Review Exercises for Chapter 2

1. $f(x) = 12$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{12 - 12}{\Delta x} = \lim_{\Delta x \to 0} \frac{0}{\Delta x} = 0$$

2. $f(x) = 5x - 4$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{[5(x + \Delta x) - 4] - (5x - 4)}{\Delta x} = \lim_{\Delta x \to 0} \frac{5x + 5\Delta x - 4 - 5x + 4}{\Delta x} = \lim_{\Delta x \to 0} \frac{5\Delta x}{\Delta x} = 5$$

3. $f(x) = x^2 - 4x + 5$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 - 4(x + \Delta x) + 5\right] - \left[x^2 - 4x + 5\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left(x^2 + 2x(\Delta x) + (\Delta x)^2 - 4x - 4(\Delta x) + 5\right) - \left(x^2 - 4x + 5\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 - 4(\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 4) = 2x - 4$$

4. $f(x) = \dfrac{6}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{6}{x + \Delta x} - \dfrac{6}{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{6x - (6x + 6\,\Delta x)}{\Delta x(x + \Delta x)x} = \lim_{\Delta x \to 0} \frac{-6\,\Delta x}{\Delta x(x + \Delta x)x} = \lim_{\Delta x \to 0} \frac{-6}{(x + \Delta x)x} = \frac{-6}{x^2}$$

5. $g(x) = 2x^2 - 3x$, $c = 2$

$$g'(2) = \lim_{x \to 2} \frac{g(x) - g(2)}{x - 2} = \lim_{x \to 2} \frac{(2x^2 - 3x) - 2}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(2x + 1)}{x - 2} = \lim_{x \to 2} (2x + 1) = 2(2) + 1 = 5$$

6. $f(x) = \dfrac{1}{x + 4}$, $c = 3$

$$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3} \frac{\dfrac{1}{x + 4} - \dfrac{1}{7}}{x - 3} = \lim_{x \to 3} \frac{7 - x - 4}{(x - 3)(x + 4)7} = \lim_{x \to 3} \frac{-1}{(x + 4)7} = -\frac{1}{49}$$

7. f is differentiable for all $x \neq 3$.

8. f is differentiable for all $x \neq -1$.

9. $y = 25$

$y' = 0$

10. $f(t) = 4t^4$
$f'(t) = 16t^3$

11. $f(x) = x^3 - 11x^2$
$f'(x) = 3x^2 - 22x$

12. $g(s) = 3s^5 - 2s^4$
$g'(s) = 15s^4 - 8s^3$

13. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x} = 6x^{1/2} + 3x^{1/3}$
$h'(x) = 3x^{-1/2} + x^{-2/3} = \dfrac{3}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^2}}$

14. $f(x) = x^{1/2} - x^{-1/2}$
$f'(x) = \dfrac{1}{2}x^{-1/2} + \dfrac{1}{2}x^{-3/2} = \dfrac{x+1}{2x^{3/2}}$

15. $g(t) = \dfrac{2}{3}t^{-2}$
$g'(t) = \dfrac{-4}{3}t^{-3} = -\dfrac{4}{3t^3}$

16. $h(x) = \dfrac{8}{5x^4} = \dfrac{8}{5}x^{-4}$
$h'(x) = -\dfrac{32}{5}x^{-5} = -\dfrac{32}{5x^5}$

17. $f(\theta) = 4\theta - 5\sin\theta$
$f'(\theta) = 4 - 5\cos\theta$

18. $g(\alpha) = 4\cos\alpha + 6$
$g'(\alpha) = -4\sin\alpha$

19. $f(\theta) = 3\cos\theta - \dfrac{\sin\theta}{4}$
$f'(\theta) = -3\sin\theta - \dfrac{\cos\theta}{4}$

20. $g(\alpha) = \dfrac{5\sin\alpha}{3} - 2\alpha$
$g'(\alpha) = \dfrac{5\cos\alpha}{3} - 2$

21. $f(x) = \dfrac{27}{x^3} = 27x^{-3}, (3, 1)$
$f'(x) = 27(-3)x^{-4} = -\dfrac{81}{x^4}$
$f'(3) = -\dfrac{81}{3^4} = -1$

22. $f(x) = 3x^2 - 4x, (1, -1)$
$f'(x) = 6x - 4$
$f'(1) = 6 - 4 = 2$

23. $f(x) = 2x^4 - 8, (0, -8)$
$f'(x) = 8x^3$
$f'(0) = 0$

24. $f(\theta) = 3\cos\theta - 2\theta, (0, 3)$
$f'(\theta) = -3\sin\theta - 2$
$f'(0) = -3\sin(0) - 2 = -2$

25. $F = 200\sqrt{T}$
$F'(t) = \dfrac{100}{\sqrt{T}}$

(a) When $T = 4$, $F'(4) = 50$ vibrations/sec/lb.

(b) When $T = 9$, $F'(9) = 33\frac{1}{3}$ vibrations/sec/lb.

26. $S = 6l^2$
$\dfrac{dS}{dl} = 12l$

(a) When $l = 3$, $\dfrac{dS}{dl} = 12(3) = 36$ in.^{2}/in.

(b) When $l = 5$, $\dfrac{dS}{dl} = 12(5) = 60$ in.^{2}/in.

27. $s(t) = -16t^2 + v_0t + s_0;\ s_0 = 600, v_0 = -30$

(a) $s(t) = -16t^2 - 30t + 600$
$s'(t) = v(t) = -32t - 30$

(b) Average velocity $= \dfrac{s(3) - s(1)}{3 - 1}$
$= \dfrac{366 - 554}{2}$
$= -94$ ft/sec

(c) $v(1) = -32(1) - 30 = -62$ ft/sec
$v(3) = -32(3) - 30 = -126$ ft/sec

(d) $s(t) = 0 = -16t^2 - 30t + 600$
Using a graphing utility or the Quadratic Formula, $t \approx 5.258$ seconds.

(e) When
$t \approx 5.258$, $v(t) \approx -32(5.258) - 30 \approx -198.3$ ft/sec.

