Ch. 4 Test Review

Ch. 4 Help Session Practice Problems

Even/Odd functions:

1) Given: $\int_{-2}^{-4} f(x)\,dx = -3$ $\quad \int_{0}^{4} f(x)\,dx = 7$

Find $\int_{-2}^{4} f(x)\,dx$

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a) $f(x)$ is even:

i) sketch (shorter) smaller interval first

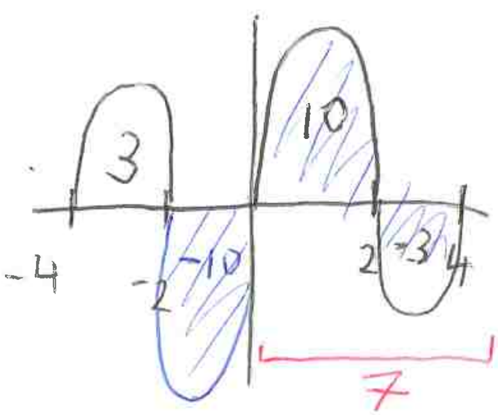
ii) Fill in other side based on even/odd properties

iii) Fill in gaps using longer (larger) interval.

$\int_{-4}^{-2} f(x)\,dx = 3$

$\boxed{\int_{-2}^{4} f(x)\,dx = 11}$

b) $f(x)$ is odd



$\boxed{\int_{-2}^{4} f(x)\,dx = -3}$

2) Second Fundamental Theorem (SFTC)

$$\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f[p(x)]\cdot p'(x)$$

$$\frac{d}{dx}\int_{q(x)}^{p(x)} f(t)\,dt = f[p(x)]\cdot p'(x) - f[q(x)]\cdot q'(x)$$

$\boxed{\text{Ex.}}$ $f(x) = \displaystyle\int_3^{\sqrt{x}} \frac{t}{5-t^2}\,dt$ Find $\frac{d}{dx}f(x)$

$x^{1/2}$

a) $$\frac{d}{dx}\int_3^{\sqrt{x}} \frac{t}{5-t^2}\,dt = \frac{\sqrt{x}}{5-(\sqrt{x})^2}\cdot\frac{1}{2}x^{-1/2} \rightarrow \frac{\cancel{\sqrt{x}}}{5-x}\cdot\frac{1}{2\cancel{\sqrt{x}}} = \boxed{\frac{1}{(5-x)2}}$$

or

$$\frac{1}{10-2x}$$

b) $$\frac{d}{dx}\int_{x^2}^{\sqrt{x}} \frac{t}{5-t^2}\,dt = \frac{\sqrt{x}}{5-(\sqrt{x})^2}\cdot\frac{1}{2}x^{-1/2} - \frac{x^2}{5-(x^2)^2}\cdot(2x)$$

$$= \boxed{\frac{1}{10-2x} - \frac{2x^3}{5-x^4}}$$

3) Find $\int_2^6 4x\sqrt{x^2+19}\,dx$

$$\int 4x\sqrt{x^2+19}\,dx$$

$$\int 4x(x^2+19)^{1/2}\,dx$$

$u = x^2+19$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int 4x \cdot u^{1/2} \cdot \frac{du}{2x}$$

$$\int \overset{2}{\cancel{4}}\cancel{x} \cdot u^{1/2} \cdot \frac{du}{\cancel{2x}}$$

$$\int 2u^{1/2}\,du$$

$$2\int u^{1/2}\,du$$

$$2 \cdot \frac{u^{3/2}}{3/2} \rightarrow 2 \cdot \frac{2}{3} u^{3/2}$$

$$\left.\frac{4}{3}u^{3/2}\right]$$

convert bounds:

