AP Calculus Chapter 4 Morning Review Session (part 2: Calculator)

1. An object moving along a horizontal line has v(t) = $t\cos\left(\frac{\pi t}{6}\right)$ measured in inches per second from [0,11]

a. Create Sign line for v(t) and a(t)

b. Find the time(s) when the object is motionless

c. Find the velocity of the object at t = 4 seconds.

d. Find the acceleration of the object at t = 4 seconds.

e. Is the object's speed increasing or decreasing at t = 4 seconds? Justify answer.

f. Find the total displacement of the object from t = 0 to t = 11 seconds (Show Integral Notation)

g. Find the total distance of the object from t = 0 to t = 11 seconds (Show Integral Notation)

h. Find the time when the object reaches minimum velocity in [0, 11]

i. Find the minimum velocity in [0, 11]

j. Given x(0) = 3, Find x(11). (Show integral notation)

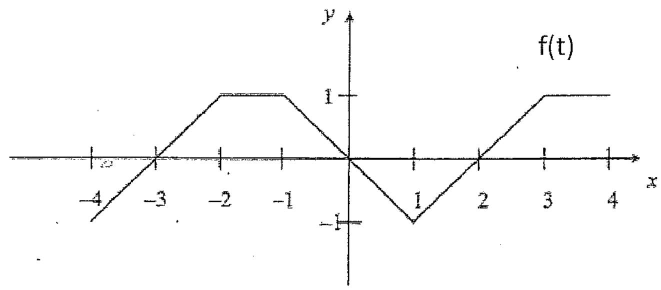
k. Find the average velocity in [0, 11]

l. Find the time(s) when object reaches average velocity.

2. The graph of f consists of line segments. Let $g(x) = \int_2^x f(t)dt$

a. Find $g'(x)$

b. Find $g''(x)$



c) Find g(4)

d) Find g(-2)

e) Find $g''(-3.5)$

f) For what values of x is g increasing? Justify Answer

g) For what values of x is $g'(x)$ decreasing?

h) Find the absolute extrema of g on the interval [-1,3].

3. The following table shows the size of an incoming wave headed towards shore at a given moment.

Distance from left of wave (x feet)	0	7	18	24	36	44	53
Height of wave h(x) (feet)	0	5	13	26	16	7	0

a) Use a trapezoidal sum with the six sub-intervals indicated by the data in the table to approximate the area of the face of the wave. Show correct units.

b) Estimate $\int_0^{53} h(x)dx$ using 3 middle rectangles

c) Find the average height on the interval [0, 53] using estimation from part b

AP Calculus **Chapter 4** **Morning Review Session (part 2: Calculator)**

* Make sure you are in Radian Mode!

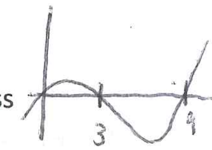
1. An object moving along a horizontal line has $v(t) = t\cos\left(\frac{\pi t}{6}\right)$ measured in inches per second from [0,11]

a. Create Sign line for v(t) and a(t)

$v(t)$: + on (0, 3), − on (3, 9), + on (9, 11); marks at 0, 3, 9, 11

$a(t)$: + (∪) on (0, 1.643), − (∩) on (1.643, 6.542), + (∪) on (6.542, 11); marks at 1.643, 6.542

b. Find the time(s) when the object is motionless



$t = 0, 3, 9$ seconds

c. Find the velocity of the object at t = 4 seconds.

$v(4) = -2$ in./sec.

calculator: $Y_1(4)$

d. Find the acceleration of the object at t = 4 seconds.

$a(4) = -2.314$ in./s^2

calculator: nDeriv(Y_1, X, 4)

e. Is the object's speed increasing or decreasing at t = 4 seconds? Justify answer.

Speed is increasing b/c $v(4) < 0$ and $a(4) < 0$ (same signs)

f. Find the total displacement of the object from t = 0 to t = 11 seconds (Show Integral Notation)

$\int_0^{11} v(t)\,dt = -10.993$ in.

calculator: fnInt(Y_1, X, 0, 11)

g. Find the total distance of the object from t = 0 to t = 11 seconds (Show Integral Notation)

$\int_0^{11} |v(t)|\,dt = 34.844$ in.

calculator: fnInt(Abs(Y_1), X, 0, 11)

h. Find the time when the object reaches minimum velocity in [0, 11]

$t = 6.542$ when $a(t)$ changes from − to +

i. Find the minimum velocity in [0, 11]

$v(6.542) = -6.28$ in./sec.

j. Given x(0) = 3, Find x(11). (Show integral notation)

$x(11) = x(0) + \int_0^{11} v(t)\,dt$

$= 3 + (-10.993) = -7.993$

$\boxed{x(11) = -7.993}$

k. Find the average velocity in [0, 11]

Avg. velocity $= \frac{1}{11-0}\int_0^{11} v(t)\,dt = \frac{1}{11}(-10.993)$

$\boxed{\text{Avg. velocity} = -0.999 \text{ in/s}}$

l. Find the time(s) when object reaches average velocity.