28. $s(t) = -16t^2 + s_0$

$s(9.2) = -16(9.2)^2 + s_0 = 0$

$s_0 = 1354.24$

The building is approximately 1354 feet high (or 415 m).

29. $f(x) = (5x^2 + 8)(x^2 - 4x - 6)$

$f'(x) = (5x^2 + 8)(2x - 4) + (x^2 - 4x - 6)(10x)$

$= 10x^3 + 16x - 20x^2 - 32 + 10x^3 - 40x^2 - 60x$

$= 20x^3 - 60x^2 - 44x - 32$

$= 4(5x^3 - 15x^2 - 11x - 8)$

30. $g(x) = (2x^3 + 5x)(3x - 4)$

$g'(x) = (2x^3 + 5x)(3) + (3x - 4)(6x^2 + 5)$

$= 6x^3 + 15x + 18x^3 - 24x^2 + 15x - 20$

$= 24x^3 - 24x^2 + 30x - 20$

31. $h(x) = \sqrt{x}\sin x = x^{1/2}\sin x$

$h'(x) = \dfrac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x$

32. $f(t) = 2t^5\cos t$

$f'(t) = 2t^5(-\sin t) + \cos t(10t^4)$

$= -2t^5\sin t + 10t^4\cos t$

33. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

$f'(x) = \dfrac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2}$

$= \dfrac{-(x^2 + 1)}{(x^2 - 1)^2}$

34. $f(x) = \dfrac{2x + 7}{x^2 + 4}$

$f'(x) = \dfrac{(x^2 + 4)(2) - (2x + 7)(2x)}{(x^2 + 4)^2}$

$= \dfrac{2x^2 + 8 - 4x^2 - 14x}{(x^2 + 4)^2}$

$= \dfrac{-2x^2 - 14x + 8}{(x^2 + 4)^2} = \dfrac{-2(x^2 + 7x - 4)}{(x^2 + 4)^2}$

35. $y = \dfrac{x^4}{\cos x}$

$y' = \dfrac{(\cos x)4x^3 - x^4(-\sin x)}{\cos^2 x}$

$= \dfrac{4x^3\cos x + x^4\sin x}{\cos^2 x}$

36. $y = \dfrac{\sin x}{x^4}$

$y' = \dfrac{(x^4)\cos x - (\sin x)(4x^3)}{(x^4)^2} = \dfrac{x\cos x - 4\sin x}{x^5}$

