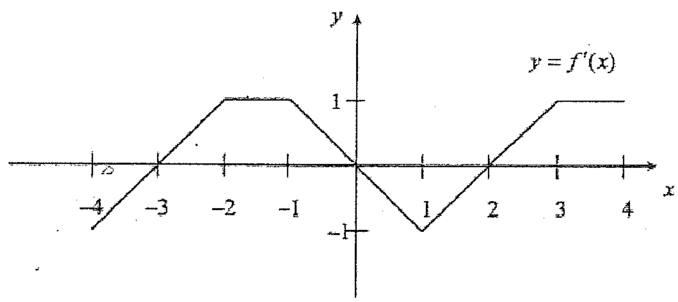
AP Calculus Chapter 4 Morning Review Session (part 1: Non-Calculator)

1. Find the average value of $f'(x)$ on [-4, 4]



2. a) Find the average value of $f(x) = 4 - x^2$ on [0, 2].

b) Find the c-value guaranteed by the average value theorem.

3. $\int \frac{x-2}{\sqrt[4]{x^2-4x}}\,dx$

4. $\int_{-5}^{2} |x+3|\,dx$

5. $\displaystyle\int_{\sqrt{7}}^{0} x\sqrt{16-x^2}\,dx$

6. $\displaystyle\int \sqrt[3]{\cos x}\,\sin x dx$

7. $\displaystyle\int x\sqrt{1-x}dx$

8. Find $\dfrac{d}{dx}\left[\displaystyle\int_{-4x}^{\sqrt{x}} 1-t^2 dt\right]$

9. Given f''(x) = 1 – 2x and f'(-1) = 6 and f(0) = 14 find the below

   a. Find the specific equation for f'(x)

   b. Find the specific equation for f(x)

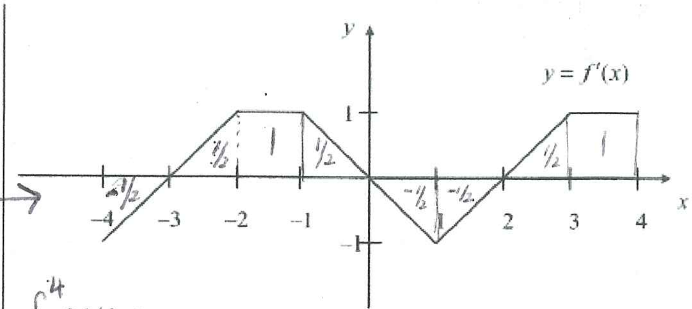
AP Calculus **Chapter 4** **Morning Review Session (part 1: Non-Calculator)**

1. Find the average value of f '(x) on [-4, 4]

$$\text{Avg. value} = \frac{1}{b-a}\int_a^b f'(x)\,dx$$

$$= \frac{1}{4-4}\int_{-4}^{4} f'(x)\,dx \qquad \int_{-4}^{4} f'(x)\,dx = 2$$

$$= \frac{1}{8}(2) = \frac{2}{8} = \boxed{\frac{1}{4}}$$



$$\int_{-4}^{4} f'(x)\,dx = -\frac{1}{2}+\frac{1}{2}+1+\frac{1}{2}-\frac{1}{2}-\frac{1}{2}+\frac{1}{2}+1 = 2$$

2. a) Find the average value of f(x) = 4 – x² on [0, 2].

$$\text{Avg. value} = \frac{1}{2-0}\int_0^2 4-x^2\,dx$$

$$\int_0^2 4-x^2\,dx = 4x-\frac{x^3}{3}\Big]_0^2 = 8-\frac{8}{3}-\left(4(0)-\frac{0^3}{3}\right)$$

$$\text{Avg. value} = \frac{1}{2}\left(8-\frac{8}{3}\right) = \frac{1}{2}\left(\frac{16}{3}\right) = \boxed{\frac{8}{3}}$$

b) Find the c-value guaranteed by the average value theorem.

set f(x) = Avg. value

$$4-x^2 = \frac{8}{3}$$

$$-x^2 = \frac{8}{3}-4$$

$$-x^2 = -\frac{4}{3}$$

$$x^2 = \frac{4}{3}$$

$$x = \pm\sqrt{\frac{4}{3}}$$

$$\boxed{c = \frac{2}{\sqrt{3}}}$$

3. $\displaystyle\int \frac{x-2}{\sqrt[4]{x^2-4x}}\,dx = \int \frac{x-2}{(x^2-4x)^{1/4}}\,dx$

