}{x^2+4}$ $(2,0)$

$$2y\frac{dy}{dx} = \frac{(2x)(x^2+4)-(x^2-4)(2x)}{(x^2+4)^2} = \frac{2x^3+8x-2x^3+8x}{(x^2+4)^2} = \frac{16x}{(x^2+4)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{8x}{y(x^2+4)^2}} \qquad \frac{dy}{dx}_{(2,0)} = \frac{8(2)}{0(8)^2} = \text{undefined}$$

25) $x^{2/3}+y^{2/3}=5$ $(8,1)$

$$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\left(\frac{dy}{dx}\right)=0$$

$$\frac{2}{3x^{1/3}}+\frac{2}{3y^{1/3}}\frac{dy}{dx}=0$$

$$\frac{2}{3y^{1/3}}\frac{dy}{dx} = \frac{-2}{3x^{1/3}}$$

$$\frac{dy}{dx} = \frac{3y^{1/3}}{2}\cdot\frac{-2}{3x^{1/3}} = \boxed{\frac{-y^{1/3}}{x^{1/3}}}$$

$$\frac{dy}{dx}_{(8,1)} = \frac{-1^{1/3}}{8^{1/3}} = \boxed{\frac{-1}{2}}$$

29) Find slope of tangent line

$(x^2+4)y=8$ $(2,1)$

*product rule

$$2xy+(x^2+4)\frac{dy}{dx}=0$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2+4}$$

$$\frac{dy}{dx}_{(2,1)} = \frac{-2(2)(1)}{2^2+4} = \frac{-4}{8} = \boxed{\frac{-1}{2}}$$

31) $(x^2+y^2)^2 = 4x^2y$ $(1,1)$ *chain rule, product rule

$$2(x^2+y^2)\left(2x+2y\left(\frac{dy}{dx}\right)\right) = 8xy+4x^2\left(\frac{dy}{dx}\right)$$

$$2(1+1)\left(2+2\frac{dy}{dx}\right) = 8(1)(1)+4(1)\frac{dy}{dx}$$

$$4\left(2+2\frac{dy}{dx}\right) = 8+4\frac{dy}{dx}$$

$$8+8\frac{dy}{dx} = 8+4\frac{dy}{dx}$$

$$4\frac{dy}{dx}=0$$

$$\boxed{\frac{dy}{dx}=0}$$