37. $y = 3x^2\sec x$

$y' = 3x^2\sec x\tan x + 6x\sec x$

38. $y = 2x - x^2\tan x$

$y' = 2 - x^2\sec^2 x - 2x\tan x$

39. $y = x\cos x - \sin x$

$y' = -x\sin x + \cos x - \cos x = -x\sin x$

40. $g(x) = 3x\sin x + x^2\cos x$

$g'(x) = 3x\cos x + 3\sin x - x^2\sin x + 2x\cos x$

$= 5x\cos x + (3 - x^2)\sin x$

41. $f(x) = (x + 2)(x^2 + 5), (-1, 6)$

$f'(x) = (x + 2)(2x) + (x^2 + 5)(1)$

$= 2x^2 + 4x + x^2 + 5 = 3x^2 + 4x + 5$

$f'(-1) = 3 - 4 + 5 = 4$

Tangent line: $y - 6 = 4(x + 1)$

$y = 4x + 10$

42. $f(x) = (x - 4)(x^2 + 6x - 1), (0, 4)$

$f'(x) = (x - 4)(2x + 6) + (x^2 + 6x - 1)(1)$

$= 2x^2 - 2x - 24 + x^2 + 6x - 1$

$= 3x^2 + 4x - 25$

$f'(0) = 0 + 0 - 25 = -25$

Tangent line: $y - 4 = -25(x - 0)$

$y = -25x + 4$

43. $f(x) = \frac{x+1}{x-1}, \left(\frac{1}{2}, -3\right)$

$f'(x) = \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$

$f'\left(\frac{1}{2}\right) = \frac{-2}{(1/4)} = -8$

Tangent line: $y + 3 = -8\left(x - \frac{1}{2}\right)$

$y = -8x + 1$

44. $f(x) = \frac{1+\cos x}{1-\cos x}, \left(\frac{\pi}{2}, 1\right)$

$f'(x) = \frac{(1-\cos x)(-\sin x) - (1+\cos x)(\sin x)}{(1-\cos x)^2}$

$= \frac{-2\sin x}{(1-\cos x)^2}$

$f'\left(\frac{\pi}{2}\right) = \frac{-2}{1} = -2$

Tangent line: $y - 1 = -2\left(x - \frac{\pi}{2}\right)$

$y = -2x + 1 + \pi$

45. $g(t) = -8t^3 - 5t + 12$

$g'(t) = -24t^2 - 5$

$g''(t) = -48t$

46. $h(x) = 6x^{-2} + 7x^2$

$h'(x) = -12x^{-3} + 14x$

$h''(x) = 36x^{-4} + 14 = \frac{36}{x^4} + 14$

47. $f(x) = 15x^{5/2}$

$f'(x) = \frac{75}{2}x^{3/2}$

$f''(x) = \frac{225}{4}x^{1/2} = \frac{225}{4}\sqrt{x}$

48. $f(x) = 20\sqrt[5]{x} = 20x^{1/5}$

$f'(x) = 4x^{-4/5}$

$f''(x) = \frac{-16}{5}x^{-9/5} = -\frac{16}{5x^{9/5}}$

49. $f(\theta) = 3\tan\theta$

$f'(\theta) = 3\sec^2\theta$

$f''(\theta) = 6\sec\theta(\sec\theta\tan\theta)$

$= 6\sec^2\theta\tan\theta$

50. $h(t) = 10\cos t - 15\sin t$

$h'(t) = -10\sin t - 15\cos t$

$h''(t) = -10\cos t + 15\sin t$

51. $v(t) = 20 - t^2, 0 \le t \le 6$

$a(t) = v'(t) = -2t$

$v(3) = 20 - 3^2 = 11$ m/sec

$a(3) = -2(3) = -6$ m/sec^2

52. $v(t) = \frac{90t}{4t + 10}$

$a(t) = \frac{(4t+10)90 - 90t(4)}{(4t+10)^2}$

$= \frac{900}{(4t+10)^2} = \frac{225}{(2t+5)^2}$

(a) $v(1) = \frac{90}{14} \approx 6.43$ ft/sec

$a(1) = \frac{225}{49} \approx 4.59$ ft/sec^2

(b) $v(5) = \frac{90(5)}{30} = 15$ ft/sec

$a(5) = \frac{225}{15^2} = 1$ ft/sec^2

(c) $v(10) = \frac{90(10)}{50} = 18$ ft/sec

$a(10) = \frac{225}{25^2} = 0.36$ ft/sec^2

53. $y = (7x+3)^4$

$y' = 4(7x+3)^3(7) = 28(7x+3)^3$

54. $y = (x^2 - 6)^3$

$y' = 3(x^2-6)^2(2x) = 6x(x^2-6)^2$

55. $y = \frac{1}{x^2+4} = (x^2+4)^{-1}$

$y' = -1(x^2+4)^{-2}(2x) = -\frac{2x}{(x^2+4)^2}$

56. $f(x) = \frac{1}{(5x+1)^2} = (5x+1)^{-2}$

$f'(x) = -2(5x+1)^{-3}(5) = -\frac{10}{(5x+1)^3}$

57. $y = 5\cos(9x+1)$

$y' = -5\sin(9x+1)(9) = -45\sin(9x+1)$

58. $y = 1 - \cos 2x + 2\cos^2 x$

$y' = 2\sin 2x - 4\cos x \sin x$

$= 2[2\sin x \cos x] - 4\sin x \cos x$

$= 0$

59. $y = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$

$y' = \dfrac{1}{2} - \dfrac{1}{4}\cos 2x(2) = \dfrac{1}{2}(1 - \cos 2x) = \sin^2 x$

60. $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

$y' = \sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x)$

$= \sec^5 x \tan x(\sec^2 x - 1)$

$= \sec^5 x \tan^3 x$

61. $y = x(6x + 1)^5$

$y' = x\,5(6x + 1)^4(6) + (6x + 1)^5(1)$

$= 30x(6x + 1)^4 + (6x + 1)^5$

$= (6x + 1)^4(30x + 6x + 1)$

$= (6x + 1)^4(36x + 1)$

62. $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$

$f'(s) = (s^2 - 1)^{5/2}(3s^2) + (s^3 + 5)\left(\tfrac{5}{2}\right)(s^2 - 1)^{3/2}(2s)$

$= s(s^2 - 1)^{3/2}\left[3s(s^2 - 1) + 5(s^3 + 5)\right]$

$= s(s^2 - 1)^{3/2}(8s^3 - 3s + 25)$

63. $f(x) = \dfrac{3x}{\sqrt{x^2 + 1}}$

$f'(x) = \dfrac{3(x^2 + 1)^{1/2} - 3x\frac{1}{2}(x^2 + 1)^{-1/2}(2x)}{x^2 + 1}$

$= \dfrac{3(x^2 + 1) - 3x^2}{(x^2 + 1)^{3/2}} = \dfrac{3}{(x^2 + 1)^{3/2}}$

64. $h(x) = \left(\dfrac{x + 5}{x^2 + 3}\right)^2$

$h'(x) = 2\left(\dfrac{x + 5}{x^2 + 3}\right)\left(\dfrac{(x^2 + 3)(1) - (x + 5)(2x)}{(x^2 + 3)^2}\right)$

