Test Form D **Name** ______________ **Date** ______________

Chapter 4 **Class** ______________ **Section** ______________

1. Evaluate the integral: $\int \sqrt{x^3}\, dx$.

2. Evaluate the integral: $\int 3 \csc x \cot x \, dx$.

3. Evaluate the integral: $\int \frac{x^3 - x^2}{x^2}\, dx$.

4. Evaluate the integral: $\int \frac{\cos^3 \theta}{2 - 2\sin^2 \theta}\, d\theta$.

5. Find the function, $y = f(x)$, if $f'(x) = 2x - 1$ and $f(1) = 3$.

6. Use $a(t) = -32$ feet per second squared as the acceleration due to gravity. An object is thrown vertically downward from the top of a 480-foot building with an initial velocity of 64 feet per second. With what velocity does the object hit the ground?

7. Let $s(n) = \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)\left(\frac{2}{n}\right)$. Find the limit of $s(n)$ as $n \to \infty$.

$4x - x^2$

8. Write the definite integral that represents the area of the region enclosed by $y = 4x - x^2$ and the x axis.

9. Evaluate: $\frac{d}{dx}\int_2^x (2t^2 + 5)^2\, dt$.

10. Use the Fundamental Theorem of Calculus to evaluate $\int_{-1}^{1} \left(\sqrt[3]{t} - 2\right) dt$.

11. Find the average value of $f(x) = \sin x$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

12. Evaluate the integral: $\int_0^1 x\sqrt{1 - x^2}\, dx$.

13. Evaluate the integral: $\int \frac{\sec^2 x}{\sqrt{\tan x}}\, dx$.

14. Evaluate the integral: $\int_0^3 |x - 2|\, dx$.

15. Find the indefinite integral: $\int \frac{x}{\sqrt{x - 1}}\, dx$.

16. Use [redacted]'s Rule with $n = 4$ to approximate $\int_1^2 \frac{1}{(x + 1)^2}\, dx$.
Trapezoid

Test Form K   Answer Key

1) $\int \sqrt{x^3}\,dx = \int x^{\frac{3}{2}}dx = \boxed{\frac{2}{5}x^{\frac{5}{2}} + C}$

2) $\int 3\csc x \cot x\,dx = \boxed{-3\csc x + C}$

3) $\int \frac{x^3-x^2}{x^2}dx = \int (x-1)\,dx = \boxed{\frac{1}{2}x^2 - x + C}$

4) $\int \frac{\cos^3\theta}{2-2\sin^2\theta}\,d\theta = \int \frac{\cos^3\theta}{2(1-\sin^2\theta)}\,d\theta = \int \frac{\cos^3\theta}{2\cos^2\theta}\,d\theta$

$= \int \frac{1}{2}\cos\theta\,d\theta = \boxed{\frac{1}{2}\sin\theta + C}$

5) $f'(x) = 2x-1$ so $f(x) = x^2 - x + C$

$f(1) = 1 - 1 + C = 3$ so $C = 3$ $\boxed{f(x) = x^2 - x + 3}$

6) $a(t) = -32\ \frac{ft}{sec^2}$ $v(0) = C = 64$

$v(t) = -32t + C$ so $v(t) = -32t + 64$

$x(t) = -16t^2 + 64t + C$ so $x(t) = -16t^2 + 64t + 480$

$x(0) = C = 480$ $t = \frac{4 \pm \sqrt{16+120}}{2} = \frac{4 \pm \sqrt{136}}{2} = \frac{4 \pm 2\sqrt{34}}{2}$

$-16t^2 + 64t + 480 = 0$ $= 2 \pm \sqrt{34}$

$t^2 - 4t - 30 = 0$

$v(2+\sqrt{34}) = -32(2+\sqrt{34}) + 64 = -64 - 32\sqrt{34} + 64 = \boxed{-32\sqrt{34}\ \frac{ft}{sec}}$

$\boxed{\text{or} \approx -186.590\ \frac{ft}{sec}}$

7) $\lim_{n\to\infty} \sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)\left(\frac{2}{n}\right) = \lim_{n\to\infty} \frac{2}{n}\left[\sum_{i=1}^{n} 1 + \sum_{i=1}^{n}\frac{2i}{n}\right] = \lim_{n\to\infty} \frac{2}{n}\left[\sum_{i=1}^{n} 1 + \frac{2}{n}\sum_{i=1}^{n} i\right]$

