AP Calculus Ch. 4 Test Review WS 2 (Non-Calculator)

1. $\int \frac{x^2 + x + 1}{\sqrt{x}}\, dx$

2. $\int 2x\sqrt{1-3x^2}\, dx$

3. $\int 5\sqrt{x}\,(4 - 3x^2)\, dx$

4. $\int 5x\sec^2(3x^2)\, dx$

5. $\int x^2\sqrt{7-x}\, dx$

6. $\int_1^2 x(1-2x^2)^3\, dx$

7. Find $f'(x)$ if $f(x) = \int_{2x^3}^{\sqrt{x}} \sqrt{1-t^2}\,dt$

8. Find $f'(x)$ if $f(x) = \int_{3x^2}^{\pi} \sqrt{1-t^2}\,dt$

9. $\int_{-5}^{6} |x+2|\,dx$

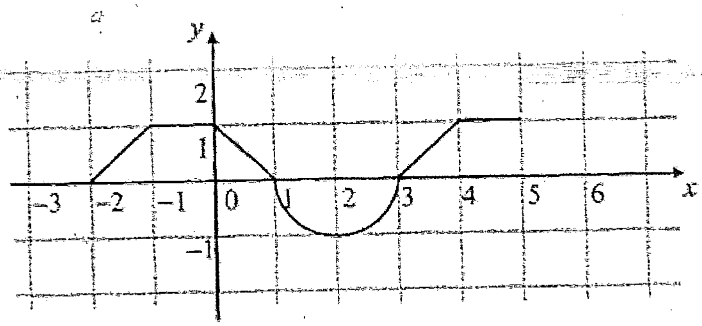
10. $\int_{-2}^{7} |x-4|\,dx$

11. If $a(t) = 12t^2 + 18t - 4$ and $x(1) = 3$ and $v(-1) = 9$, find the below:

a) Find the specific function for v(t)

b) Find the specific function for x(t)

12. The graph of f below consists of a semicircle, triangles, and squares. Find the average value of f on the interval [-2, 5]



AP Calculus Ch. 4 Test Review WS 2 (Non-Calculator) Solution Key

1. $\int \frac{x^2+x+1}{\sqrt{x}}\,dx$ $\quad \int \frac{x^2}{x^{1/2}}+\frac{x}{x^{1/2}}+\frac{1}{x^{1/2}}\,dx$

$\int x^{3/2}+x^{1/2}+x^{-1/2}\,dx$

$=\frac{x^{5/2}}{5/2}+\frac{x^{3/2}}{3/2}+\frac{x^{1/2}}{1/2}+C$

$=\boxed{\frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}+2x^{1/2}+C}$

2. $\int 2x\sqrt{1-3x^2}\,dx = \int 2x(1-3x^2)^{1/2}\,dx$

$u=1-3x^2$

$\frac{du}{dx}=-6x$

$dx=\frac{du}{-6x}$

$\int 2\cancel{x}\cdot u^{1/2}\cdot\frac{du}{-6\cancel{x}}$

$-\frac{1}{3}\int u^{1/2}\,du$

$=-\frac{1}{3}\left(\frac{u^{3/2}}{3/2}\right)=-\frac{1}{3}\left(\frac{2}{3}\right)u^{3/2}+C$