$= \dfrac{2(x + 5)(-x^2 - 10x + 3)}{(x^2 + 3)^3}$

65. $f(x) = \sqrt{1 - x^3},\ (-2, 3)$

$f'(x) = \dfrac{1}{2}(1 - x^3)^{-1/2}(-3x^2) = \dfrac{-3x^2}{2\sqrt{1 - x^3}}$

$f'(-2) = \dfrac{-12}{2(3)} = -2$

66. $f(x) = \sqrt[3]{x^2 - 1},\ (3, 2)$

$f'(x) = \dfrac{1}{3}(x^2 - 1)^{-2/3}(2x) = \dfrac{2x}{3(x^2 - 1)^{2/3}}$

$f'(3) = \dfrac{2(3)}{3(4)} = \dfrac{1}{2}$

67. $f(x) = \dfrac{4}{x^2 + 1} = 4(x^2 + 1)^{-1},\ (-1, 2)$

$f'(x) = -4(x^2 + 1)^{-2}(2x) = -\dfrac{8x}{(x^2 + 1)^2}$

$f'(-1) = -\dfrac{8(-1)}{\left[(-1)^2 + 1\right]^2} = \dfrac{8}{4} = 2$

68. $f(x) = \dfrac{3x + 1}{4x - 3},\ (4, 1)$

$f'(x) = \dfrac{(4x - 3)(3) - (3x + 1)(4)}{(4x - 3)^2}$

$= \dfrac{12x - 9 - 12x - 4}{(4x - 3)^2}$

$= -\dfrac{13}{(4x - 3)^2}$

$f'(4) = -\dfrac{13}{(16 - 3)^2} = -\dfrac{1}{13}$

69. $y = \dfrac{1}{2}\csc 2x,\ \left(\dfrac{\pi}{4}, \dfrac{1}{2}\right)$

$y' = -\csc 2x \cot 2x$

$y'\left(\dfrac{\pi}{4}\right) = 0$

70. $y = \csc 3x + \cot 3x,\ \left(\dfrac{\pi}{6}, 1\right)$

$y' = -3\csc 3x \cot 3x - 3\csc^2 3x$

$y'\left(\dfrac{\pi}{6}\right) = 0 - 3 = -3$

71. $y = (8x + 5)^3$

$y' = 3(8x + 5)^2(8) = 24(8x + 5)^2$

$y'' = 24(2)(8x + 5)(8) = 384(8x + 5)$

72. $y = \dfrac{1}{5x+1} > (5x+1)^{-1}$

$y' = (-1)(5x+1)^{-2}(5) = -5(5x+1)^{-2}$

$y'' = (-5)(-2)(5x+1)^{-3}(5) = \dfrac{50}{(5x+1)^3}$

73. $f(x) = \cot x$

$f'(x) = -\csc^2 x$

$f''(x) = -2\csc x(-\csc x \cdot \cot x)$

$= 2\csc^2 x \cot x$

74. $y = \sin^2 x$

$y' = 2\sin x \cos x = \sin 2x$

$y'' = 2\cos 2x$

75. $T = \dfrac{700}{t^2 + 4t + 10}$

$T = 700(t^2 + 4t + 10)^{-1}$

$T' = \dfrac{-1400(t+2)}{(t^2+4t+10)^2}$

(a) When $t = 1$,

$T' = \dfrac{-1400(1+2)}{(1+4+10)^2} \approx -18.667$ deg/h.

(b) When $t = 3$,

$T' = \dfrac{-1400(3+2)}{(9+12+10)^2} \approx -7.284$ deg/h.

(c) When $t = 5$,

$T' = \dfrac{-1400(5+2)}{(25+20+10)^2} \approx -3.240$ deg/h.

(d) When $t = 10$,

$T' = \dfrac{-1400(10+2)}{(100+40+10)^2} \approx -0.747$ deg/h.

76. $y = \dfrac{1}{4}\cos 8t - \dfrac{1}{4}\sin 8t$

$y' = \dfrac{1}{4}(-\sin 8t)8 - \dfrac{1}{4}(\cos 8t)8$

$= -2\sin 8t - 2\cos 8t$

At time $t = \dfrac{\pi}{4}$,

$y\left(\dfrac{\pi}{4}\right) = \dfrac{1}{4}\cos\left[8\left(\dfrac{\pi}{4}\right)\right] - \dfrac{1}{4}\sin\left[8\left(\dfrac{\pi}{4}\right)\right]$

$= \dfrac{1}{4}(1) = \dfrac{1}{4}$ ft.

$v(t) = y'\left(\dfrac{\pi}{4}\right) = -2\sin\left[8\left(\dfrac{\pi}{4}\right)\right] - 2\cos\left[8\left(\dfrac{\pi}{4}\right)\right]$

$= -2(0) - 2(1) = -2$ ft/sec

77. $x^2 + y^2 = 64$

$2x + 2yy' = 0$

$2yy' = -2x$

$y' = -\dfrac{x}{y}$

78. $x^2 + 4xy - y^3 = 6$

$2x + 4xy' + 4y - 3y^2y' = 0$

$(4x - 3y^2)y' = -2x - 4y$

$y' = \dfrac{2x+4y}{3y^2 - 4x}$

79. $x^3y - xy^3 = 4$

$x^3y' + 3x^2y - x3y^2y' - y^3 = 0$

$x^3y' - 3xy^2y' = y^3 - 3x^2y$

$y'(x^3 - 3xy^2) = y^3 - 3x^2y$

$y' = \dfrac{y^3 - 3x^2y}{x^3 - 3xy^2}$

$y' = \dfrac{y(y^2 - 3x^2)}{x(x^2 - 3y^2)}$

80. $\sqrt{xy} = x - 4y$

$\dfrac{\sqrt{x}}{2\sqrt{y}}y' + \dfrac{\sqrt{y}}{2\sqrt{x}} = 1 - 4y'$

$xy' + y = 2\sqrt{xy} - 8\sqrt{xy}y'$

$x + 8\sqrt{xy}y' = 2\sqrt{xy} - y$

$y' = \dfrac{2\sqrt{xy} - y}{x + 8\sqrt{xy}} = \dfrac{2(x-4y) - y}{x + 8(x-4y)} = \dfrac{2x - 9y}{9x - 32y}$

81.
$$x \sin y = y \cos x$$
$$(x \cos y)y' + \sin y = -y \sin x + y' \cos x$$
$$y'(x \cos y - \cos x) = -y \sin x - \sin y$$
$$y' = \frac{y \sin x + \sin y}{\cos x - x \cos y}$$

82.
$$\cos(x + y) = x$$
$$-(1 + y')\sin(x + y) = 1$$
$$-y' \sin(x + y) = 1 + \sin(x + y)$$
$$y' = -\frac{1 + \sin(x + y)}{\sin(x + y)} = -\csc(x + 1) - 1$$

83. $x^2 + y^2 = 10$

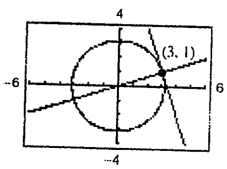
$2x + 2yy' = 0$

$y' = \frac{-x}{y}$

At $(3, 1)$, $y' = -3$

Tangent line: $y - 1 = -3(x - 3) \Rightarrow 3x + y - 10 = 0$

Normal line: $y - 1 = \frac{1}{3}(x - 3) \Rightarrow x - 3y = 0$



84. $x^2 - y^2 = 20$

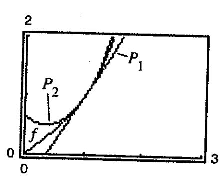
$2x - 2yy' = 0$

$y' = \frac{x}{y}$

At $(6, 4)$, $y' = \frac{3}{2}$

Tangent line:
$$y - 4 = \frac{3}{2}(x - 6)$$
$$y = \frac{3}{2}x - 5$$
$$2y - 3x + 10 = 0$$

Normal line:
$$y - 4 = -\frac{2}{3}(x - 6)$$
$$y = -\frac{2}{3}x + 8$$
$$3y + 2x - 24 = 0$$



85. $y = \sqrt{x}$

$\frac{dy}{dt} = 2$ units/sec

$\frac{dy}{dt} = \frac{1}{2\sqrt{x}}\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 2\sqrt{x}\frac{dy}{dt} = 4\sqrt{x}$

(a) When $x = \frac{1}{2}$, $\frac{dx}{dt} = 2\sqrt{2}$ units/sec.