$\lim_{n\to\infty} \frac{2}{n}\left[n + \frac{2}{n}\cdot\frac{n(n+1)}{2}\right] = \lim_{n\to\infty} \frac{2}{n}\left[n + n + 1\right] = \lim_{n\to\infty} \frac{2}{n}(2n+1)$

$= \lim_{n\to\infty} \frac{4n+2}{n} = \boxed{4}$

8) $4x - x^2 = 0$ $\boxed{\int_0^4 (4x - x^2)\,dx}$

$x(4-x) = 0$

$x = 0, 4$

9) $\frac{d}{dx}\int_2^x (2t^2+5)^2\,dt = \boxed{(2x^2+5)^2}$

10) $\int_{-1}^{1} (\sqrt[3]{t} - 2)\,dt = \frac{3}{4}t^{\frac{4}{3}} - 2t\Big]_{-1}^{1} = \frac{3}{4} - 2 - \left(\frac{3}{4} + 2\right) = \boxed{-4}$

11) $\bar{y} = \frac{1}{\frac{\pi}{2} - \frac{\pi}{4}}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x\,dx \rightarrow \bar{y} = -\frac{4}{\pi}\cos x\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$\bar{y} = -\frac{4}{\pi}\left(\cos\frac{\pi}{2} - -\cos\frac{\pi}{4}\right) = \frac{4}{\pi}\left(0 + \frac{\sqrt{2}}{2}\right) = \frac{4}{\pi}\left(\frac{\sqrt{2}}{2}\right) = \frac{2\sqrt{2}}{\pi}$

12) $\int_0^1 x\sqrt{1-x^2}\,dx = -\frac{1}{2}\int_1^0 u^{\frac{1}{2}}du = \frac{1}{2}\int_0^1 u^{\frac{1}{2}}du = \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\Big]_0^1\right) = \frac{1}{2}\left(\frac{2}{3} - 0\right) = \boxed{\frac{1}{3}}$

$u = 1 - x^2$

$du = -2x\,dx$

13) $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx = \int u^{-\frac{1}{2}}du = 2u^{\frac{1}{2}} + C = \boxed{2\sqrt{\tan x} + C}$

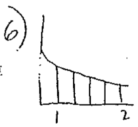
$u = \tan x$, $du = \sec^2 x\,dx$

14) $\int_0^3 |x-2|\,dx = \int_0^2 -(x-2)\,dx + \int_2^3 (x-2)\,dx$

$= -\frac{1}{2}x^2 + 2x\Big]_0^2 + \frac{1}{2}x^2 - 2x\Big]_2^3 = -2 + 4 + \frac{9}{2} - 6 - (2 - 4) = \boxed{\frac{5}{2}}$

15) $\int \frac{x}{\sqrt{x-1}}\,dx = \int u^{-\frac{1}{2}}(u+1)\,du = \int \left(u^{\frac{1}{2}} + u^{-\frac{1}{2}}\right)du = \frac{2}{3}u^{\frac{3}{2}} + 2u^{\frac{1}{2}} + C$

$u = x - 1$, $du = dx$ $= \boxed{\frac{2}{3}\sqrt{(x-1)^3} + 2\sqrt{x-1} + C}$

16) 

$f(x) = \frac{1}{(x+1)^2}\,dx$ $\text{Area} \approx \frac{1}{2}\cdot\frac{1}{4}\left[f(1) + f\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + f\left(\frac{3}{2}\right) + f\left(\frac{3}{2}\right) + f\left(\frac{7}{4}\right) + f\left(\frac{7}{4}\right) + f(2)\right]$

width $= \frac{1}{4}$

$= \frac{1}{2}\cdot\frac{1}{4}\left(\frac{1}{4} + \frac{16}{81} + \frac{16}{81} + \frac{4}{25} + \frac{4}{25} + \frac{16}{121} + \frac{16}{121} + \frac{1}{9}\right)$

$= \frac{1313957}{[illegible]} \approx \boxed{0.168}$