Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding Extrema on a Closed Interval In Exercises 1–8, find the absolute extrema of the function on the closed interval.

1. $f(x) = x^2 + 5x, \quad [-4, 0]$

2. $f(x) = x^3 + 6x^2, \quad [-6, 1]$

3. $f(x) = \sqrt{x} - 2, \quad [0, 4]$

4. $h(x) = 3\sqrt{x} - x, \quad [0, 9]$

5. $f(x) = \dfrac{4x}{x^2 + 9}, \quad [-4, 4]$

6. $f(x) = \dfrac{x}{\sqrt{x^2 + 1}}, \quad [0, 2]$

7. $g(x) = 2x + 5\cos x, \quad [0, 2\pi]$

8. $f(x) = \sin 2x, \quad [0, 2\pi]$

Using Rolle's Theorem In Exercises 9–12, determine whether Rolle's Theorem can be applied to f on the closed interval $[a, b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

9. $f(x) = 2x^2 - 7, \quad [0, 4]$

10. $f(x) = (x - 2)(x + 3)^2, \quad [-3, 2]$

11. $f(x) = \dfrac{x^2}{1 - x^2}, \quad [-2, 2]$

12. $f(x) = \sin 2x, \quad [-\pi, \pi]$

Using the Mean Value Theorem In Exercises 13–18, determine whether the Mean Value Theorem can be applied to f on the closed interval $[a, b]$. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

If the Mean Value Theorem cannot be applied, explain why not.

13. $f(x) = x^{2/3}, \quad [1, 8]$

14. $f(x) = \dfrac{1}{x}, \quad [1, 4]$

15. $f(x) = |5 - x|, \quad [2, 6]$

16. $f(x) = 2x - 3\sqrt{x}, \quad [-1, 1]$

17. $f(x) = x - \cos x, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

18. $f(x) = \sqrt{x} - 2x, \quad [0, 4]$

19. Mean Value Theorem Can the Mean Value Theorem be applied to the function

$$f(x) = \frac{1}{x^2}$$

on the interval $[-2, 1]$? Explain.

20. Using the Mean Value Theorem

(a) For the function $f(x) = Ax^2 + Bx + C$, determine the value of c guaranteed by the Mean Value Theorem on the interval $[x_1, x_2]$.

(b) Demonstrate the result of part (a) for $f(x) = 2x^2 - 3x + 1$ on the interval $[0, 4]$.

Intervals on Which f Is Increasing or Decreasing In Exercises 21–26, identify the open intervals on which the function is increasing or decreasing.

21. $f(x) = x^2 + 3x - 12$

22. $h(x) = (x + 2)^{1/3} + 8$

23. $f(x) = (x - 1)^2(x - 3)$

24. $g(x) = (x + 1)^3$

25. $h(x) = \sqrt{x}(x - 3), \quad x > 0$

26. $f(x) = \sin x + \cos x, \quad [0, 2\pi]$

Applying the First Derivative Test In Exercises 27–34, (a) find the critical numbers of f (if any), (b) find the open interval(s) on which the function is increasing or decreasing, (c) apply the First Derivative Test to identify all relative extrema, and (d) use a graphing utility to confirm your results.

27. $f(x) = x^2 - 6x + 5$

28. $f(x) = 4x^3 - 5x$

29. $h(t) = \dfrac{1}{4}t^4 - 8t$

30. $g(x) = \dfrac{x^3 - 8x}{4}$

31. $f(x) = \dfrac{x + 4}{x^2}$

32. $f(x) = \dfrac{x^2 - 3x - 4}{x - 2}$

33. $f(x) = \cos x - \sin x, \quad (0, 2\pi)$

34. $g(x) = \dfrac{3}{2}\sin\left(\dfrac{\pi x}{2} - 1\right), \quad [0, 4]$

Finding Points of Inflection In Exercises 35–40, find the points of inflection and discuss the concavity of the graph of the function.

35. $f(x) = x^3 - 9x^2$

36. $f(x) = 6x^4 - x^2$

37. $g(x) = x\sqrt{x + 5}$

38. $f(x) = 3x - 5x^3$

39. $f(x) = x + \cos x, \quad [0, 2\pi]$

40. $f(x) = \tan\dfrac{x}{4}, \quad (0, 2\pi)$

Using the Second Derivative Test In Exercises 41–46, find all relative extrema. Use the Second Derivative Test where applicable.