$u = x^2-4x$

$\frac{du}{dx} = 2x-4$

$dx = \frac{du}{2x-4}$

$$\int \frac{x-2}{u^{1/4}}\cdot\frac{du}{2x-4}$$

$$\int \frac{\cancel{x-2}}{u^{1/4}}\cdot\frac{du}{2\cancel{(x-2)}}$$

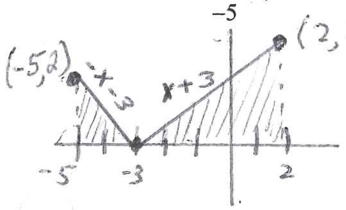
$$\frac{1}{2}\int u^{-1/4}\,du$$

$$= \frac{1}{2}\left(\frac{u^{3/4}}{3/4}\right)$$

$$= \frac{1}{2}\cdot\frac{4}{3}u^{3/4}+C$$

$$= \boxed{\frac{2}{3}(x^2-4x)^{3/4}+C}$$

4. $\displaystyle\int_{-5}^{2} |x+3|\,dx$



$$\frac{1}{2}(2)(2)+\frac{1}{2}(5)(5)$$

$$2+\frac{25}{2} = \boxed{14.5 \text{ or } \frac{29}{2}}$$

$\boxed{\text{OR}}$ $$\int_{-5}^{-3} -x-3\,dx + \int_{-3}^{2} x+3\,dx$$

$$-\frac{x^2}{2}-3x\Big]_{-5}^{-3} + \frac{x^2}{2}+3x\Big]_{-3}^{2}$$

$$-\frac{9}{2}+9-\left(-\frac{25}{2}+15\right) \qquad \frac{4}{2}+6-\left(\frac{9}{2}-9\right)$$

$$2 + 12.5 = \boxed{14.5 \text{ or } \frac{29}{2}}$$

5. $\int_{\sqrt{7}}^{0} x\sqrt{16-x^2}\,dx$

if $x=\sqrt{7}$, $u=16-\sqrt{7}^2=9$
if $x=0$, $u=16-0=16$

$u=16-x^2$

$\frac{du}{dx}=-2x$

$dx=\frac{du}{-2x}$

$\int x\cdot u^{1/2}\cdot\frac{du}{-2x}$

$\frac{-1}{2}\int u^{1/2}du$

$\frac{-1}{2}\left(\frac{u^{3/2}}{3/2}\right)$

$\frac{-1}{2}\cdot\frac{2}{3}u^{3/2}$

$-\frac{1}{3}u^{3/2}\Big]_9^{16} = \frac{-1}{3}(16)^{3/2}-\left(\frac{-1}{3}(9)^{3/2}\right)$

$=\frac{-1}{3}(64)+\frac{1}{3}(27)$

$=\boxed{\frac{-37}{3}}$

6. $\int \sqrt[3]{\cos x}\sin x\,dx$

$u=\cos x$

$\frac{du}{dx}=-\sin x$

$dx=\frac{du}{-\sin x}$

$\int u^{1/3}\cdot\cancel{\sin x}\cdot\frac{du}{-\cancel{\sin x}}$

$-1\int u^{1/3}du$

$-\frac{u^{4/3}}{4/3}+C$

$\boxed{\frac{-3}{4}(\cos x)^{4/3}+C}$

$\boxed{\text{use SFTC}}$

7. $\int x\sqrt{1-x}\,dx$

$u=1-x \mapsto x=1-u$

$\frac{du}{dx}=-1$

$dx=-du$

$\int x\cdot u^{1/2}(-du)$

$\int(1-u)u^{1/2}(-du)$

$\int -u^{1/2}+u^{3/2}du$

$-\frac{u^{3/2}}{3/2}+\frac{u^{5/2}}{5/2}+C$

$\frac{-2}{3}u^{3/2}+\frac{2}{5}u^{5/2}+C$

$\boxed{\frac{-2}{3}(1-x)^{3/2}+\frac{2}{5}(1-x)^{5/2}+C}$

8. Find $\frac{d}{dx}\left[\int_{-4x}^{\sqrt{x}} 1-t^2\,dt\right]$

$=\left(1-(\sqrt{x})^2\right)\cdot\frac{1}{2}x^{-1/2}-\left(1-(-4x)^2\right)(-4)$

$=\boxed{\frac{1-x}{2\sqrt{x}}+4-64x^2}$

9. Given f''(x) = 1 – 2x and f'(-1) = 6 and f(0) = 14 find the below

a. Find the specific equation for f'(x)

$f'(x)=\int f''(x)dx=\int 1-2x\,dx = x-\frac{2x^2}{2}+C$

$f'(x)=x-x^2+C$

$6=(-1)-(-1)^2+C$

$6=-1-1+C$

$8=C$

$\boxed{f'(x)=x-x^2+8}$

b. Find the specific equation for f(x)

$f(x)=\int f'(x)dx=\int x-x^2+8\,dx=\frac{x^2}{2}-\frac{x^3}{3}+8x+K$

$f(x)=\frac{x^2}{2}-\frac{x^3}{3}+8x+K$

$14=0-0+0+K$

$14=K$

$\boxed{f(x)=\frac{x^2}{2}-\frac{x^3}{3}+8x+14}$