(b) When $x = 1$, $\frac{dx}{dt} = 4$ units/sec.

(c) When $x = 4$, $\frac{dx}{dt} = 8$ units/sec.

86. Surface area $= A = 6x^2$, $x =$ length of edge

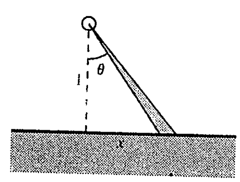
$\frac{dx}{dt} = 8$

$\frac{dA}{dt} = 12x\frac{dx}{dt} = 12(6.5)(8) = 624 \text{ cm}^2/\text{sec}$

87.
$$\tan\theta = x$$
$$\frac{d\theta}{dt} = 3(2\pi) \text{ rad/min}$$
$$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$$
$$\frac{dx}{dt} = (\tan^2\theta + 1)(6\pi) = 6\pi(x^2 + 1)$$

When $x = \frac{1}{2}$,

$\frac{dx}{dt} = 6\pi\left(\frac{1}{4} + 1\right) = \frac{15\pi}{2}$ km/min $= 450\pi$ km/h.



88. $s(t) = 60 - 4.9t^2$

$s'(t) = -9.8t$

$s = 35 = 60 - 4.9t^2$

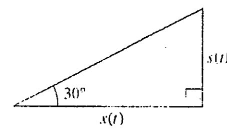
$4.9t^2 = 25$

$t = \frac{5}{\sqrt{4.9}}$

$\tan 30 = \frac{1}{\sqrt{3}} = \frac{s(t)}{x(t)}$

$x(t) = \sqrt{3}s(t)$

$\frac{dx}{dt} = \sqrt{3}\frac{ds}{dt} = \sqrt{3}(-9.8)\frac{5}{\sqrt{4.9}} \approx -38.34$ m/sec



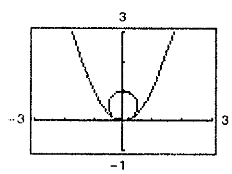
Problem Solving for Chapter 2

1. (a) $x^2 + (y - r)^2 = r^2$, Circle

$x^2 = y$, Parabola

Substituting:

$$(y - r)^2 = r^2 - y$$
$$y^2 - 2ry + r^2 = r^2 - y$$
$$y^2 - 2ry + y = 0$$
$$y(y - 2r + 1) = 0$$



Because you want only one solution, let $1 - 2r = 0 \Rightarrow r = \frac{1}{2}$. Graph $y = x^2$ and $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$.

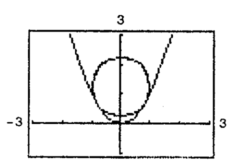
(b) Let (x, y) be a point of tangency:

$x^2 + (y - b)^2 = 1 \Rightarrow 2x + 2(y - b)y' = 0 \Rightarrow y' = \frac{x}{b - y}$, Circle

$y = x^2 \Rightarrow y' = 2x$, Parabola

Equating:

$$2x = \frac{x}{b - y}$$
$$2(b - y) = 1$$
$$b - y = \frac{1}{2} \Rightarrow b = y + \frac{1}{2}$$



Also, $x^2 + (y - b)^2 = 1$ and $y = x^2$ imply:

$$y + (y - b)^2 = 1 \Rightarrow y + \left[y - \left(y + \frac{1}{2}\right)\right]^2 = 1 \Rightarrow y + \frac{1}{4} = 1 \Rightarrow y = \frac{3}{4} \text{ and } b = \frac{5}{4}$$

Center: $\left(0, \frac{5}{4}\right)$

Graph $y = x^2$ and $x^2 + \left(y - \frac{5}{4}\right)^2 = 1$.

2. Let (a, a^2) and $(b, -b^2 + 2b - 5)$ be the points of tangency. For $y = x^2$, $y' = 2x$ and for $y = -x^2 + 2x - 5$, $y' = -2x + 2$. So, $2a = -2b + 2 \Rightarrow a + b = 1$, or $a = 1 - b$. Furthermore, the slope of the common tangent line is

$$\frac{a^2 - (-b^2 + 2b - 5)}{a - b} = \frac{(1 - b)^2 + b^2 - 2b + 5}{(1 - b) - b} = -2b + 2$$

$$\Rightarrow \frac{1 - 2b + b^2 + b^2 - 2b + 5}{1 - 2b} = -2b + 2$$

$$\Rightarrow 2b^2 - 4b + 6 = 4b^2 - 6b + 2$$

$$\Rightarrow 2b^2 - 2b - 4 = 0$$

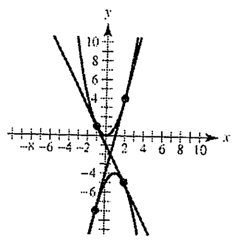
$$\Rightarrow b^2 - b - 2 = 0$$

$$\Rightarrow (b - 2)(b + 1) = 0$$

$$b = 2, -1$$

For $b = 2$, $a = 1 - b = -1$ and the points of tangency are $(-1, 1)$ and $(2, -5)$. The tangent line has slope -2: $y - 1 = -2(x = 1) \Rightarrow y = -2x - 1$

For $b = -1$, $a = 1 - b = 2$ and the points of tangency are $(2, 4)$ and $(-1, -8)$. The tangent line has slope 4: $y - 4 = 4(x - 2) \Rightarrow y = 4x - 4$



3. (a) $f(x) = \cos x$ $\quad P_1(x) = a_0 + a_1x$

$f(0) = 1$ $\quad P_1(0) = a_0 \Rightarrow a_0 = 1$

$f'(0) = 0$ $\quad P_1'(0) = a_1 \Rightarrow a_1 = 0$

$P_1(x) = 1$

(b) $f(x) = \cos x$ $\quad P_2(x) = a_0 + a_1x + a_2x^2$

$f(0) = 1$ $\quad P_2(0) = a_0 \Rightarrow a_0 = 1$

$f'(0) = 0$ $\quad P_2'(0) = a_1 \Rightarrow a_1 = 0$

$f''(0) = -1$ $\quad P_2''(0) = 2a_2 \Rightarrow a_2 = -\frac{1}{2}$

$P_2(x) = 1 - \frac{1}{2}x^2$

(c)

x	-1.0	-0.1	-0.001	0	0.001	0.1	1.0
$\cos x$	0.5403	0.9950	≈1	1	≈1	0.9950	0.5403
$P_2(x)$	0.5	0.9950	≈1	1	≈1	0.9950	0.5

$P_2(x)$ is a good approximation of $f(x) = \cos x$ when x is near 0.

