Ch. 4 Free Response WS #1

1. 1999 #1 (Calculators permitted):

A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

b) Find the acceleration of the particle at time $t = 1.5$.
Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

2. 2001 #2 (Calculators permitted):

t (days)	0	3	6	9	12	15
W(t) (°C)	20	31	28	24	22	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

a) Use data from the table to find an approximation for $W'(12)$.
Show the computations that lead to your answer. Indicate units of measure.

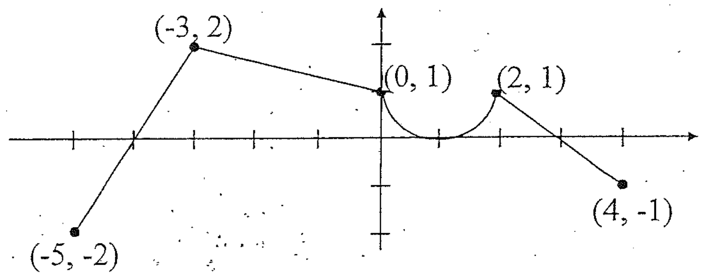
b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

c) A student proposes the function P, given by $P(t) = 20 + 10te^{\left(-\frac{t}{3}\right)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

d) Use the function P defined in part c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

3. **2004 #5 (No Calculators)**

The graph of the function *f* shown to the right consists of a semicircle and three line segments. Let *g* be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



a) Find g'(x) and g''(x)

b) Find g(0), g'(0), and g''(-1)

c) Find all values of x in the open interval (-5, 4) at which g attains a relative maximum. Justify your answer.

d) Find the absolute minimum value of g on the closed interval [-5, 4]. Show work and justify your answer.

e) Find all values of x in the open interval (-5, 4) at which the graph of g has a point of inflection.

KEY

Ch. 4 Free Response WS #1 $\boxed{\text{FRQ}}$

calculator: Radian mode!

1. 1999 #1 (Calculators permitted):

A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

$v(1.5) = 1.167$ $\quad v(1.5) > 0$ so particle is moving up.

(Math→8)

b) Find the acceleration of the particle at time $t = 1.5$.
Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

calculator: nDeriv(Y_1, X, 1.5)

$v'(1.5) = -2.049 < 0$, so velocity is decreasing.

c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

$y(2) = y(0) + \int_0^2 v(t)\,dt = 3 + 0.827 = \boxed{3.827}$

(Math→9)

d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

calculator: fnInt(Y_1, X, 0, 2)

distance $= \int_0^2 |v(t)|\,dt = \boxed{1.173}$

2. 2001 #2 (Calculators permitted):

"x" t (days)	"y" $W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table to the right shows the water temperature as recorded every 3 days over a 15-day period.

a) Use data from the table to find an approximation for $W'(12)$.
Show the computations that lead to your answer. Indicate units of measure.

$W'(12) = \dfrac{W(15) - W(12)}{15 - 12} = \dfrac{21 - 22}{15 - 12} = -\dfrac{1}{3}\ °C/\text{day}$

b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

Avg. temp $= \dfrac{1}{b-a}\int_a^b w(t)\,dt$ $\quad \int_0^{15} w(t)\,dt \approx \frac{W}{2}[h_1 + 2h_2 + 2h_3 + 2h_4 + \ldots + h_n] = \underbrace{\tfrac{3}{2}[20 + 2(31) + 2(28) + 2(24) + 2(22) + 21]}_{= 376.5}$

$= \dfrac{1}{15-0}\int_0^{15} w(t)\,dt = \frac{1}{15}(376.5) = \boxed{25.1\ °C}$

c) A student proposes the function P, given by $P(t) = 20 + 10te^{\left(-\frac{t}{3}\right)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

$P'(12) = -0.549\ °C/\text{day}$ $\quad$ This is the rate of change in temperature at $t = 12$ days.

d) Use the function P defined in part c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

Avg. temperature $= \dfrac{1}{15-0}\int_0^{15} P(t)\,dt = \frac{1}{15}(386.362) = \boxed{25.757\ °C}$

Ch. 4 Review WS #1 (continued)

Area(rectangle) − Area(semicircle)

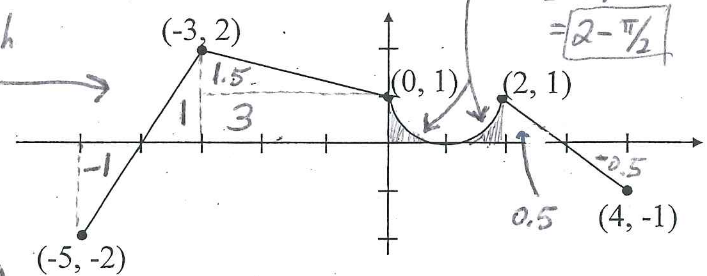
$b(h) - \frac{1}{2}\pi r^2$

$2(1) - \frac{1}{2}\pi(1)^2$

$= \boxed{2 - \pi/2}$

3. **2004 #5 (No Calculators)**

*Let $f(t)$ graph represent velocity →



The graph of the function f shown to the right consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)dt$.

SFTC

a) Find $g'(x)$ and $g''(x)$

$g'(x) = \frac{d}{dx}\int_{-3}^{x} f(t)dt = f(x)\cdot(1)$

$g'(x) = f(x)$

$g''(x) = f'(x)$

b)

$g(0) = \int_{-3}^{0} f(t)dt = \boxed{4.5}$

$g'(0) = f(0) = \boxed{1}$

$g''(-1) = f'(-1) = \frac{2-1}{-3-0} = \frac{1}{-3} = \boxed{\frac{-1}{3}}$

slope of line segment using points (-3,2) and (0,1)

c) Find all values of x in the open interval (-5, 4) at which g attains a relative maximum. Justify your answer.

$g'(x)$: −5 [−] −4 [+] 1 [+] 3 [−] 4

Rel. max at $x = 3$ b/c $g'(x)$ changes from + to −.

(y-value!)

d) Find the absolute minimum value of g on the closed interval [-5, 4]. Justify your answer.

EVT:
1) Test endpoints
2) Test rel. mins.

$g(-5) = \int_{-3}^{-5} f(t)dt = -\int_{-5}^{-3} f(t)dt = -(-1+1) = 0$

$g(-4) = \int_{-3}^{-4} f(t)dt = -\int_{-4}^{-3} f(t)dt = -(1) = -1$

$g(4) = \int_{-3}^{4} f(t)dt = 3 + 1.5 + 2 - \pi/2 + 0.5 - 0.5 = 6.5 - \frac{\pi}{2} \approx 5$

$\boxed{\text{Abs. min is } -1 \text{ at } x = -4}$

e) Find all values of x in the open interval (-5, 4) at which the graph of g has a point of inflection.

$g''(x)$: −5 [+ ∪] −3 [− ∩] 1 [+ ∪] 2 [− ∩] 4

POI occur at $x = -3, 1, 2$ b/c $g''(x)$ changes signs