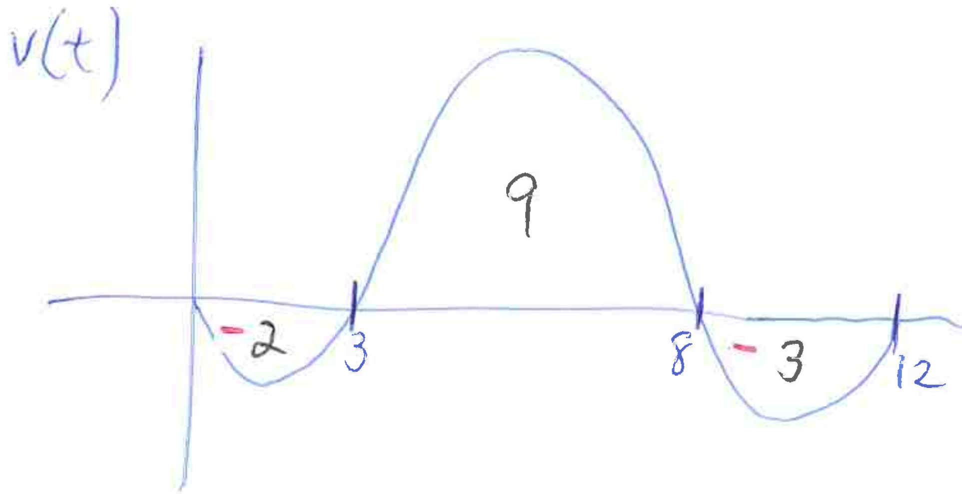
If $x=2$, $u = \cancel{x}^2+19 = 23$

If $x=6$, $u = x^2+19 = 36+19 = 55$

Method 1: $\left.\frac{4}{3}u^{3/2}\right]_{23}^{55} = \boxed{\frac{4}{3}(55)^{3/2} - \frac{4}{3}(23)^{3/2}}$ ✓

Method 2: $\left.\frac{4}{3}(x^2+19)^{3/2}\right]_2^6 = \frac{4}{3}(6^2+19)^{3/2} - \frac{4}{3}(2^2+19)^{3/2} =$ ✓

4 Interpreting velocity graph:



Given: $x(0) = 4$

Find $x(12) = \_\_\_\_?$

$x(b) = x(a) + \int_a^b v(t)\,dt$

* $\underset{\text{position}}{\text{Final}} = \underset{\text{position}}{\text{Given}} + \text{displacement}$

$$x(12) = x(0) + \int_0^{12} v(t)\,dt$$

$$4 + (-2+9-3) = \boxed{8}$$

Given: $x(8) = 6$ Find $x(0)$

$$x(0) = x(8) + \int_8^0 v(t)\,dt$$

$$x(0) + x(8) - \int_0^8 v(t)\,dt$$

$6 - (-2+9)$

$6 - (7)$

$= \boxed{-1}$

5) $\int 3x^2\left(\sqrt{x+5}\right)dx$

$\int 3x^2(x+5)^{1/2}\,dx$

$\boxed{u=x+5}$

$\frac{du}{dx}=1$

$dx=du$

$\int 3x^2 \cdot u^{1/2}\,du$

$x=u-5$

$\int 3(u-5)^2 u^{1/2}\,du$

$\int 3u^{1/2}(u-5)^2\,du$

$(u-5)(u-5)$

$\int 3u^{1/2}(u^2-10u+25)\,du$

$\int 3u^{5/2}-30u^{3/2}+75u^{1/2}\,du$

$3\cdot\frac{u^{7/2}}{7/2}-30\cdot\frac{u^{5/2}}{5/2}+75\cdot\frac{u^{3/2}}{3/2}$

$\frac{6}{7}u^{7/2}-\frac{60}{5}u^{5/2}+\frac{150}{3}u^{3/2}+C$

$$\boxed{\frac{6}{7}(x+5)^{7/2}-12(x+5)^{5/2}+50(x+5)^{3/2}+C}$$

6) particle motion (PVA)

calculator: $v(t) = 2\cos(t^2 - 3t)$ $[0, 4]$

a) when is object changing direction?

* ~~when~~ look for when $v(t)$ change signs ($v(t) = 0$)

b/t 0.5 and 1 → 0.67
b/t 2 and 2.5 → 2.324
b/t 3 and 3.5 3.455

} b/c $v(t)$ change signs

b) Find $a(3.5) = -7.872$

c) Find $v(3.5) = -0.356$

d) Is velocity inc/dec at $t = 3.5$?