(d) $f(x) = \sin x$ $\quad P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

$f(0) = 0$ $\quad P_3(0) = a_0 \Rightarrow a_0 = 0$

$f'(0) = 1$ $\quad P_3'(0) = a_1 \Rightarrow a_1 = 1$

$f''(0) = 0$ $\quad P_3''(0) = 2a_2 \Rightarrow a_2 = 0$

$f'''(0) = -1$ $\quad P_3'''(0) = 6a_3 \Rightarrow a_3 = -\frac{1}{6}$

$P_3(x) = x - \frac{1}{6}x^3$

4. (a) $y = x^2, y' = 2x$, Slope $= 4$ at $(2, 4)$

Tangent line: $y - 4 = 4(x - 2)$

$$y = 4x - 4$$

(b) Slope of normal line: $-\frac{1}{4}$

Normal line: $y - 4 = -\frac{1}{4}(x - 2)$

$$y = -\frac{1}{4}x + \frac{9}{2}$$

$$y = -\frac{1}{4}x + \frac{9}{2} = x^2$$

$$\Rightarrow 4x^2 + x - 18 = 0$$

$$\Rightarrow (4x + 9)(x - 2) = 0$$

$$x = 2, -\frac{9}{4}$$

Second intersection point: $\left(-\frac{9}{4}, \frac{81}{16}\right)$

(c) Tangent line: $y = 0$

Normal line: $x = 0$

(d) Let (a, a^2), $a \neq 0$, be a point on the parabola $y = x^2$.

Tangent line at (a, a^2) is $y = 2a(x - a) + a^2$.

Normal line at (a, a^2) is $y = -(1/2a)(x - a) + a^2$.

To find points of intersection, solve:

$$x^2 = -\frac{1}{2a}(x - a) + a^2$$

$$x^2 + \frac{1}{2a}x = a^2 + \frac{1}{2}$$

$$x^2 + \frac{1}{2a}x + \frac{1}{16a^2} = a^2 + \frac{1}{2} + \frac{1}{16a^2}$$

$$\left(x + \frac{1}{4a}\right)^2 = \left(a + \frac{1}{4a}\right)^2$$

$$x + \frac{1}{4a} = \pm\left(a + \frac{1}{4a}\right)$$

$$x + \frac{1}{4a} = a + \frac{1}{4a} \Rightarrow x = a \quad \text{(Point of tangency)}$$

$$x + \frac{1}{4a} = -\left(a + \frac{1}{4a}\right) \Rightarrow x = -a - \frac{1}{2a} = -\frac{2a^2 + 1}{2a}$$

The normal line intersects a second time at $x = -\frac{2a^2 + 1}{2a}$.

5. Let $p(x) = Ax^3 + Bx^2 + Cx + D$

$$p'(x) = 3Ax^2 + 2Bx + C.$$

At $(1, 1)$:

$A + B + C + D = 1$ Equation 1

$3A + 2B + C = 14$ Equation 2

At $(-1, -3)$:

$A + B - C + D = -3$ Equation 3

$3A + 2B + C = -2$ Equation 4

Adding Equations 1 and 3: $2B + 2D = -2$

Subtracting Equations 1 and 3: $2A + 2C = 4$

Adding Equations 2 and 4: $6A + 2C = 12$

Subtracting Equations 2 and 4: $4B = 16$

So, $B = 4$ and $D = \frac{1}{2}(-2 - 2B) = -5$. Subtracting $2A + 2C = 4$ and $6A + 2C = 12$, you obtain $4A = 8 \Rightarrow A = 2$. Finally, $C = \frac{1}{2}(4 - 2A) = 0$. So, $p(x) = 2x^3 + 4x^2 - 5$.

6. $f(x) = a + b\cos cx$

$f'(x) = -bc\sin cx$

At $(0, 1)$: $a + b = 1$ Equation 1

At $\left(\frac{\pi}{4}, \frac{3}{2}\right)$: $a + b\cos\left(\frac{c\pi}{4}\right) = \frac{3}{2}$ Equation 2

$-bc\sin\left(\frac{c\pi}{4}\right) = 1$ Equation 3

From Equation 1, $a = 1 - b$. Equation 2 becomes

$$(1 - b) + b\cos\left(\frac{c\pi}{4}\right) = \frac{3}{2} \Rightarrow -b + b\cos\frac{c\pi}{4} = \frac{1}{2}.$$

From Equation 3, $b = \frac{-1}{c\sin(c\pi/4)}$. So:

$$\frac{1}{c\sin(c\pi/4)} + \frac{-1}{c\sin(c\pi/4)}\cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}$$

$$1 - \cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}c\sin\left(\frac{c\pi}{4}\right)$$

Graphing the equation

$$g(c) = \frac{1}{2}c\sin\left(\frac{c\pi}{4}\right) + \cos\left(\frac{c\pi}{4}\right) - 1,$$

you see that many values of c will work. One answer:

$$c = 2, b = -\frac{1}{2}, a = \frac{3}{2} \Rightarrow f(x) = \frac{3}{2} - \frac{1}{2}\cos 2x$$

7. (a) $x^4 = a^2x^2 - a^2y^2$

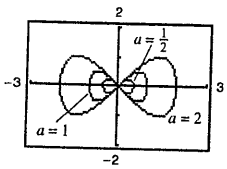
$a^2y^2 = a^2x^2 - x^4$

$y = \dfrac{\pm\sqrt{a^2x^2 - x^4}}{a}$

Graph: $y_1 = \dfrac{\sqrt{a^2x^2 - x^4}}{a}$ and

$y_2 = -\dfrac{\sqrt{a^2x^2 - x^4}}{a}$.