$=\boxed{\frac{-2}{9}(1-3x^2)^{3/2}+C}$

3. $\int 5\sqrt{x}\,(4-3x^2)\,dx = \int 5x^{1/2}(4-3x^2)\,dx$

$=\int 20x^{1/2}-15x^{5/2}\,dx$

$=20\left(\frac{x^{3/2}}{3/2}\right)-15\left(\frac{x^{7/2}}{7/2}\right)+C$

$=20\cdot\frac{2}{3}x^{3/2}-15\cdot\frac{2}{7}x^{7/2}+C$

$=\boxed{\frac{40}{3}x^{3/2}-\frac{30}{7}x^{7/2}+C}$

4. $\int 5x\sec^2(3x^2)\,dx$

$u=3x^2$

$\frac{du}{dx}=6x$

$dx=\frac{du}{6x}$

$\int 5\cancel{x}\cdot\sec^2(u)\cdot\frac{du}{6\cancel{x}}$

$\frac{5}{6}\int\sec^2u\,du$

$\frac{5}{6}\tan u+C$

$=\boxed{\frac{5}{6}\tan(3x^2)+C}$

5. $\int x^2\sqrt{7-x}\,dx = \int x^2(7-x)^{1/2}\,dx$

$u=7-x \rightarrow x=7-u$

$\frac{du}{dx}=-1$

$dx=-du$

$\int x^2\cdot u^{1/2}(-du)$

$\int (7-u)^2u^{1/2}(-du)$

$\int -u^{1/2}(49-14u+u^2)\,du$

$\int -49u^{1/2}+14u^{3/2}-u^{5/2}\,du$

$-49\left(\frac{u^{3/2}}{3/2}\right)+14\left(\frac{u^{5/2}}{5/2}\right)-\frac{u^{7/2}}{7/2}+C$

$-49\cdot\frac{2}{3}u^{3/2}+14\cdot\frac{2}{5}u^{5/2}-\frac{2}{7}u^{7/2}+C$

$\boxed{-\frac{98}{3}(7-x)^{3/2}+\frac{28}{5}(7-x)^{5/2}-\frac{2}{7}(7-x)^{7/2}+C}$

6. $\int_1^2 x(1-2x^2)^3\,dx$ $\quad \int \cancel{x}\cdot u^3\cdot\frac{du}{-4\cancel{x}}=\frac{-1}{4}\int u^3\,du$

$u=1-2x^2$

$\frac{du}{dx}=-4x$

$dx=\frac{du}{-4x}$

if $x=1$, $u=1-2(1)^2=-1$

if $x=2$, $u=1-2(2)^2=-7$

$=-\frac{1}{4}\cdot\frac{u^4}{4}\Big]_{-1}^{-7}=-\frac{1}{16}(-7)^4-\left(-\frac{1}{16}(1)\right)$

$=\frac{-2401}{16}+\frac{1}{16}$

$=\frac{-2400}{16}=\boxed{-150}$

$\boxed{\text{SFTC}}$

7. Find $f'(x)$ if $f(x) = \int_{2x^3}^{\sqrt{x}} \sqrt{1-t^2}\,dt$ $\sqrt{x} \rightarrow x^{1/2}$

$f'(x) = \sqrt{1-(\sqrt{x})^2}\cdot \frac{1}{2}x^{-1/2} - \sqrt{1-(2x^3)^2}\cdot 6x^2$

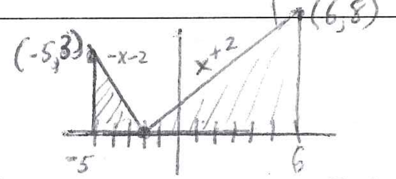
$= \boxed{\dfrac{\sqrt{1-x}}{2\sqrt{x}} - 6x^2\sqrt{1-4x^6}}$

$\boxed{\text{SFTC}}$ $\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$

8. Find $f'(x)$ if $f(x) = \int_{3x^2}^{\pi} \sqrt{1-t^2}\,dt = \int_{\pi}^{3x^2} -\sqrt{1-t^2}\,dt$

$f'(x) = -\sqrt{1-(3x^2)^2}\cdot 6x = \boxed{-6x\sqrt{1-9x^4}}$

9. $\int_{-5}^{6} |x+2|\,dx$



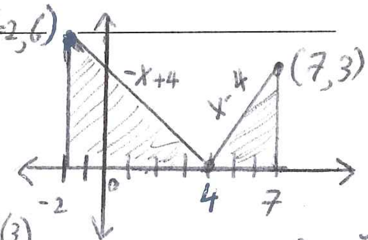
$\frac{1}{2}bh = \frac{1}{2}(3)(3) + \frac{1}{2}(8)(8) = \frac{9}{2} + \frac{64}{2} = \boxed{\frac{73}{2}}$

