AP Calculus AB **Abs. Extrema on an Open Interval** **Name:** ____________________

Remember your steps for finding absolute extrema on an open interval are very similar to those for finding relative extrema.

1) Find the critical numbers for $f(x)$ (where $f'(x)$ is 0 or undefined)
2) Make a sign line
3) Write your answers with because statements:

- The abs maximum is (y-value) and occurs at $x = a$ because $f'(x) > 0$ for all $x < a$ and $f'(x) < 0$ for all $x > a$. (Or because $f'(x)$ changes from positive to negative and $x = a$ is the only critical number of $f(x)$.)
- The abs minimum is (y-value) and occurs at $x = a$ because $f'(x) < 0$ for all $x < a$ and $f'(x) > 0$ for all $x > a$. (Or because $f'(x)$ changes from negative to positive and $x = a$ is the only critical number of $f(x)$.)

Find the absolute extrema for the following functions. Justify your answers.

1. $f(x) = 3x^2 - 12x + 16$

2. $f(x) = x^4 - 4$

3. $f(x) = \frac{4}{x+5}$

4. $f(x) = \frac{18}{x^2+9}$

5. $f(x) = (x+2)^{\frac{2}{3}}$

6. $f(x) = -3x\sqrt{x+1}$

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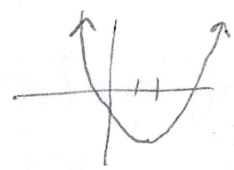
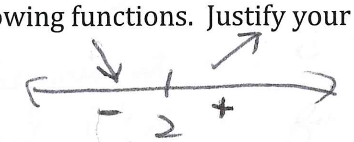


1. $f(x) = 3x^2 - 12x + 16$



$\boxed{f'(x) = 6x - 12}$

$0 = 6x - 12$

$6x = 12$

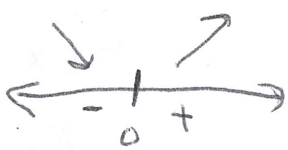
$x = 2$

$(2, 4)$

Abs. min. occurs at $(2, 4)$ b/c $f'(x) < 0$ for all $x < 2$ and $f'(x) > 0$ for all $x > 2$

2. $f(x) = x^4 - 4$

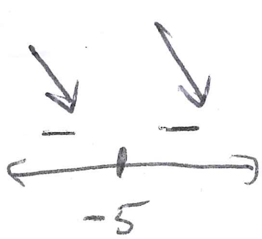
$f'(x) = 4x^3$



Abs min at $(0, -4)$ b/c $f'(x) < 0$ for all $x < 0$ and $f'(x) > 0$ for all $x > 0$

3. $f(x) = \frac{4}{x+5}$

$f'(x) = \dfrac{0(x+5) - 4(1)}{(x+5)^2}$



$f'(x) = \dfrac{-4}{(x+5)^2}$

No Absolute extrema since $f'(x)$ does not change signs.

$f(x) = 4(x+5)^{-1}$

$f'(x) = 4(-1)(x+5)^{-2}(1) = \dfrac{-4}{(x+5)^2}$

4. $f(x) = \frac{18}{x^2+9}$

$$f'(x) = \frac{0(x^2+9) - 18(2x)}{(x^2+9)^2} = \frac{-36x}{(x^2+9)^2}$$

$+ \quad | \quad -$

0

Abs max at $(0, 2)$ b/c $f'(x) > 0$ for all $x < 0$ and $f'(x) < 0$ for all $x > 0$

5. $f(x) = (x+2)^{\frac{2}{3}}$ $^{-1/3}$

$- \quad | \quad +$

-2

$$f'(x) = \frac{2}{3}(x+2)^{-1/3}(1)$$

$$f'(x) = \frac{2}{3(x+2)^{1/3}}$$

Abs. min at $(-2, 0)$ b/c $f'(x) < 0$ for all $x < -2$ and $f'(x) > 0$ for all $x > -2$

6. $f(x) = -3x\sqrt{x+1}$ $\quad x \geq -1$

$$f(x) = (-3x)(x+1)^{1/2}$$

$$f'(x) = (-3)(x+1)^{1/2} + (-3x)\frac{1}{2}(x+1)^{-1/2}(1)$$

$$f'(x) = -3\sqrt{x+1} - \frac{3x}{2\sqrt{(x+1)}}$$

$$f'(x) = \frac{-3(2)(x+1) - 3x}{2\sqrt{x+1}}$$

$$f'(x) = \frac{-6x - 6 - 3x}{2\sqrt{x+1}}$$

$$f'(x) = \frac{-9x - 6}{2\sqrt{x+1}}$$

$-9x - 6 = 0$

$-9x = 6$

$x = -2/3$

$| \quad + \quad | \quad -$

$-1 \qquad -2/3$

Abs max at $x = -\frac{2}{3}$ b/c $f'(x) > 0$ for $-1 < x < -2/3$ and $f'(x) < 0$ for all $x > -2/3$