(b)



$(\pm a, 0)$ are the x-intercepts, along with $(0, 0)$.

(c) Differentiating implicitly:

$4x^3 = 2a^2x - 2a^2yy'$

$y' = \dfrac{2a^2x - 4x^3}{2a^2y}$

$= \dfrac{x(a^2 - 2x^2)}{a^2y} = 0 \Rightarrow 2x^2 = a^2 \Rightarrow x = \dfrac{\pm a}{\sqrt{2}}$

$\left(\dfrac{a^2}{2}\right)^2 = a^2\left(\dfrac{a^2}{2}\right) - a^2y^2$

$\dfrac{a^4}{4} = \dfrac{a^4}{2} - a^2y^2$

$a^2y^2 = \dfrac{a^4}{4}$

$y^2 = \dfrac{a^2}{4}$

$y = \pm\dfrac{a}{2}$

Four points: $\left(\dfrac{a}{\sqrt{2}}, \dfrac{a}{2}\right), \left(\dfrac{a}{\sqrt{2}}, -\dfrac{a}{2}\right), \left(-\dfrac{a}{\sqrt{2}}, \dfrac{a}{2}\right),$ $\left(\dfrac{-a}{\sqrt{2}}, -\dfrac{a}{2}\right)$

8. (a) $b^2y^2 = x^3(a - x); \quad a, b > 0$

$y^2 = \dfrac{x^3(a - x)}{b^2}$

Graph $y_1 = \dfrac{\sqrt{x^3(a - x)}}{b}$ and

$y_2 = -\dfrac{\sqrt{x^3(a - x)}}{b}$.

(b) a determines the x-intercept on the right: $(a, 0)$. b affects the height.

(c) Differentiating implicitly:

$2b^2yy' = 3x^2(a - x) - x^3 = 3ax^2 - 4x^3$

$y' = \dfrac{(3ax^2 - 4x^3)}{2b^2y} = 0$

$\Rightarrow 3ax^2 = 4x^3$

$3a = 4x$

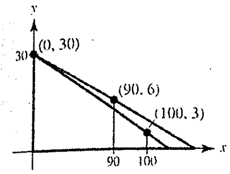
$x = \dfrac{3a}{4}$

$b^2y^2 = \left(\dfrac{3a}{4}\right)^3\left(a - \dfrac{3a}{4}\right) = \dfrac{27a^3}{64}\left(\dfrac{1}{4}a\right)$

$y^2 = \dfrac{27a^4}{256b^2} \Rightarrow y = \pm\dfrac{3\sqrt{3}a^2}{16b}$

Two points: $\left(\dfrac{3a}{4}, \dfrac{3\sqrt{3}a^2}{16b}\right), \left(\dfrac{3a}{4}, \dfrac{-3\sqrt{3}a^2}{16b}\right)$

9. (a)



Not drawn to scale

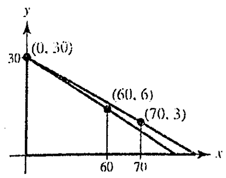
Line determined by $(0, 30)$ and $(90, 6)$:

$$y - 30 = \frac{30 - 6}{0 - 90}(x - 0) = -\frac{24}{90}x = -\frac{4}{15}x \Rightarrow y = -\frac{4}{15}x + 30$$

When $x = 100$: $y = -\frac{4}{15}(100) + 30 = \frac{10}{3} > 3$

As you can see from the figure, the shadow determined by the man extends beyond the shadow determined by the child.

(b)



Not drawn to scale

Line determined by $(0, 30)$ and $(60, 6)$:

$$y - 30 = \frac{30 - 6}{0 - 60}(x - 0) = -\frac{2}{5}x \Rightarrow y = -\frac{2}{5}x + 30$$

When $x = 70$: $y = -\frac{2}{5}(70) + 30 = 2 < 3$

As you can see from the figure, the shadow determined by the child extends beyond the shadow determined by the man.

(c) Need $(0, 30), (d, 6), (d + 10, 3)$ collinear.

$$\frac{30 - 6}{0 - d} = \frac{6 - 3}{d - (d + 10)} \Rightarrow \frac{24}{d} = \frac{3}{10} \Rightarrow d = 80 \text{ feet}$$

(d) Let y be the distance from the base of the street light to the tip of the shadow. You know that $dx/dt = -5$.

For $x > 80$, the shadow is determined by the man.

$$\frac{y}{30} = \frac{y - x}{6} \Rightarrow y = \frac{5}{4}x \text{ and } \frac{dy}{dt} = \frac{5}{4}\frac{dx}{dt} = \frac{-25}{4}$$

For $x < 80$, the shadow is determined by the child.

$$\frac{y}{30} = \frac{y - x - 10}{3} \Rightarrow y = \frac{10}{9}x + \frac{100}{9} \text{ and } \frac{dy}{dt} = \frac{10}{9}\frac{dx}{dt} = -\frac{50}{9}$$

Therefore:

$$\frac{dy}{dt} = \begin{cases} -\frac{25}{4}, & x > 80 \\ -\frac{50}{9}, & 0 < x < 80 \end{cases}$$

dy/dt is not continuous at $x = 80$.

ALTERNATE SOLUTION for parts (a) and (b):

(a) As before, the line determined by the man's shadow is

$$y_m = -\frac{4}{15}x + 30$$

The line determined by the child's shadow is obtained by finding the line through $(0, 30)$ and $(100, 3)$:

$$y - 30 = \frac{30 - 3}{0 - 100}(x - 0) \Rightarrow y_c = -\frac{27}{100}x + 30$$

By setting $y_m = y_c = 0$, you can determine how far the shadows extend:

Man: $y_m = 0 \Rightarrow \frac{4}{15}x = 30 \Rightarrow x = 112.5 = 112\frac{1}{2}$

Child: $y_c = 0 \Rightarrow \frac{27}{100}x = 30 \Rightarrow x = 111.\overline{11} = 111\frac{1}{9}$

The man's shadow is $112\frac{1}{2} - 111\frac{1}{9} = 1\frac{7}{18}$ ft beyond the child's shadow.