OR

$\int_{-5}^{-2} -x-2\,dx + \int_{-2}^{6} x+2\,dx = \left.-\frac{x^2}{2} - 2x\right]_{-5}^{-2} = \frac{9}{2}$

$+ \left.\frac{x^2}{2} + 2x\right]_{-2}^{6} = 64/2 \rightarrow \frac{9}{2} + \frac{64}{2} = \boxed{\frac{73}{2}}$

10. $\int_{-2}^{7} |x-4|\,dx$



$\frac{1}{2}bh = \frac{1}{2}(6)(6) + \frac{1}{2}(3)(3)$

$= \frac{36}{2} + \frac{9}{2} = \boxed{\frac{45}{2}}$

OR

$\int_{-2}^{4} -x+4\,dx + \int_{4}^{7} x-4\,dx$

$\left.-\frac{x^2}{2} + 4x\right]_{-2}^{4} = 18$

$\left.\frac{x^2}{2} - 4x\right]_{4}^{7} = 9/2$

$18 + 9/2 = \boxed{\frac{45}{2}}$

11. If $a(t) = 12t^2 + 18t - 4$ and $x(1) = 3$ and $v(-1) = 9$, find the below:

$v(t) = \int a(t)\,dt = \int 12t^2 + 18t - 4\,dt = \frac{12t^3}{3} + \frac{18t^2}{2} - 4t + C$

$v(t) = 4t^3 + 9t^2 - 4t + C$

$9 = 4(-1)^3 + 9(-1)^2 - 4(-1) + C$

$9 = -4 + 9 + 4 + C$

$0 = C$

$v(t) = 4t^3 + 9t^2 - 4t$

$x(t) = \int v(t)\,dt = \int 4t^3 + 9t^2 - 4t\,dt$

$x(t) = \frac{4t^4}{4} + \frac{9t^3}{3} - \frac{4t^2}{2} + K$

$x(t) = t^4 + 3t^3 - 2t^2 + K$

a) Find the specific function for v(t)

$\boxed{v(t) = 4t^3 + 9t^2 - 4t}$

b) Find the specific function for x(t)

$3 = 1^4 + 3(1)^3 - 2(1)^2 + K$

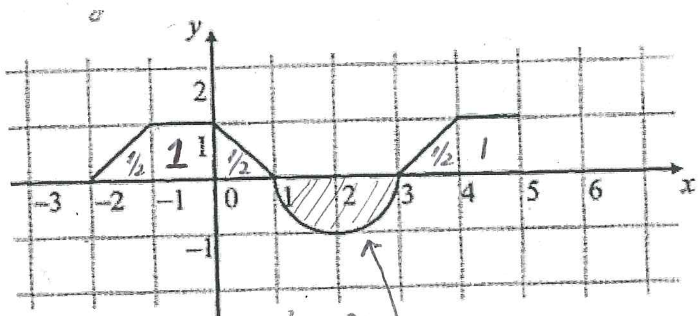
$3 = 1 + 3 - 2 + K$

$3 = 4 - 2 + K$

$1 = K$

$\boxed{x(t) = t^4 + 3t^3 - 2t^2 + 1}$

12. The graph of f below consists of a semicircle, triangles, and squares. Find the average value of f on the interval [-2, 5]



Avg. value $= \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{5--2}\int_{-2}^{5} f(x)\,dx$

$= \frac{1}{7}\left(\frac{7-\pi}{2}\right)$

$\boxed{\text{Avg. value} = \frac{7-\pi}{14} \text{ or } \frac{1}{2} - \frac{\pi}{14}}$

$= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(1) = \boxed{-\pi/2}$

$\int_{-2}^{5} f(x)\,dx = \frac{1}{2} + 1 + \frac{1}{2} - \pi/2 + \frac{1}{2} + 1 = \frac{7}{2} - \frac{\pi}{2} = \frac{7-\pi}{2}$