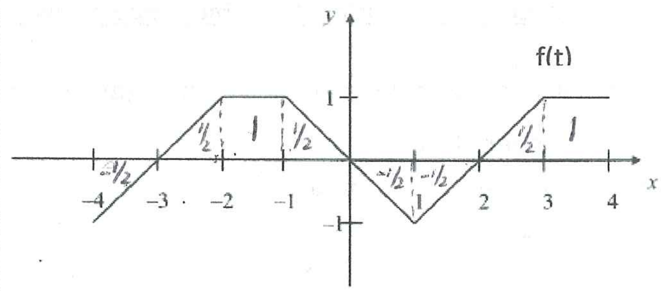
set $v(t) = -0.999$

$t\cos(\pi t/6) = -0.999$

$t\cos(\pi t/6) + 0.999 = 0$

$\boxed{t = 3.546 \text{ sec and } t = 8.782 \text{ sec.}}$

2. The graph of f consists of line segments. Let g(x) = $\int_2^x f(t)\,dt$



a. Find g'(x) $= f(x)\cdot 1 = f(x)$

$g'(x) = f(x)$

b. Find g''(x) $= f'(x)$

c) Find g(4)

$h(4) = \int_2^4 f(t)\,dt = \boxed{1.5}$

d) Find g(-2)

$h(-2) = \int_2^{-2} f(t)\,dt = -\int_{-2}^{2} f(t)\,dt$

$= -(\tfrac{1}{2}) = \boxed{-\tfrac{1}{2}}$

e) Find g''(-3.5) $= f'(-3.5)$

Find slope b/t (-4,-1) and (-3,0)

$g''(-3.5) = \frac{-1-0}{-4+3} = \frac{-1}{-1} = \boxed{1}$

f) For what values of x is g increasing? Justify Answer

$g'(x)$: − | + | − | + on −4, −3, 0, 2, 4

g(x) is increasing on $(-3,0) \cup (2,4)$ b/c $g'(x) > 0$

g) For what values of x is g'(x) decreasing?

$g'(x)$ is decreasing on $(-1,1)$ b/c $g''(x) < 0$

h) Find the absolute extrema of g on the interval [-1, 3]
Show Work

*Test endpts and critical pts.

$g(-1) = \int_2^{-1} f(t)\,dt = -\int_{-1}^{2} f(t)\,dt = -(-\tfrac{1}{2}) = \tfrac{1}{2}$

$g(0) = \int_2^0 f(t)\,dt = -\int_0^2 f(t)\,dt = -(-1) = 1$

$g(2) = \int_2^2 f(t)\,dt = 0$

$g(3) = \int_2^3 f(t)\,dt = \tfrac{1}{2}$

Abs. max is 1 at $x = 0$

Abs. min is 0 at $x = 2$

3. The following table shows the size of an incoming wave headed towards shore at a given moment.

Distance from left of wave (feet) x	0	7	18	24	36	44	53
Height of wave H(x) (feet)	0	5	13	26	16	7	0

a) Use a trapezoidal sum with the six sub-intervals indicated by the data in the table to approximate the area of the face of the wave. Show correct units. $\frac{w}{2}[h_1 + h_2]$

$A \approx \frac{7}{2}[0+5] + \frac{11}{2}[5+13] + \frac{6}{2}[13+26] + \frac{12}{2}[26+16] + \frac{8}{2}[16+7] + \frac{9}{2}[7+0] = 609$

$17.5 + 99 + 117 + 252 + 92 + 31.5 = \boxed{609}\ ft^2$

b) Estimate $\int_0^{53} h(x)dx$ using 3 middle rectangles

$\int_0^{53} h(x)dx \approx 18\cdot h(7) + 18\cdot h(24) + 17\cdot h(44)$

$= 18(5) + 18(26) + 17(7) = \boxed{677}\ ft^2$

c) Find the average height on the interval [0, 53] using estimation from part b

Avg. height $= \frac{1}{53-0}\int_0^{53} h(x)\,dx$

$= \frac{1}{53}(677) = \boxed{12.774\text{ ft.}}$