(b) As before, the line determined by the man's shadow is

$$y_m = -\frac{2}{5}x + 30$$

For the child's shadow,

$$y - 30 = \frac{30 - 3}{0 - 70}(x - 0) \Rightarrow y_c = -\frac{27}{70}x + 30$$

Man: $y_m = 0 \Rightarrow \frac{2}{5}x = 30 \Rightarrow x = 75$

Child: $y_c = 0 \Rightarrow \frac{27}{70}x = 30 \Rightarrow x = \frac{700}{9} = 77\frac{7}{9}$

So the child's shadow is $77\frac{7}{9} - 75 = 2\frac{7}{9}$ ft beyond the man's shadow.

10. (a) $y = x^{1/3} \Rightarrow \frac{dy}{dt} = \frac{1}{3}x^{-2/3}\frac{dx}{dt}$

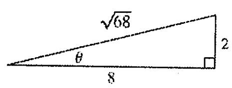
$$1 = \frac{1}{3}(8)^{-2/3}\frac{dx}{dt}$$

$$\frac{dx}{dt} = 12 \text{ cm/sec}$$

(b) $D = \sqrt{x^2 + y^2} \Rightarrow \frac{dD}{dt} = \frac{1}{2}(x^2 + y^2)\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x(dx/dt) + y(dy/dt)}{\sqrt{x^2 + y^2}}$

$$= \frac{8(12) + 2(1)}{\sqrt{64 + 4}} = \frac{98}{\sqrt{68}} = \frac{49}{\sqrt{17}} \text{ cm/sec}$$

(c) $\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{x(dy/dt) - y(dx/dt)}{x^2}$



From the triangle, $\sec\theta = \sqrt{68}/8$. So $\frac{d\theta}{dt} = \frac{8(1) - 2(12)}{64(68/64)} = \frac{-16}{68} = -\frac{4}{17}$ rad/sec.

11. (a) $v(t) = -\frac{27}{5}t + 27$ ft/sec

$a(t) = -\frac{27}{5}$ ft/sec^2

(b) $v(t) = -\frac{27}{5}t + 27 = 0 \Rightarrow \frac{27}{5}t = 27 \Rightarrow t = 5$ seconds

$S(5) = -\frac{27}{10}(5)^2 + 27(5) + 6 = 73.5$ feet

(c) The acceleration due to gravity on Earth is greater in magnitude than that on the moon.

12. $E'(x) = \lim_{\Delta x \to 0} \frac{E(x + \Delta x) - E(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{E(x)E(\Delta x) - E(x)}{\Delta x} = \lim_{\Delta x \to 0} E(x)\left(\frac{E(\Delta x) - 1}{\Delta x}\right) = E(x)\lim_{\Delta x \to 0}\frac{E(\Delta x) - 1}{\Delta x}$

But, $E'(0) = \lim_{\Delta x \to 0}\frac{E(\Delta x) - E(0)}{\Delta x} = \lim_{\Delta x \to 0}\frac{E(\Delta x) - 1}{\Delta x} = 1$. So, $E'(x) = E(x)E'(0) = E(x)$ exists for all x.

For example: $E(x) = e^x$.

13. $L'(x) = \lim_{\Delta x \to 0} \frac{L(x + \Delta x) - L(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{L(x) + L(\Delta x) - L(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{L(\Delta x)}{\Delta x}$

Also, $L'(0) = \lim_{\Delta x \to 0} \frac{L(\Delta x) - L(0)}{\Delta x}$. But, $L(0) = 0$ because $L(0) = L(0 + 0) = L(0) + L(0) \Rightarrow L(0) = 0$.

So, $L'(x) = L'(0)$ for all x. The graph of L is a line through the origin of slope $L'(0)$.

14. (a)

z (degrees)	0.1	0.01	0.0001
$\frac{\sin z}{z}$	0.0174524	0.0174533	0.0174533

(b) $\lim_{z \to 0} \frac{\sin z}{z} \approx 0.0174533$

In fact, $\lim_{z \to 0} \frac{\sin z}{z} = \frac{\pi}{180}$.

(c)
$$\begin{aligned}\frac{d}{dz}(\sin z) &= \lim_{\Delta z \to 0} \frac{\sin(z + \Delta z) - \sin z}{\Delta z} \\ &= \lim_{\Delta z \to 0} \frac{\sin z \cdot \cos \Delta z + \sin \Delta z \cdot \cos z - \sin z}{\Delta z} \\ &= \lim_{\Delta z \to 0}\left[\sin z\left(\frac{\cos \Delta z - 1}{\Delta z}\right)\right] + \lim_{\Delta z \to 0}\left[\cos z\left(\frac{\sin \Delta z}{\Delta z}\right)\right] \\ &= (\sin z)(0) + (\cos z)\left(\frac{\pi}{180}\right) = \frac{\pi}{180}\cos z\end{aligned}$$

(d) $S(90) = \sin\left(\frac{\pi}{180}90\right) = \sin\frac{\pi}{2} = 1$

$C(180) = \cos\left(\frac{\pi}{180}180\right) = -1$

$\frac{d}{dz}S(z) = \frac{d}{dz}\sin(cz) = c \cdot \cos(cz) = \frac{\pi}{180}C(z)$

(e) The formulas for the derivatives are more complicated in degrees.

15. $j(t) = a'(t)$

(a) $j(t)$ is the rate of change of acceleration.

(b) $s(t) = -8.25t^2 + 66t$

$v(t) = -16.5t + 66$

$a(t) = -16.5$

$a'(t) = j(t) = 0$

The acceleration is constant, so $j(t) = 0$.

(c) a is position.
b is acceleration.
c is jerk.
d is velocity.