$a(t)$ pos or neg? velocity decreasing at $t = 3.5$ b/c $a(t) < 0$.

e) Find displacement $t=0$ to $t=3$? $\int_0^3 v(t)\,dt = \boxed{0.163}$

f) Total distance $t=1$ to $t=4$? $\int_1^4 |v(t)|\,dt =$

g) Find $x(3)$ if $x(1) = -2$

* $x(b) = x(a) + \int_a^b v(t)\,dt$

$x(3) = x(1) + \int_1^3 v(t)\,dt \rightarrow -2 + (-0.478) = \boxed{-2.478}$

6b) $\int \frac{4}{\sqrt{x}} \sin\left(\frac{\sqrt{x}}{2}\right) dx$

$\int \sin u\, du = -\cos u + C$

$u = \frac{1}{2}x^{1/2}$

$\frac{du}{dx} = \frac{1}{2} \cdot \frac{1}{2} x^{-1/2}$

$\frac{du}{dx} = \frac{1}{4\sqrt{x}}$

$du(4\sqrt{x}) = dx$

$\int \frac{4}{\cancel{\sqrt{x}}} \sin u \cdot 4\cancel{\sqrt{x}}\, du$

$16 \int \sin u\, du$

$-16 \cos u + C$

$\boxed{-16\cos\left(\frac{\sqrt{x}}{2}\right) + C}$

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7) $\frac{d}{dx} \int_{-4x}^{\sqrt{x}} 1 - t^2\, dt$ $\quad \sqrt{x} \to x^{1/2}$

$\frac{d}{dx} \int_{q(x)}^{p(x)} f(t)\,dt = f(p(x)) \cdot p'(x) - f(q(x)) \cdot q'(x)$

$= \left(1 - (\sqrt{x})^2\right) \cdot \frac{1}{2} x^{-1/2} - \left(1 - (-4x)^2\right) \cdot (-4)$

$= (1 - x)\left(\frac{1}{2\sqrt{x}}\right) + (1 - 16x^2)(4)$

8) $\displaystyle\int_{\sqrt{7}}^{0} x\sqrt{16-x^2}\,dx$

$\displaystyle\int x\sqrt{16-x^2}\,dx$

$\displaystyle\int x(16-x^2)^{1/2}\,dx$

$u=16-x^2$

$\dfrac{du}{dx}=-2x$

$du=-2x\,dx$

$\dfrac{du}{-2x}=dx$

$\displaystyle\int \cancel{x}\cdot u^{1/2}\cdot\frac{du}{-2\cancel{x}}$

$\displaystyle-\frac{1}{2}\int u^{1/2}\,du$

$-\dfrac{1}{2}\cdot\dfrac{u^{3/2}}{3/2}$

$-\dfrac{1}{2}\cdot\dfrac{2}{3}u^{3/2}$

$-\dfrac{1}{3}u^{3/2}$

Method 1:

If $x=\sqrt{7}$, $u=16-\sqrt{7}^2=9$

If $x=0$, $u=16-0=16$

$\displaystyle -\frac{1}{3}u^{3/2}\Big]_{9}^{16}$

$\boxed{-\dfrac{1}{3}(16)^{3/2}-\left(-\dfrac{1}{3}(9)^{3/2}\right)=-\dfrac{37}{3}}$

Method 2:

$\displaystyle -\frac{1}{3}(16-x^2)^{3/2}\Big]_{\sqrt{7}}^{0}$ $\boxed{=-\dfrac{1}{3}(16-0)^{3/2}-\left(-\dfrac{1}{3}(16-7)^{3/2}\right)=-\dfrac{37}{3}}$

