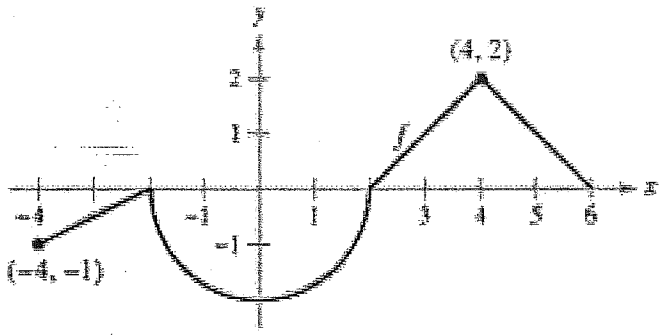
## Ch. 4 Test Review (Calculator Portion)

1. The graph of f consists of line segments and a semicircle, as shown: Let g(x) = $\int_{-2}^{x} f(t)dt$




a) Find g(2)

b) Find g(-4)

c) Find g(6)

d) Find g'(4)

e) Find g'(-2)

f) Find g''(5)

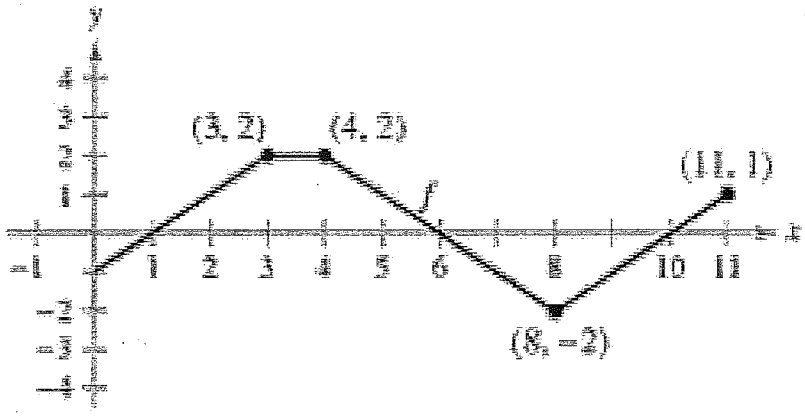
g) For what values of x is g increasing? Justify Answer

h) For what values of x is g decreasing? Justify Answer

i) Find the x-values of all points of inflection of g. Justify Answer

j) Find the absolute extrema of g on the interval [-4,6]

2. The graph of f consists of line segments. Let h(x) = $\int_6^x f(t)dt$




a) Find h(0)

b) Find h(6)

c) Find h(10)

d) Find h(11)

e) Find h'(3)

f) Find h''(9.5)

g) For what values of x is h increasing? Justify Answer

h) For what values of x is h'(x) decreasing?

i) Find the absolute extrema of g on the interval [0,11]

3. The table below shows the speed of a sprinter at the time intervals (in seconds) in the 200 meter race

| time t (seconds) | 0 | 2 | 5 | 8 | 11 | 17 | 20 |
|---|---|---|---|---|---|---|---|
| Velocity V(t) (m/s) | 5 | 6.5 | 7 | 8.5 | 9 | 8 | 7.5 |

a. Estimate $\int_0^{20} v(t)dt$ using the following methods

i. 6 trapezoids

ii. 3 left-handed rectangles

iii. 3 right-handed rectangles

iv. 3 middle rectangles

b. Find the average velocity on the interval [0, 20] using estimation from 6 trapezoids.