41. $f(x) = (x + 9)^2$

42. $f(x) = 2x^3 + 11x^2 - 8x - 12$

43. $g(x) = 2x^2(1 - x^2)$

44. $h(t) = t - 4\sqrt{t + 1}$

45. $f(x) = 2x + \dfrac{18}{x}$

46. $h(x) = x - 2\cos x, \quad [0, 4\pi]$

Think About It **In Exercises 47 and 48, sketch the graph of a function f having the given characteristics.**

47. $f(0) = f(6) = 0$
$f'(3) = f'(5) = 0$
$f'(x) > 0$ for $x < 3$
$f'(x) > 0$ for $3 < x < 5$
$f'(x) < 0$ for $x > 5$
$f''(x) < 0$ for $x < 3$ or $x > 4$
$f''(x) > 0$ for $3 < x < 4$

48. $f(0) = 4, \ f(6) = 0$
$f'(x) < 0$ for $x < 2$ or $x > 4$
$f'(2)$ does not exist.
$f'(4) = 0$
$f'(x) > 0$ for $2 < x < 4$
$f''(x) < 0$ for $x \neq 2$

49. Writing A newspaper headline states that "The rate of growth of the national deficit is decreasing." What does this mean? What does it imply about the graph of the deficit as a function of time?

50. Inventory Cost The cost of inventory C depends on the ordering and storage costs according to the inventory model

$$C = \left(\frac{Q}{x}\right)s + \left(\frac{x}{2}\right)r.$$

Determine the order size that will minimize the cost, assuming that sales occur at a constant rate, Q is the number of units sold per year, r is the cost of storing one unit for one year, s is the cost of placing an order, and x is the number of units per order.

51. Modeling Data Outlays for national defense D (in billions of dollars) for selected years from 1970 through 2010 are shown in the table, where t is time in years, with $t = 0$ corresponding to 1970. *(Source: U.S. Office of Management and Budget)*

t	0	5	10	15	20	25	30	35	40
D	81.7	86.5	134.0	252.7	299.3	272.1	294.4	495.3	693.6

(a) Use the regression capabilities of a graphing utility to find a model of the form

$$D = at^4 + bt^3 + ct^2 + dt + e$$

for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) For the years shown in the table, when does the model indicate that the outlay for national defense was at a maximum? When was it at a minimum?

(d) For the years shown in the table, when does the model indicate that the outlay for national defense was increasing at the greatest rate?

52. Modeling Data The manager of a store recorded the annual sales S (in thousands of dollars) of a product over a period of 7 years, as shown in the table, where t is the time in years, with $t = 6$ corresponding to 2006.

t	6	7	8	9	10	11	12
S	5.4	6.9	11.5	15.5	19.0	22.0	23.6

(a) Use the regression capabilities of a graphing utility to find a model of the form

$$S = at^3 + bt^2 + ct + d$$

for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) Use calculus and the model to find the time t when sales were increasing at the greatest rate.

(d) Do you think the model would be accurate for predicting future sales? Explain.

Finding a Limit **In Exercises 53–62, find the limit.**

53. $\displaystyle\lim_{x \to \infty} \left(8 + \frac{1}{x}\right)$

54. $\displaystyle\lim_{x \to -\infty} \frac{1 - 4x}{x + 1}$

55. $\displaystyle\lim_{x \to \infty} \frac{2x^2}{3x^2 + 5}$

56. $\displaystyle\lim_{x \to \infty} \frac{4x^3}{x^4 + 3}$

57. $\displaystyle\lim_{x \to -\infty} \frac{3x^2}{x + 5}$

58. $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{x^2 + x}}{-2x}$

59. $\displaystyle\lim_{x \to \infty} \frac{5\cos x}{x}$

60. $\displaystyle\lim_{x \to \infty} \frac{x^3}{\sqrt{x^2 + 2}}$

61. $\displaystyle\lim_{x \to -\infty} \frac{6x}{x + \cos x}$

62. $\displaystyle\lim_{x \to -\infty} \frac{x}{2\sin x}$

Horizontal Asymptotes **In Exercises 63–66, use a graphing utility to graph the function and identify any horizontal asymptotes.**

63. $f(x) = \dfrac{3}{x} - 2$

64. $g(x) = \dfrac{5x^2}{x^2 + 2}$

65. $h(x) = \dfrac{2x + 3}{x - 4}$

66. $f(x) = \dfrac{3x}{\sqrt{x^2 + 2}}$

Analyzing the Graph of a Function **In Exercises 67–76, analyze and sketch a graph of the function. Label any intercepts, relative extrema, points of inflection, and asymptotes. Use a graphing utility to verify your results.**