9) $\int \frac{x-2}{\sqrt[4]{x^2-4x}}dx$

$\int \frac{x-2}{(x^2-4x)^{1/4}}dx$

$u=x^2-4x$

$\frac{du}{dx}=2x-4$

$du=(2x-4)dx$

$\frac{du}{2x-4}=dx$

$\int \frac{\cancel{x-2}}{u^{1/4}} \cdot \frac{du}{2x-4}$

$2\cancel{(x-2)}$

$\frac{1}{2}\int \frac{1}{u^{1/4}}du$

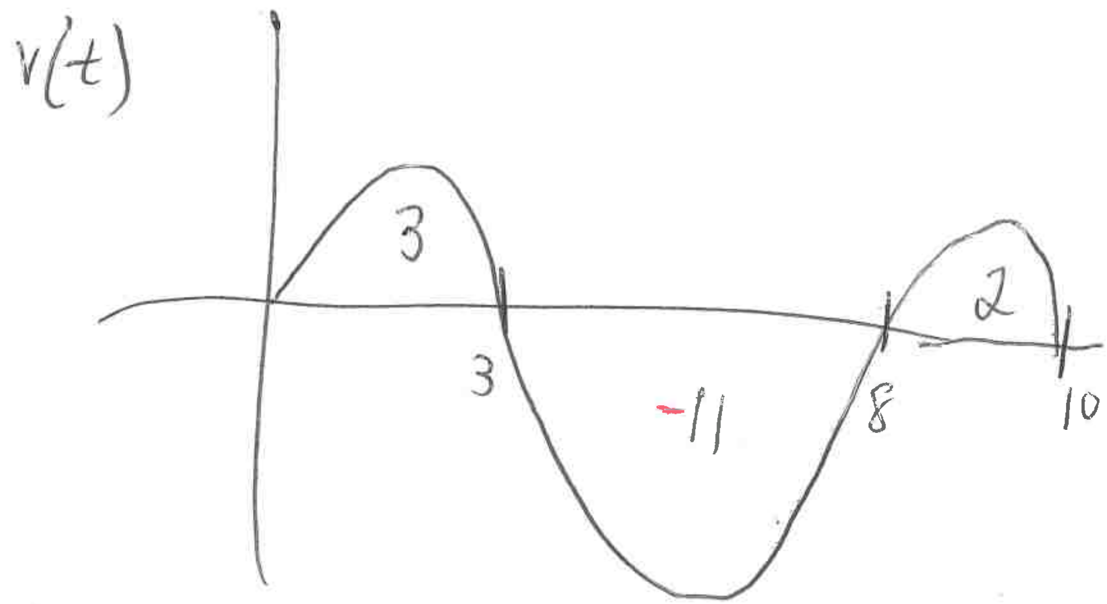
$\frac{1}{2}\int u^{-1/4}du$

$\frac{1}{2} \cdot \frac{u^{3/4}}{3/4}+C$

$\frac{1}{2} \cdot \frac{4}{3}u^{3/4}+C$

$\boxed{\frac{2}{3}(x^2-4x)^{3/4}+C}$

Reading / Interpret velocity graph:



Given: $x(3)=5$ Find $x(10)$

* Final position = given position + displacement

$$x(b) = x(a) + \int_a^b v(t)\,dt$$

$$x(10) = x(3) + \int_3^{10} v(t)\,dt$$

$$= 5 + (-11+2) = 5-9 = \boxed{-4}$$

Given: $x(10)=8$ Find $x(0)$

$$x(0) = x(10) + \int_{10}^{0} v(t)\,dt$$

$$x(0) = x(10) - \int_0^{10} v(t)\,dt$$

$$8 - (3-11+2)$$

$$8-(-6)$$

$$= \boxed{14}$$

4) $v(t) = 4\sin(t^2 - 2t + 2)$ $[0, 4]$

* x-ints are locations on $v(t)$ graph where $v(t) = 0$ particle motionless.

$t = \cancel{2.44}, \cancel{3.322}, \cancel{3.866}$

$t = 2.463$, $t = 3.368$, 3.91 b/c $v(t) = 0$

b) change direction at $t = 2.463, 3.368, 3.91$ b/c $v(t)$ change signs.

c) $v(3) = -3.836$

d) $a(3) = 4.539$

e) Speed inc/dec at $t = 3$? speed decreasing at $t = 3$ b/c $v(t), a(t)$ have opposite signs.

f) Displacement $\rightarrow \int_0^4 v(t)\,dt = 7.753$

g) Total distance $\rightarrow \int_0^4 |v(t)|\,dt =$