4. An object moving along a horizontal line has $v(t) = 4\sin(t^2 - 2t + 2)$ measured in meters per second from [0,4] (hint: set windows to x-values [-1, 5] and y-values [-6, 6] )
*Round answers to 3 decimal places

a. Find the time(s) when the object is motionless

b. When does the object change directions in $0 < t < 4$ ?

c. Find the velocity of the object at t = 3 seconds.

d. Find the acceleration of the object at t = 3 seconds.

e. Is the object's speed increasing or decreasing at t = 3 seconds? Justify answer.

f. Find the total displacement of the object from t = 0 to t = 4 seconds (Show Integral Notation)

g. Find the total distance of the object from t = 0 to t = 4 seconds (Show Integral Notation)

h. Find the time when the object reaches minimum velocity in [0, 3]

i. Find the minimum velocity in [0, 3]

j. Given x(0) = 2, Find x(4) . (Show integral notation)

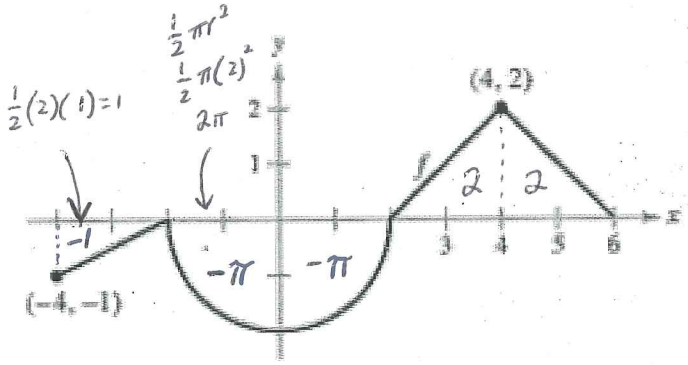
k. Find the average velocity in [0, 4]

l. Find the time(s) when object reaches average velocity.

KEY 1/4

**Ch. 4 Test Review (Calculator Portion)** WS #3

1. The graph of f consists of line segments and a semicircle, as shown: Let $g(x) = \int_{-2}^{x} f(t)\,dt$




$g'(x) = \frac{d}{dx}\int_{-2}^{x} f(t)\,dt = f(x)\cdot 1$

$g'(x) = f(x)$

$g''(x) = f'(x)$

a) Find g(2)

$g(2) = \int_{-2}^{2} f(t)\,dt = \boxed{-2\pi}$

b) Find g(-4)

$g(-4) = \int_{-2}^{-4} f(t)\,dt = -\int_{-4}^{-2} f(t)\,dt$

$= -(-1) = \boxed{1}$

c) Find g(6)

$g(6) = \int_{-2}^{6} f(t)\,dt = \boxed{-2\pi + 4}$

d) Find g'(4)

$g'(4) = f(4) = \boxed{2}$

e) Find g'(-2)

$g'(-2) = f(-2) = \boxed{0}$

f) Find g''(5)

$g''(5) = f'(5) = \frac{2-0}{4-6} = \frac{2}{-2} = \boxed{-1}$

find slope of line segment using (4, 2) and (6, 0)

g) For what values of x is g increasing? Justify Answer

$g'(x)$: − | − | + on intervals $(-4,-2)$, $(-2,2)$, $(2,6)$

$g(x)$ is increasing $(2,6)$ b/c $g'(x) > 0$

h) For what values of x is g decreasing? Justify Answer

$g(x)$ is decreasing $(-4,-2) \cup (-2,2)$

b/c $g'(x) < 0$

i) Find the x-values of all points of inflection of g. Justify Answer

$g''(x)$: + | − | + | − on intervals $(-4,-2)$, $(-2,0)$, $(0,4)$, $(4,6)$ (∪ ∩ ∪ ∩)

POI at $x = -2, 0, 4$ b/c $g''(x)$ change signs.

j) Find the absolute extrema of g on the interval [-4,6] (EVT) Test endpts and critical pts of $g(x)$