67. $f(x) = 4x - x^2$

68. $f(x) = 4x^3 - x^4$

69. $f(x) = x\sqrt{16 - x^2}$

70. $f(x) = (x^2 - 4)^2$

71. $f(x) = x^{1/3}(x + 3)^{2/3}$

72. $f(x) = (x - 3)(x + 2)^3$

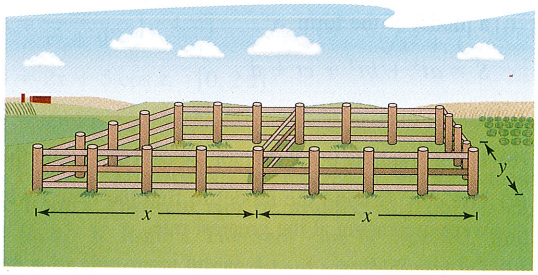
73. $f(x) = \dfrac{5 - 3x}{x - 2}$

74. $f(x) = \dfrac{2x}{1 + x^2}$

75. $f(x) = x^3 + x + \dfrac{4}{x}$

76. $f(x) = x^2 + \dfrac{1}{x}$

77. Maximum Area A rancher has 400 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?



78. Maximum Area Find the dimensions of the rectangle of maximum area, with sides parallel to the coordinate axes, that can be inscribed in the ellipse given by

$$\frac{x^2}{144} + \frac{y^2}{16} = 1.$$

79. Minimum Length A right triangle in the first quadrant has the coordinate axes as sides, and the hypotenuse passes through the point $(1, 8)$. Find the vertices of the triangle such that the length of the hypotenuse is minimum.

80. Minimum Length The wall of a building is to be braced by a beam that must pass over a parallel fence 5 feet high and 4 feet from the building. Find the length of the shortest beam that can be used.

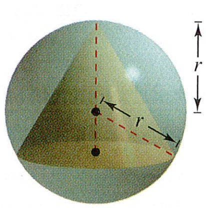
81. Maximum Length Find the length of the longest pipe that can be carried level around a right-angle corner at the intersection of two corridors of widths 4 feet and 6 feet.

82. Maximum Length A hallway of width 6 feet meets a hallway of width 9 feet at right angles. Find the length of the longest pipe that can be carried level around this corner. [*Hint:* If L is the length of the pipe, show that

$$L = 6 \csc \theta + 9 \csc\left(\frac{\pi}{2} - \theta\right)$$

where θ is the angle between the pipe and the wall of the narrower hallway.]

83. Maximum Volume Find the volume of the largest right circular cone that can be inscribed in a sphere of radius r.



84. Maximum Volume Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius r.

Using Newton's Method **In Exercises 85–88, approximate the zero(s) of the function. Use Newton's Method and continue the process until two successive approximations differ by less than 0.001. Then find the zero(s) using a graphing utility and compare the results.**

85. $f(x) = x^3 - 3x - 1$

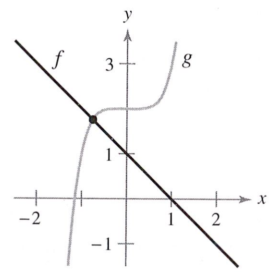
86. $f(x) = x^3 + 2x + 1$

87. $f(x) = x^4 + x^3 - 3x^2 + 2$

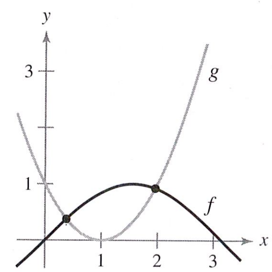
88. $f(x) = 3\sqrt{x - 1} - x$

Finding Point(s) of Intersection **In Exercises 89 and 90, apply Newton's Method to approximate the x-value(s) of the indicated point(s) of intersection of the two graphs. Continue the process until two successive approximations differ by less than 0.001. [*Hint:* Let $h(x) = f(x) - g(x)$.]**

89. $f(x) = 1 - x$
$g(x) = x^5 + 2$



90. $f(x) = \sin x$
$g(x) = x^2 - 2x + 1$



Comparing Δy and dy **In Exercises 91 and 92, use the information to evaluate and compare Δy and dy.**

	Function	x-Value	Differential of x
91.	$y = 0.5x^2$	$x = 3$	$\Delta x = dx = 0.01$
92.	$y = x^3 - 6x$	$x = 2$	$\Delta x = dx = 0.1$

Finding a Differential **In Exercises 93 and 94, find the differential dy of the given function.**

93. $y = x(1 - \cos x)$

94. $y = \sqrt{36 - x^2}$

95. Volume and Surface Area The radius of a sphere is measured as 9 centimeters, with a possible error of 0.025 centimeter.

(a) Use differentials to approximate the possible propagated error in computing the volume of the sphere.

(b) Use differentials to approximate the possible propagated error in computing the surface area of the sphere.

(c) Approximate the percent errors in parts (a) and (b).

96. Demand Function A company finds that the demand for its commodity is

$$p = 75 - \frac{1}{4}x$$

where p is the price in dollars and x is the number of units. Find and compare the values of Δp and dp as x changes from 7 to 8.