$g(-4) = 1$

$g(-2) = \int_{-2}^{-2} f(t)\,dt = 0$

$g(2) = -2\pi \approx -6$

$g(6) = -2\pi + 4 \approx -2$

Abs min is $-2\pi$ at $x = 2$

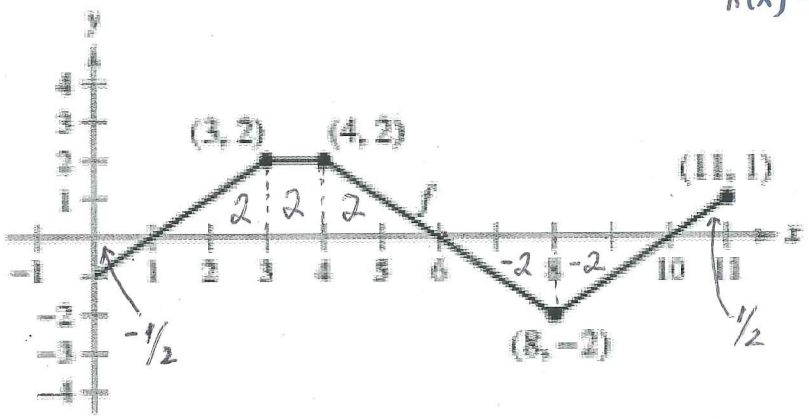
Abs. max is 1 at $x = -4$

2. The graph of f consists of line segments. Let $h(x) = \int_6^x f(t)dt$

$h'(x) = \frac{d}{dx}\int_6^x f(t)dt = f(x)\cdot 1 = f(x)$

$h'(x) = f(x)$

$h''(x) = f'(x)$




a) Find h(0)

$h(0) = \int_6^0 f(t)dt = -\int_0^6 f(t)dt$

$= -\left(\frac{-1}{2} + 2 + 2 + 2\right)$

$= -(5.5) =$ **$-5.5$ or $-11/2$**

b) Find h(6)

$h(6) = \int_6^6 f(t)dt =$ **0**

c) Find h(10)

$h(10) = \int_6^{10} f(t)dt =$ **$-4$**

d) Find h(11)

$h(11) = \int_6^{11} f(t)dt = -2 - 2 + 1/2$

$=$ **$-3.5$ or $-7/2$**

e) Find h'(3)

$h'(3) = f(3) =$ **2**

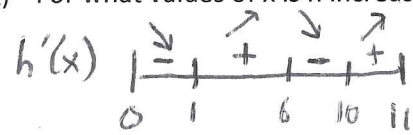
f) Find h''(9.5)

$h''(9.5) = f'(9.5) = \frac{-2-1}{8-11}$

find slope of line segment using (8, −2) and (11, 1)

$= \frac{-3}{-3} =$ **1**

g) For what values of x is h increasing? Justify Answer




h(x) is increasing $(1,6) \cup (10,11)$

b/c $h'(x) > 0$

h) For what values of x is

h'(x) decreasing?

h(x) decreasing (4, 8) b/c $h''(x) < 0$

i) Find the absolute extrema of g on the interval [0,11]

Test endpts and critical pts.

$h(0) = -5.5$

$h(1) = \int_6^1 f(t)dt = -\int_1^6 f(t)dt = -(6) = -6$

$h(6) = 0$

$h(10) = -4$

$h(11) = -3.5$

**Abs. max is 0 at x = 6**

**Abs. min is −6 at x = 1**

3. The table below shows the speed of a sprinter at the time intervals (in seconds) in the 200 meter race

| time $t$ (seconds) | 0 | 2 | 5 | 8 | 11 | 17 | 20 |
|---|---|---|---|---|---|---|---|
| Velocity (m/s) $v(t)$ | 5 | 6.5 | 7 | 8.5 | 9 | 8 | 7.5 |

a. Estimate $\int_0^{20} v(t)dt$ using the following methods

i. 6 trapezoids $A = \frac{w}{2}[h_1+h_2]$

$A \approx \frac{2}{2}[5+6.5] + \frac{3}{2}[6.5+7] + \frac{3}{2}[7+8.5]$
$+ \frac{3}{2}[8.5+9] + \frac{6}{2}[9+8] + \frac{3}{2}[8+7.5]$

$= 11.5 + 20.25 + 23.25 + 26.25$
$+ 51 + 23.25$

$= \boxed{155.5}$ m

ii. 3 left-handed rectangles $A = b \cdot h$

$= (5) \cdot v(0) + 6 \cdot v(5) + 9 \cdot v(11)$

$= 5(5) + 6(7) + 9(9)$

$= \boxed{148}$ m

iii. 3 right-handed rectangles

$= 5 \cdot v(5) + 6 \cdot v(11) + 9 \cdot v(20)$

$= 5(7) + 6(9) + 9(7.5)$

$= \boxed{156.5 \text{ m}}$

iv. 3 middle rectangles

$= 5 \cdot v(2) + 6 \cdot v(8) + 9 \cdot v(17)$

$= 5(6.5) + 6(8.5) + 9(8)$

$32.5 + 51 + 72$

$= \boxed{155.5 \text{ m}}$

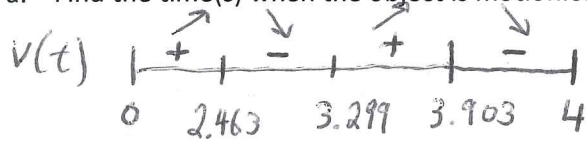
b. Find the average velocity on the interval [0, 20] using estimation from 6 trapezoids.

$\text{Avg. velocity} = \frac{1}{b-a}\int_a^b v(t)dt = \frac{1}{20-0}\int_0^{20} v(t)dt$

$\text{Avg. velocity} = \frac{1}{20}(155.5) = \boxed{7.775 \text{ m/s}}$

4. An object moving along a horizontal line has $v(t) = 4\sin(t^2 - 2t + 2)$ measured in meters per second from [0,4] (hint: set windows to x-values [-1, 5] and y-values [-6, 6] )
*Round answers to 3 decimal places

a. Find the time(s) when the object is motionless




object motionless at $t = 2.463, 3.299, 3.903$ secs b/c $v(t) = 0$

b. When does the object change directions in $0 < t < 4$?

object changes direction at $t = 2.463$, 3.299, and 3.903 seconds b/c $v(t)$ change signs.

c. Find the velocity of the object at t = 3 seconds.

$v(3) = -3.836$ m/s

d. Find the acceleration of the object at t = 3 seconds.

$v'(3) = 4.539$ m/s$^2$

or

$a(3) = 4.539$ m/s$^2$

calculator syntax: nderiv(Y1, X, 3)

e. Is the object's speed increasing or decreasing at t = 3 seconds? Justify answer.

Speed is decreasing b/c $v(3) < 0$ and $a(3) > 0$ (opposite signs)

f. Find the total displacement of the object from t = 0 to t = 4 seconds (show integral setup)

$$\int_0^4 v(t)\,dt = \boxed{7.753 \text{ m}}$$

fnInt(Y1, X, 0, 4)

g. Find the total distance of the object from t = 0 to t = 4 seconds (show integral setup)

$$\int_0^4 |v(t)|\,dt = \boxed{12.178 \text{ m}}$$

h. Find the time when the object reaches minimum velocity in [0, 3]

$t = 2.927$ seconds

i. Find the minimum velocity in [0, 3]

$v(2.927) = -4$ m/s

j. Given x(0) = 2, Find x(4). (Show integral notation)

$$x(4) = x(0) + \int_0^4 v(t)\,dt$$

$$= 2 + 7.753 = \boxed{9.753}$$

k. Find the average velocity in [0, 4]

$$\text{Avg. velocity} = \frac{1}{b-a}\int_a^b v(t)\,dt = \frac{1}{4-0}\int_0^4 v(t)\,dt$$

$$= \frac{1}{4}(7.753) = \boxed{1.938 \text{ m/s}}$$

l. Find the time(s) when object reaches average velocity.

set $4\sin(t^2 - 2t + 2) = 1.938$

$4\sin(t^2 - 2t + 2) - 1.938 = 0$

$t = 2.279, 3.406, 3.814$ seconds